

The Mathematical Association of Victoria

FURTHER MATHEMATICS 2012

Trial Written Examination 1-SOLUTIONS

SECTION A: Core--Data analysis

Answers:

- | | | | | |
|-------|-------|-------|------|-------|
| 1. D | 2. D | 3. C | 4. D | 5. A |
| 6. B | 7. D | 8. B | 9. A | 10. A |
| 11. D | 12. C | 13. A | | |

SECTION B: MODULES

Module 1: Number Patterns

- | | | | | |
|------|------|------|------|------|
| 1. B | 2. E | 3. A | 4. C | 5. C |
| 6. A | 7. C | 8. E | 9. C | |

Module 2: Geometry and trigonometry

- | | | | | |
|------|------|------|------|------|
| 1. C | 2. C | 3. D | 4. A | 5. B |
| 6. A | 7. C | 8. B | 9. B | |

Module 3: Graphs and relations

- | | | | | |
|------|------|------|------|------|
| 1. C | 2. E | 3. B | 4. A | 5. B |
| 6. D | 7. C | 8. D | 9. B | |

Module 4: Business-related mathematics

- | | | | | |
|------|------|------|------|------|
| 1. D | 2. E | 3. B | 4. C | 5. D |
| 6. B | 7. B | 8. B | 9. C | |

Module 5: Networks and decision mathematics

- | | | | | |
|------|------|------|------|------|
| 1. D | 2. D | 3. B | 4. B | 5. C |
| 6. C | 7. B | 8. B | 9. E | |

Module 6: Matrices

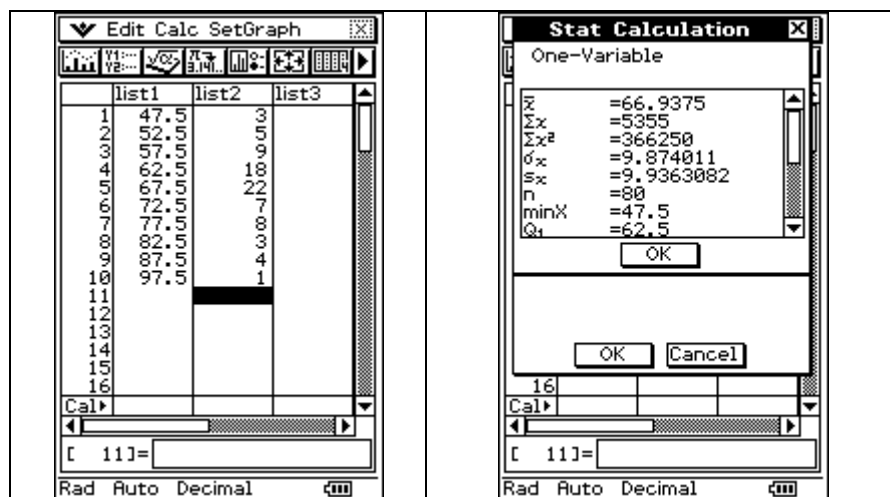
- | | | | | |
|------|------|------|------|------|
| 1. D | 2. D | 3. C | 4. B | 5. E |
| 6. E | 7. A | 8. A | 9. C | |

Worked solutions--Core: Data analysis**Question 1**

Eight people have a heart rate over 80, so $\frac{8}{80} \times 100 = 10\%$ are at risk.

*Answer D***Question 2**

Entering the middle heart rate value of each class interval and the corresponding frequency



will give a mean value of 66.9

*Answer D***Question 3**

Parallel boxplots are used to display bivariate data where one set is categorical and the other is numerical.

Options A, B and E are incorrect – these refer to two categories.

Options D is incorrect – this refers to two numerical data.

Answer C

Question 4

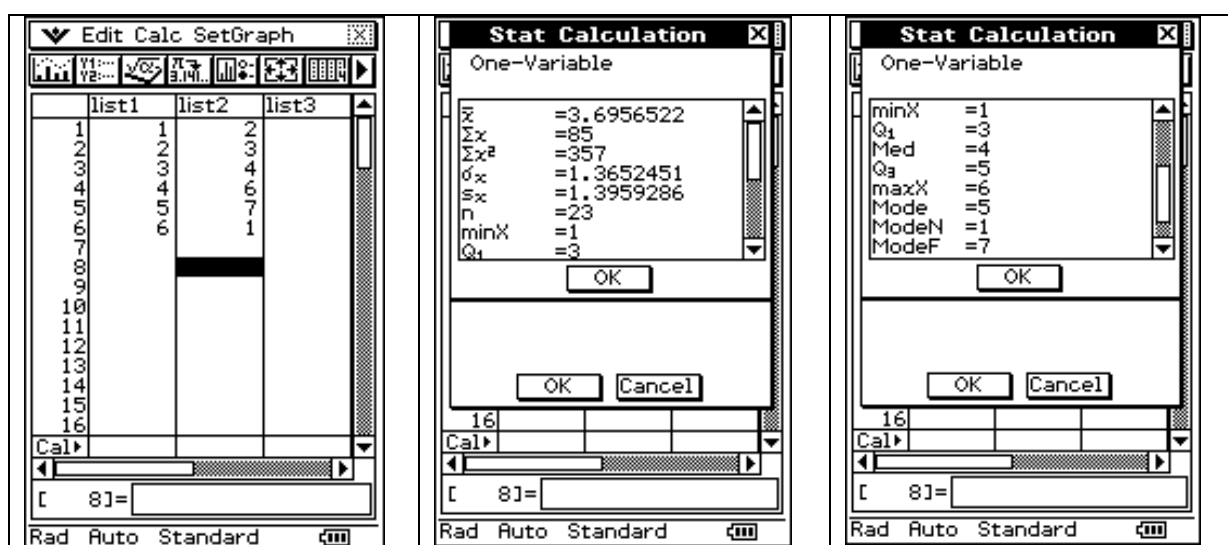
Option A is incorrect because $\bar{x} - 3s = 64 - 3 \times 12 = 28$ means that 0.15% of the population of students will score less than 28.

Option B is incorrect because $\bar{x} - s = 64 - 12 = 52$ means that 16% of the population of students will score less than 52.

Option C is incorrect because $\bar{x} + 2s = 64 + 2 \times 12 = 88$ means that 2.5% of the population of students will score more than 88.

Option D is **correct** because $\bar{x} - 2s = 40$ and $\bar{x} + s = 78$ means that half of 95% plus half of 68% of the population of students scored between 40 and 76. That is $47.5 + 34 = 81.5\%$ (more than 80%) lie between 40 and 76.

Option E is incorrect because 99.7% of scores lie in the interval between $\bar{x} - 3s$ and $\bar{x} + 3s$, which in this case is between 28 and 100.

*Answer D***Question 5**

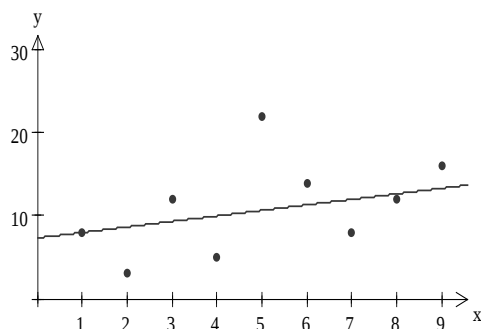
Mean is 3.7, Median is 4 and mode is 5 so the order from lowest to highest is mean, median, mode.

*Answer A***Question 6**

The gradient of the line is $m = \frac{10}{4} = 2.5$, so the interpretation of the gradient on the graph is:

For every hour worked \$2.50 is earned.

Answer B

Question 7

The three median line shown on the graph passes above the left and right median points and has a positive y-intercept which is less than 10. It appears to pass through one data point on the scatterplot, not one – third of the points. The median point will always pass through the average point of the three median points, that is through the point

$$\left(\frac{x_L + x_M + x_R}{3}, \frac{y_L + y_M + y_R}{3} \right)$$

*Answer D***Question 8**

The IQR is $35.7 - 24.3 = 11.4$ so option A is correct

Upper Boundary = $35.7 + 1.5 \times 11.4 = 52.8$ so option B is **incorrect**

75% if the data lie above the lower Quartile of 24.3 so option C is correct

The median lies anywhere between the two quartiles 24.3 and 35.7 so option D is correct

Lower Boundary = $24.3 - 1.5 \times 11.4 = 7.2$ so option E is correct

*Answer B***Question 9**

Option A is false because $r^2 = 0.757$ so $r = -\sqrt{0.757} = -0.87$ (see graph for direction - negative)

Option B is true because there is a negative gradient

Option C is true because $m = -0.8 = \frac{-8}{10}$, so for every increase in 10 minutes of talking, text time

decreases by 8 minutes

Option D is true because $y = -0.8 \times 100 + 166.4 = 86.4$ so residual = $60 - 86.4 = -26.4$

Option E is correct interpretation of coefficient of determination.

*Answer A***Question 10**

To linearise the data, stretch the x scale by applying an x^2 transformation or stretch the y scale by applying a y^2 transformation

Answer A

Question 11

Day	Sales	Median (4 point)	Median centred
Monday	23		
Tuesday	32		
Wednesday	18		
		Median (32, 18, 34, 40) is 33	
Thursday	34		$\frac{33+37}{2} = 35$
		Median (18, 34, 40, 48) is 37	
Friday	40		
Saturday	48		
Sunday	33		

*Answer D***Question 12**

The sum of the seasonal indices is equal to the number of seasons. $x + 0.83 + y + 1.06 = 4$

Hence the sum of the seasonal indices for summer and winter is $4 - 1.89 = 2.11$

Option A $0.83 + 1.06 = 1.89$

Option B $1.06 + 0.83 = 1.89$

Option C $0.83 + 1.28 = 2.11$

Option D $1.06 + 1.28 = 2.34$

Option E $1.28 + 1.06 = 1.34$

*Answer C***Question 13**

Given that $a = \bar{y} - b\bar{x}$ and $b = r \frac{s_y}{s_x}$ will give the y-intercept and gradient of the least squares

regression line $y = a + bx$ and $a = 148$, $\bar{x} = 21$, $s_x = 4$, $b = 3.5$ and $r = 0.7685$

Substituting in $a = \bar{y} - b\bar{x}$ gives $148 = \bar{y} - 3.5 \times 21$ so $\bar{y} = 148 + 73.5 = 221.5$

Substituting in $b = r \frac{s_y}{s_x}$ gives $3.5 = 0.7685 \times \frac{s_y}{4}$ so $s_y = \frac{3.5 \times 4}{0.7685} = 18.22$

Answer A

Module 1: Number patterns**Question 1**

$$t_1 = -20, \quad t_2 = -20 \times -1.5 = 30, \quad t_3 = -20 \times (-1.5)^2 = -45$$

so the difference between 30 and -45 is 75

Answer B

Question 2

The worm population is increasing by the same amount therefore the sequence is arithmetic

$$t_6 = 85$$

so $a + 5d = 85$ (equation 1)

$$t_{15} = 184$$

so $a + 14d = 184$ (equation 2)

equation 2 – equation 1

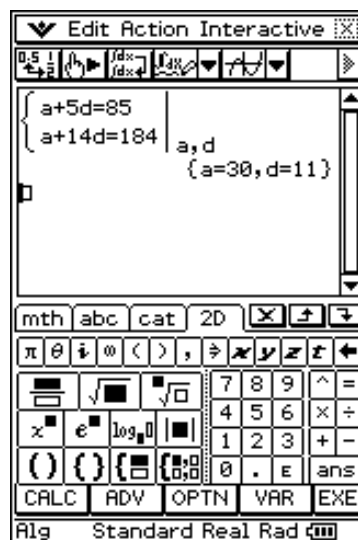
$$9d = 99$$

$$d = 11$$

sub in equation 1

$$a + 5 \times 11 = 85$$

so $a = 30$



Answer E

Question 3

Substitute $n = 2$ in $t_{n+1} = 2t_n - 1$

$$t_3 = 2t_2 - 1 \quad \text{since } t_3 = 9$$

$$9 = 2 \times t_2 - 1$$

so $t_2 = 5$

$$t_2 = 2k - 1$$

$$5 = 2k - 1$$

$$k = 3$$

Answer A

Question 4

If $a = 3$ and $b = 0$ then $B_{n+1} = 3B_n$, where $B_1 = 1$ generates the sequence 1, 3, 9, 27, ... This is a geometric sequence (not arithmetic).

If $a = 3$ and $b = 1$ then $B_{n+1} = 3B_n + 1$, where $B_1 = 1$ generates the sequence 1, 4, 13, 40, ... This sequence is neither arithmetic nor geometric.

If $a = 1$ and $b = 3$ then $B_{n+1} = B_n + 3$, where $B_1 = 1$ generates the sequence 1, 4, 7, 10, ... This sequence is arithmetic

*Answer C***Question 5**

The arithmetic sequence is given by $a, a + d, a + 2d, a + 3d, a + 4d$

$$S_1 = a = 3$$

$$S_2 = 2a + d = 11$$

$$S_3 = 3a + 3d = 24 \text{ etc}$$

$$\begin{aligned} d &= S_2 - 2S_1 \\ &= 11 - 2 \times 3 \\ &= 5 \end{aligned}$$

so the original sequence is 3, 8, 13, 18, 23 where $d = 5$

*Answer C***Question 6**

From the graph it is observed that the sum of an even number of terms (S_2, S_4, S_6) is less than the sum of an odd number of terms (S_1, S_3, S_5) which must mean that every second term of the original sequence is a negative number. The sequence must be geometric with the first term a positive number and the common ratio a negative number. So options A and D are possible answers.

Since the difference between each sum is getting smaller then the original sequence must be converging to zero so $-1 < r < 0$

Answer A

Question 7

For the sequence 2, 5, 8, ... $t_n > 100$ $a + (n-1)d > 100$ $2 + (n-1)(3) > 100$ using solve on CAS gives $n > 33\frac{2}{3}$ when $n = 34$	For the sequence -20, -13, -6, ... $t_n > 100$ $a + (n-1)d > 100$ $-20 + (n-1)(7) > 100$ using solve on CAS gives $n > 18.14$ ie when $n = 19$	For the sequence 0.2, 0.3, 0.45, ... $t_n > 100$ $ar^{n-1} > 100$ $0.2(1.5)^{n-1} > 100$ using solve on CAS gives $n > 16.327$ ie when $n = 17$
For the sequence 99, 99.05, 99.1, ... $t_n > 100$ $a + (n-1)d > 100$ $99 + (n-1)(0.05) > 100$ using solve on CAS gives $n > 20$ ie when $n = 21$	For the sequence 0.5, -0.6, 0.72, ... $t_n > 100$ $ar^{n-1} > 100$ $0.5(-1.2)^{n-1} > 100$ generate sequence and scroll when $n = 31$	

Answer C

Question 8

For a Fibonacci sequence $f_6 + f_7 = f_8$ $b + f_7 = a$ $f_7 = a - b$ option A is true	Also $f_7 + f_8 = f_9$ $a - b + a = f_9$ $2a - b = f_9$ option B is true	Also $f_5 + f_6 = f_7$ $f_5 + b = a - b$ $f_5 = a - 2b$ option C is true
$f_6 + f_7 = f_8$ $f_6 + f_7 = a$ option D is true	$f_4 + f_5 = f_6$ $f_6 + f_7 = b$ so option E is not true	

*Answer E***Question 9**

Diameter is *increasing* by 0.8 of previous increase where the first increase is $a = 6$

The series generated is $6 + 6 \times (0.8) + 6 \times (0.8)^2 + 6 \times (0.8)^3 + \dots$

So maximum increase is given by $S_\infty = \frac{a}{1-r} = \frac{6}{1-0.8} = 30$

Maximum diameter is $15 + 30 = 45$ cm

Answer C

Module 2: Geometry and trigonometry

Question 1

$$2x^{\circ} + 58^{\circ} = 180^{\circ}$$

$$x = 61^{\circ}$$

Answer C

Question 2

The interior angles of a regular polygon with n sides is given by $180 - \frac{360}{n}$

solving $180 - \frac{360}{n} = 140$ gives $n = 9$

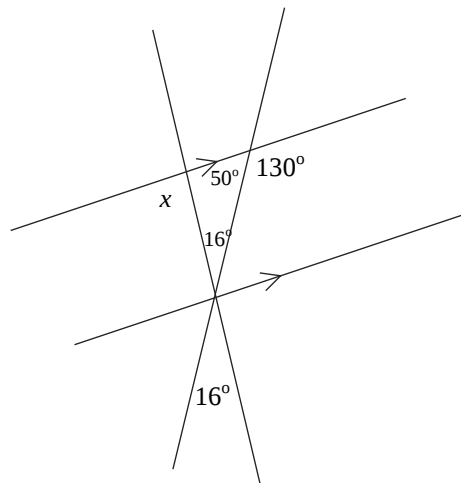
Answer C

Question 3

Inside the triangle:

Using the opposite angle of 16°
and the supplementary angle of 130°
(50°) inside the triangle then

$$x = 50^{\circ} + 16^{\circ} = 66^{\circ}$$



Answer D

Question 4

Using triangle AOC

Let x be the distance AO

$$\tan(43^\circ) = \frac{x}{15}$$

$$x = 15 \tan(43^\circ)$$

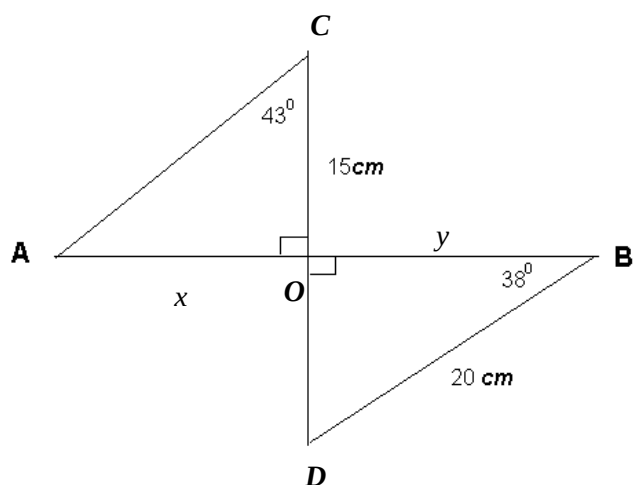
Using triangle BOD

Let y be the distance OB

$$\cos(38^\circ) = \frac{y}{20}$$

$$y = 20 \cos(38^\circ)$$

$$AB = x + y = 15 \tan(43^\circ) + 20 \cos(38^\circ)$$



Answer A

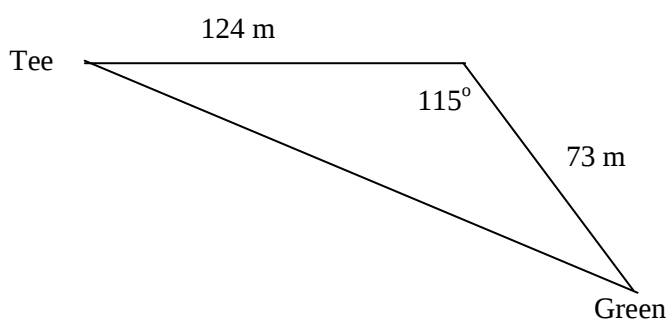
Question 5

Let x be the distance from the Tee to the Green

Using the cosine rule

$$x^2 = 124^2 + 73^2 - 2(124)(73)\cos 115^\circ$$

$$x = \sqrt{124^2 + 73^2 - 2(124)(73)\cos 115^\circ}$$



Answer B

Question 6

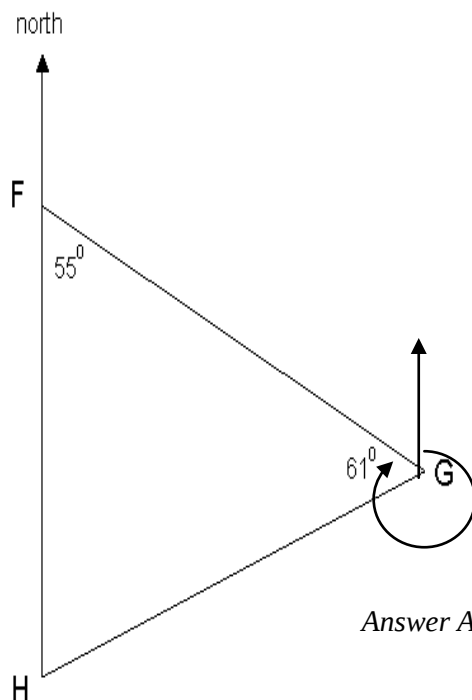
The bearing of F from G is more than 270°

The bearing of G from F is less than 180°

The bearing of G from H is less than 90°

The bearing of H from G is more than 180° but less than 270°

The bearing of H from F is 180°

**Question 7**

Volume ratio = 27:125

$$= 3^3 : 5^3$$

Length ratio = 3 : 5

Area ratio = $3^2 : 5^2$

$$\times 10 \quad \begin{array}{c} \text{---} 9 : 25 \\ \text{---} 90 : x \end{array} \quad \times 10$$

If area of smaller cone is 90 cm^2 then area of larger is 250 cm^2

Answer C

Question 8

Let x be the depth of the desk

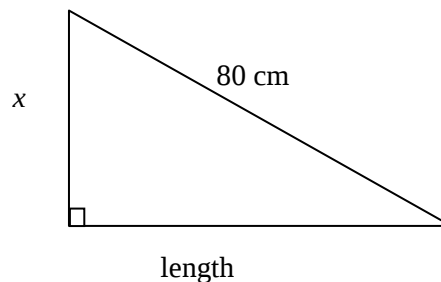
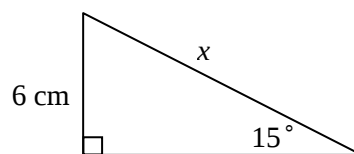
$$\sin(15^\circ) = \frac{6}{x}$$

$$x = \frac{6}{\sin(15^\circ)} = 23.18$$

using Pythagoras' Theorem

$$\text{length}^2 + x^2 = 80^2$$

$$\text{length} = \sqrt{80^2 - 23.18^2} = 76.6$$



Answer B

Question 9Method 1

Find BD using triangle ABD

$$\text{Angle ABD} = 180^\circ - 35^\circ = 145^\circ$$

$$\text{Angle ADB} = 180^\circ - 145^\circ - 28^\circ = 7^\circ$$

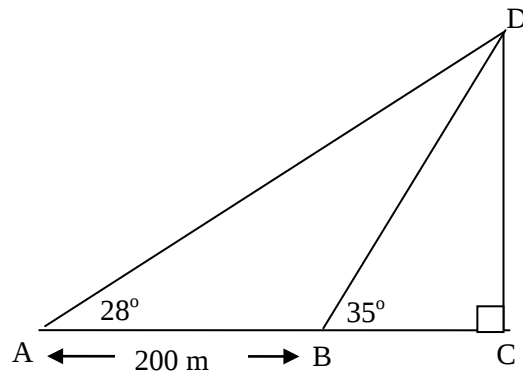
$$\frac{200}{\sin(7^\circ)} = \frac{BD}{\sin(28^\circ)}$$

$$BD = \frac{200 \sin(28^\circ)}{\sin(7^\circ)} = 770.45$$

Find CD using triangle BCD

$$\sin(35^\circ) = \frac{CD}{770.45}$$

$$CD = 770.45 \sin(35^\circ) = 441.9 \text{ m}$$

Method 2

Using triangle ACD

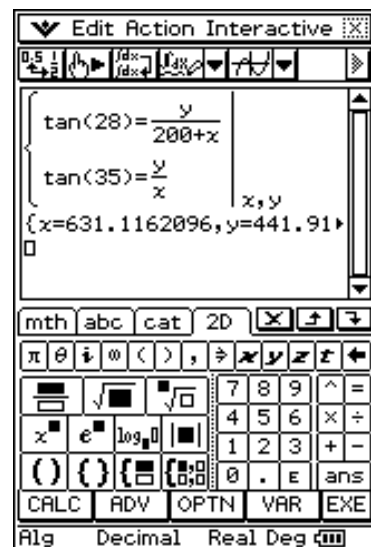
Let $BC = x$ and let the height $= CD = y$

$$\tan(28^\circ) = \frac{y}{200 + x} \quad \text{equation 1}$$

Using triangle BCD

$$\tan(35^\circ) = \frac{y}{x} \quad \text{equation 2}$$

solving on the calculator gives height = 441.9 m



Answer B

Module 3: Graphs and relations**Question 1**

45 litres of petrol was used over 8 days. Average Rate = $\frac{45}{8} = 5.6$ litres per day

*Answer C***Question 2**

$3x - 4y - 2 = 0$ can be written as $4y = 3x - 2$

$$\text{so } y = \frac{3x}{4} - \frac{2}{4}$$

The line has a positive gradient of $\frac{3}{4}$ and a y-intercept of $-\frac{1}{2}$. So options A, B and C are not true

When $x = 3$, $y = \frac{3 \times 3}{4} - \frac{2}{4} = \frac{7}{4}$ so the line does not pass through (3, 2)

When $y = 0$, $x = \frac{2}{3}$ so the x intercept is at $x = \frac{2}{3}$

For the line $6x - y - 4 = 0$ (option E)

when $y = 0$, $6x - 4 = 0 \therefore x = \frac{2}{3}$ so its x intercept is also at $x = \frac{2}{3}$

*Answer E***Question 3**

Let m = cost of a muesli bar; f = fruit strap

$$8m + 6f = 14.12 \quad \text{I}$$

$$12m + 5f = 17.90 \quad \text{II}$$

$$1.5 \times \text{I} \quad 12m + 9f = 21.18 \quad \text{III}$$

or solve on calculator

$$\text{III} - \text{II} \quad 4f = 3.28$$

$f = 0.82$ substitute into II to find m .

$$12m + 5 \times 0.82 = 17.90$$

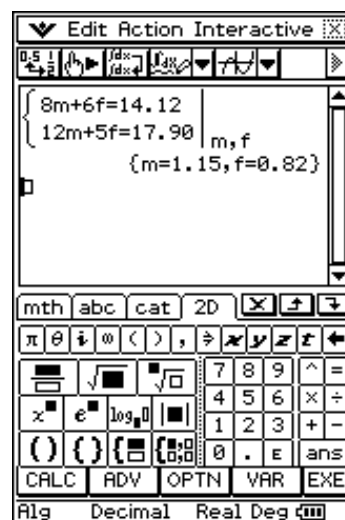
$$12m + 4.10 = 17.90$$

$$12m = 22.00$$

$$m = \$1.15$$

For three muesli bars and two fruit straps Zoe will pay $3(1.15) + 2(0.82) = \$5.09$

and will receive \$4.90 change from \$10.00

*Answer B*

Question 4

The graph shown has the equation $y = \frac{k}{x^2}$ where k = gradient of the line

$$k = \frac{6}{2} = 3 \text{ so the equation is } y = \frac{3}{x^2}$$

*Answer A***Question 5**

To break even Profit = 0. This can be read from the graph i.e. when $n = 150$

*Answer B***Question 6**

From the graph shown the gradient can be found by the points $(0, -1200)$ and $(150, 0)$

$$m = \frac{1200}{150} = 8 \quad y\text{-intercept is } -1200$$

$$\text{Profit} = 8n - 1200$$

The selling price is \$35 per T-Shirt so

$$\text{Revenue} = 35n$$

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

Rearranging this gives

$$\text{Cost} = \text{Revenue} - \text{Profit}$$

$$\text{Cost} = 35n - (8n - 1200)$$

$$\text{Cost} = 27n + 1200$$

*Answer D***Question 7**

John's travel card was used over a 2.5 hour period so the charge is \$5.

The 10 year old child used their travel card over a 2.5 hour period so the charge is also \$5.

The 7 year old child used their travel card for exactly 2.5 hours so $2.5 \times 0.5 = \$1.25$.

Sarah's travel card was used over a 6.5 hour period so \$8 was charged.

$$\text{Total charge} = \$5 + \$5 + 1.25 + 8 = \$19.25$$

Answer C

Question 8

Taxi company A has a fixed charge of \$3 (y-intercept) and a rate of \$4 per kilometre (gradient)
so $C = 4x + 3$ eliminate option A and E

Taxi company B has two charges based on the distance travelled.

- \$5 per kilometre is charged for the first 6 km so $C = 5x$, $0 \leq x \leq 6$ end point is (6, 30) and
- \$3 per kilometre is charged after the first 6 km

e.g If $x = 10$ then the first 6 km is charged \$5 each and the remaining 4 km is charged \$3 each.

$$C = 5 \times 6 + 3 \times 4 = 42 \quad (10, 42)$$

If $x = 15$ then the first 6 km is charged \$5 each and the remaining 9 km is charged \$3 each

$$C = 5 \times 6 + 3 \times 9 = 57 \quad (15, 57)$$

Using the two points (10, 42) and (15, 57) gives the linear equation $C = 3x + 12$

Generally, if x is greater than 6 km then the first 6 km is charged \$5 each and the remaining $(x - 6)$ km is charged \$3 so

$$\begin{aligned} C &= 5 \times 6 + (x - 6) \times 3 \\ &= 30 + 3x - 18 \\ &= 3x + 12 \end{aligned}$$

Answer D

Question 9

The equation passing through (0, 1) and (-1, 0) is $y = x + 1$ and

the equation passing through (0, 4) and (8, 0) is $y = -\frac{1}{2}x + 4$

The lines $y = x + 1$ and $y = -\frac{1}{2}x + 4$ intersect at the point B (2, 3)

The extreme points of the feasible region are:

A (0, 1) where the objective function $M = k \times 0 + 1 = 1$

B (0, 4) where the objective function $M = k \times 0 + 4 = 4$

C (2, 3) where the objective function $M = k \times 2 + 3 = 2k + 3$

For M to be a maximum at B

$$2k + 3 > 4$$

$$2k > 1$$

$$k > \frac{1}{2}$$

Answer B

Module 4: Business-related mathematics**Question 1**

Minimum monthly balance was brought forward from August into September \$326.36

Monthly interest on minimum balance is $\frac{3.4}{1200} \times \$326.26 = \$0.92$

Answer D

Question 2

The interest paid is $\$8250 - \$6000 = \$2250$

Let T be the time of the investment in years.

$$I = \frac{PRT}{100}$$

$$2250 = \frac{6000 \times 7.5 \times T}{100}$$

$$T = \frac{2250 \times 100}{6000 \times 7.5}$$

$$T = 5 \text{ years}$$

$$5 \times 12 = 60 \text{ months}$$

Answer E

Question 3

Vickie's investment is determined using

$$A = PR^n \quad \text{where } R = \left(1 + \frac{10}{100}\right) = 1.1 \quad \text{and} \quad n = 3 \times 1 = 3$$

$$A = 1000 \times 1.1^3$$

$$I = 1000 \times 1.1^3 - 1,000 = 331$$

Rob's investment is determined using

$$I = \frac{PRT}{100}$$

$$I = \frac{1000 \times 10 \times 3}{100} = 300$$

Vickie's interest exceeds Rob's by $\$331 - \$300 = \$31$

Answer B

Question 4

10% of \$50 = \$5 so the charge per session is \$55

The only value listed that is exactly divisible by \$55 is \$1100

Answer C

Question 5

Photocopier has depreciated by $\frac{5\,000\,000 \times 12}{100} = 600\,000$ cents

Depreciated by \$6 000

Value = 9 400 – 6 000 = \$3 400

Answer D

Question 6

The shirt is discounted at $\frac{8}{100} \times 150 = \12 and the jacket is discounted at $\frac{6}{100} \times 250 = \15

The total discount is \$12 + \$15 = \$27 for a total marked price of \$150 + \$250 = \$400

The percentage discount for the two items combined = $\frac{27}{400} \times 100 = 6.75\%$

Answer B

Question 7

Guilia's investment is determined using

$$A = PR^n \quad \text{where } R = \left(1 + \frac{7.2}{1200}\right) = 1.006 \quad \text{and} \quad n = 2 \times 12 = 24$$

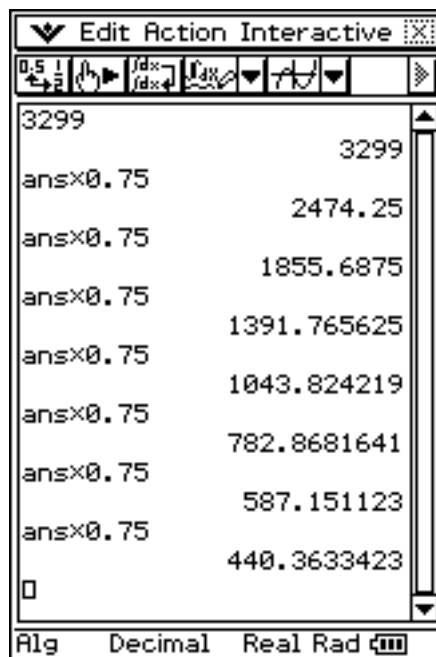
$$A = 2000 \times 1.006^{24}$$

Answer B

Question 8

A depreciation of 25% means that the television will retain 75% of its value

June 1 2011	\$3299
June 1 2012	$\$3299 \times 0.75 = \2474.25
June 1 2013	$\$3299 \times 0.75^2 = \1855.69
June 1 2014	$\$3299 \times 0.75^3 = \1391.77
June 1 2015	$\$3299 \times 0.75^4 = \1043.82
June 1 2016	$\$3299 \times 0.75^5 = \782.87
June 1 2017	$\$3299 \times 0.75^6 = \587.15
June 1 2018	$\$3299 \times 0.75^7 = \440.36

*Answer B***Question 9**

$$R = 1 + \frac{6}{1200} = 1.005$$

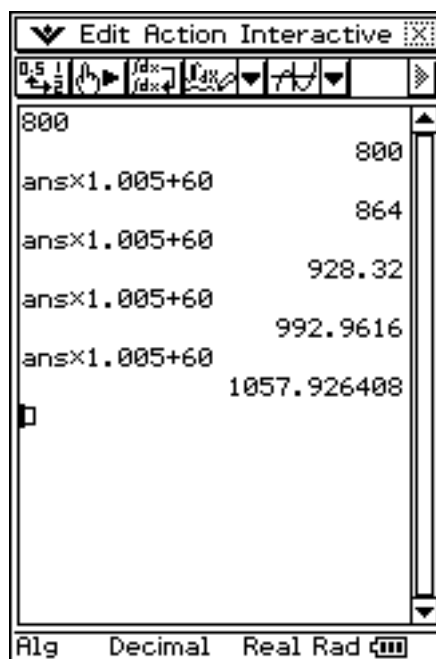
The amount of the investment at the end of:

April is $\$800 \times 1.005 + 60 = \864

May is $\$864 \times 1.005 + 60 = \928.32

June is $\$928.32 \times 1.005 + 60 = \992.96

July is $\$928.32 \times 1.005 + 60 = \1057.93

*Answer C*

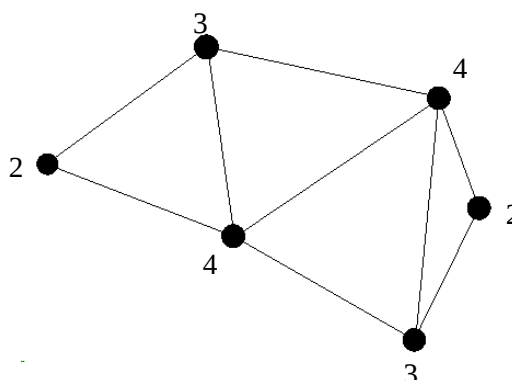
Module 5: Networks and decision mathematics**Question 1**

There are two vertices of degree 2

Two vertices of degree 4 and

Two vertices of degree 3

So four are even and two are odd

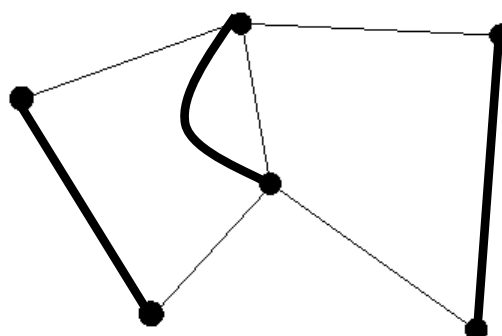


Answer D

Question 2

The degree of all vertices must be even on a connected graph for it to be an Eulerian circuit.

Adding the three paths as shown will create a graph with four vertices of degree two and two vertices of degree four hence satisfying the requirements of an Eulerian circuit.



Answer D

Question 3

The dominance matrix for the tournament is

$$\begin{array}{c}
 1 \quad 2 \quad 3 \quad 4 \\
 \begin{array}{c}
 1 \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix} \\
 2 \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \\
 3 \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \\
 4 \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}
 \end{array}
 \end{array}$$

This gives the dominance vector $\begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$ so the final will be between Teams 1 and 4

Answer B

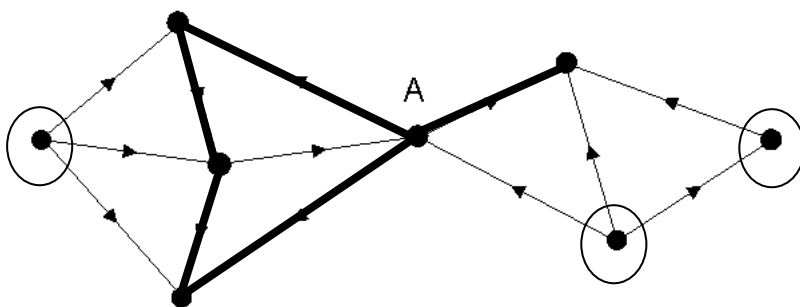
Question 4

An illustration of the relationship between two categorical variables: *player* and *position* on basketball court needs to be displayed. The only graph that is used for this purpose is a bipartite graph.

Answer B

Question 5

Three vertices (circled) cannot be reached from vertex A



Answer C

Question 6

For any connected planar graph $v + f = e + 2$. All graphs except C satisfy this relationship

Graph	Edges	Faces	Vertices	$v + f - e = 2$
A	5	4	3	$4 + 3 - 5 = 2$
B	7	5	4	$5 + 4 - 7 = 2$
C	10	10	5	$10 + 5 - 10 = 5$
D	11	7	6	$6 + 7 - 11 = 2$
E	13	8	7	$7 + 8 - 13 = 2$

Answer C

Question 7

A complete graph with five vertices has $4 + 3 + 2 + 1 = 10$ edges to be a complete graph.

If the graph is already connected with the minimum number of edges then it already has 4 edges.

It will require an extra 6 edges to make it complete

Answer B

Question 8

The dominance matrix for the water polo competition is

$$\begin{array}{c} A \quad B \quad C \quad D \\ A \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \\ B \begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix} \\ C \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \\ D \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

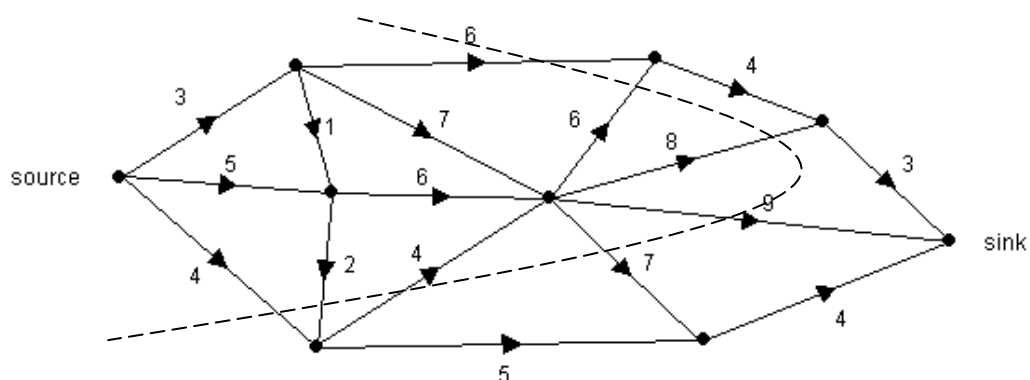
The two step dominance matrix is

$$D^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

This gives the two step dominance vector

$$\begin{bmatrix} 2 \\ 3 \\ 1 \\ 2 \end{bmatrix}$$

Answer B

Question 9

A maximum of seven edges.

Answer E

Module 6: Matrices**Question 1**

Determinant

$$= ad - bc$$

$$= 2 \times 2 - 3 \times 3$$

$$= 4 - 9$$

$$= -5$$

Answer D

Question 2

If $CX = D$

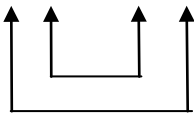
To solve for X then pre-multiply by the inverse of C

$$X = C^{-1}D$$

Answer D

Question 3

$$A \times B = AB$$

$$3 \times 4 \times m \times n = 3 \times 1$$


For the Matrix AB to exist the number of columns in matrix A must equal the number of rows in matrix B, therefore $m = 4$

The resulting matrix has the same number of columns as B, therefore $n = 1$

So B is a 4×1 matrix

Answer C

Question 4

The inverse of the matrix $\begin{bmatrix} 2 & 6 \\ 1 & 4 \end{bmatrix}$ is

$$\begin{aligned}
 &= \frac{1}{2 \times 4 - 6 \times 1} \begin{bmatrix} 4 & -6 \\ -1 & 2 \end{bmatrix} \\
 &= \frac{1}{2} \begin{bmatrix} 4 & -6 \\ -1 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & -3 \\ -\frac{1}{2} & 1 \end{bmatrix}
 \end{aligned}$$

The only element not in the inverse matrix is 3

Answer B

Question 5

A^{-1} will exist if the determinant does not equal zero

$$ad - bc \neq 0$$

$$xy - 24 \neq 0$$

$$xy \neq 24$$

The product of x and y for all options listed, except E, give 24

Answer E

Question 6

$$\begin{array}{rcl}
 t_1 - 2t_2 = 0 & & t_1 - 2t_2 + 0a = 0 \\
 t_1 - 3a = 87 & \text{can be represented as} & t_1 + 0t_2 - 3a = 87 \\
 t_2 + 2a = 29 & & 0t_1 + t_2 + 2a = 29
 \end{array}$$

so the matrix can be set up as

$$\begin{bmatrix} 1 & -2 & 0 \\ 1 & 0 & -3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ a \end{bmatrix} = \begin{bmatrix} 0 \\ 87 \\ 29 \end{bmatrix}$$

*Answer E***Question 7**

The transition matrix $\begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix}$

The steady state matrix is $\begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix}^{50} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.33333 \\ 0.66666 \end{bmatrix}$ Mario's
Sienna's

*Answer A***Question 8**

The initial state does not have an affect on the long term outcome. The transition matrix dictates the final outcome so Mario's percentage of customers will be unaltered.

*Answer A***Question 9**

$$\begin{bmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.8 & 0.3 \\ 0.2 & 0.1 & 0.6 \end{bmatrix} \begin{bmatrix} 300 \\ 200 \\ 100 \end{bmatrix} = \begin{bmatrix} 210 \\ 250 \\ 140 \end{bmatrix}$$

Labour lost 90 votes, Liberal gained 50 votes and the other party gained 40 votes.

The total change in votes was $90 + 50 + 40 = 180$ votes.

Answer C