

FURTHER MATHEMATICS

Written Examination 2



2011 Trial Examination

SOLUTIONS

Core

Question 1

- a. Training time
1 mark
- b. $r = 0.7897$, data to be entered in calculator
1 mark
- c. Circle (4,60) on graph
1 mark
- d. linear, moderate, positive
3 marks
- e. $r^2 = 0.6236$, 62.36%, Training hours, from calculator
3 marks
- f. $\text{runs} = 8.618 + 10.600 \times \text{Training time}$, calculator
2 marks

Question 2

- a. Time = $-33.490 + 59.087 \times \log(\text{distance})$, calculator

2 marks

- b. Time = $-33.490 + 59.087 \times \log 50 = 67$ minutes

1 mark

- c. Unreliable as the distance 50 kms is outside the given data range used to calculate regression equation, extrapolating

1 mark

Total 15 marks

Module 1: Number patterns

Question 1

- a. Arithmetic as $37 - 30 = 44 - 37 = 7$

2 marks

- b. $T_{10} = 30 + (10 - 1) \times 7 = 93$

2 marks

- c. $S_{10} = \frac{10}{2}(2 \times 30 + 9 \times 7) = 615$

2 marks

Question 2

- a. 20, 27, 34.7 round down to 34

1 mark

- b. Kolara-Noorat 30, 37, 44, 51, 58, 65, 72, 79
Terang-Mortlake 20, 27, 34, 42, 51, 61, 72, 84
Year 2013

2 marks

Question 3

a. $50 \times 1.1 = 55$

1 mark

b. 1.1

1 mark

c. $S_7 = \frac{50(1.1^7 - 1)}{1.1 - 1} = 474$

2 marks

d. $N_{n+1} = 1.1N_n$ $N_1 = 50$

2 marks

Total 15 marks

Module 2: Geometry and trigonometry

Question 1

a. $\tan A = \frac{5}{2}; A = \tan^{-1}\left(\frac{5}{2}\right) = 68^\circ$

1 mark

b. $\tan^{-1}\left(\frac{20}{100}\right) = 11^\circ$

2 marks

Question 2

a. $AC = \sqrt{12^2 + 8^2} = 14.4 \text{ cm}$

1 mark

b. $\text{Length} = \sqrt{208 + 3^2} = 14.7 \text{ cm}$

2 marks

c. $\text{Surface Area} = 312 \text{ cm}^2, \text{Volume} = 288 \text{ cm}^3$

2 marks

d.

i. Heron's Formula $s = 11$, Area $= \sqrt{11(11-10)(11-6)(11-6)} = 16.6 \text{ cm}^2$

ii. $\frac{16.6 \times 100}{96} = 17\%$

Total 3 marks

Question 3

a. $18^2 = 12^2 + 14^2 - 2 \times 12 \times 14 \times \cos C$
 $C = 87.3^\circ$

2 marks

b. Bearing $= 90 + 87.3 = 177.3^\circ \text{ T}$

1 mark

c. Volume $= \left(\frac{8}{4}\right)^3 \times 6000 = 48000L$

4 marks

Total 15 marks

Module 3: Graphs and relations

Question 1

a. \$9

1 mark

b. $T = \left(\frac{80}{100}\right) \times 60 = 48 \text{ minutes}$

1 mark

c. $s = \frac{80 \times 60}{50} = 96 \text{ kmh}^{-1}$

2 marks

Question 2

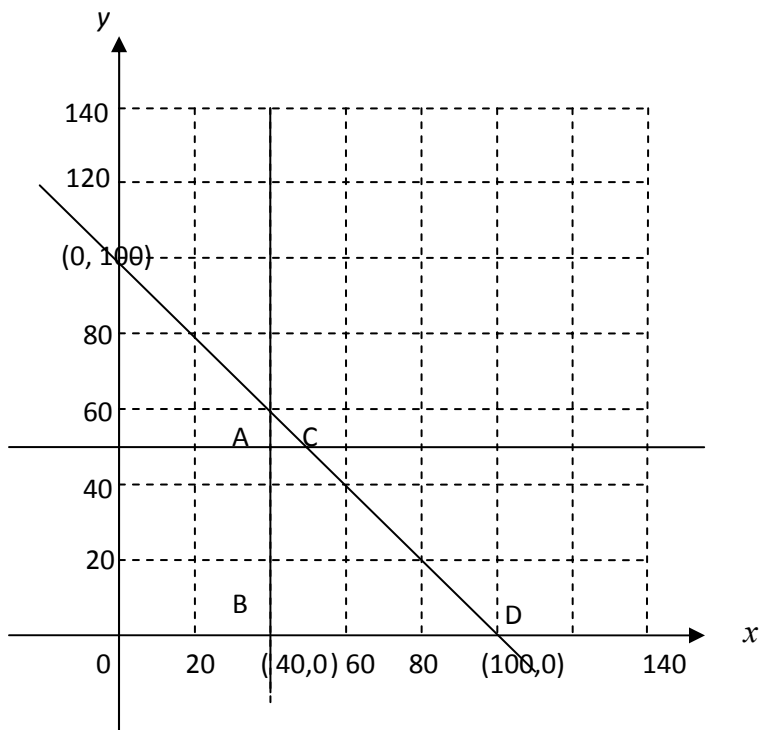
a. Constraint 4 is $x \geq 40$ and constraint 5 is $y \leq 50$

2 marks

b. $P = 0.2x + 0.3y$

1 mark

- c. Area bordered by BACD is feasible region



4 marks

- d. $(40,0)$ $P=8$; $(40,50)$ $P=23$; $(100,0)$ $P=20$; (50.50) $P=25$ maximum

2 marks

- e. Ages= 50 and Herald-Suns=50

2 marks

Total 15 marks

Module 4: Business-related mathematics

Question 1

- a. 21326.7%

1 mark

- b. Profit = $\frac{500}{100} \times 75000 = \375000
 Sell price = $375000 + 75000 = \$450000$

1 mark

c. $\frac{13.20}{1.1} = \$12.00$

1 mark

Question 2

a. $150 + 50 \times 12 = \$750$

2 marks

b. $r = \frac{100 \times 100}{500 \times 1} = 20\%$

2 marks

c. $r_{eff} = \frac{20 \times 2 \times 12}{12 + 1} = 36.9\%$

2 marks

d. $\text{Value} = 650(1 - 0.15)^3 = \399.18

2 marks

e. $N=5$ $I=?$ $PV= - 650$ $PMT= 0$ $FV= 200$ $P/Y=C/Y= 1$
 $I=21.0\%$

1 mark

Question 3

a. $N= 240$ $I=8.57$ $PV= 500000$ $PMT= ?$ $FV= 0$ $P/Y=C/Y= 12$
 $PMT= \$4361.29$

1 mark

b. $N= ?$ $I=8.57$ $PV= 500000$ $PMT= - 5000$ $FV= 0$ $P/Y=C/Y= 12$
 $N=176$ months

1 mark

c. $N=\text{any number}$ $I=8.57$ $PV= 500000$ $PMT= ?$ $FV= - 500000$ $P/Y=C/Y= 12$
 $PMT= \$3570.83$

1 mark

Total 15 marks

Module 5: Networks and decision mathematics

Question 1

- a. A 2 ,B 3 ,C 3, D 4 ,E 2 1 mark
- b. There are 2 vertices of odd degree 1 mark
- c. Draw edge from B to C 1 mark
- d. ABCBDCEDA or others 1 mark

Question 2

- a. 20 hours 1 mark
- b. ACEHI 1 mark
- c. 9 hours, $LST=11-EST=2$ 1 mark
- d. D,F,G 2 marks
- e. 16 hours. 1 mark
- f. \$240 1 mark

Question 3

a. A,C

1 mark

b. D

1 mark

c. A and C equal first, D, B

2 marks

Total 15 marks

Module 6: Matrices

Question 1

a. Order is 3×2

1 mark

b. i. $\begin{bmatrix} 350 \\ 349 \\ 257 \end{bmatrix}$

ii. Represents the takings from selling of Herald-Suns and Ages from sub-agencies X,Y,Z

1 + 1 = 2 marks

Question 2

a.
$$\begin{bmatrix} 4 & 3 & 7 \\ 9 & 5 & 2 \\ 0 & 7 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 57 \\ 64.50 \\ 47 \end{bmatrix}$$

1 mark

b. Yes, as the determinant = 364, ie. Does not equal zero, hence a unique solution

2 marks

c.

$$\text{inverse} = \begin{bmatrix} \frac{1}{364} & \frac{10}{91} & -\frac{29}{364} \\ -\frac{27}{364} & \frac{3}{91} & \frac{55}{364} \\ \frac{9}{52} & -\frac{1}{13} & -\frac{1}{52} \end{bmatrix}$$

1 mark

d.

$$\begin{bmatrix} \frac{1}{364} & \frac{10}{91} & -\frac{29}{364} \\ -\frac{27}{364} & \frac{3}{91} & \frac{55}{364} \\ \frac{9}{52} & -\frac{1}{13} & -\frac{1}{52} \end{bmatrix} \begin{bmatrix} 57 \\ 64.5 \\ 47 \end{bmatrix} = \begin{bmatrix} 3.5 \\ 5 \\ 4 \end{bmatrix}$$

Prices of X,Y,Z respectively are \$3.50, \$5, and \$4

2 marks

Question 3

a.

$$T = \begin{bmatrix} 0.80 & 0.08 & 0.075 \\ 0.05 & 0.90 & 0.005 \\ 0.15 & 0.02 & 0.920 \end{bmatrix}$$

2 marks

b. $S_0 = \begin{bmatrix} 200 \\ 250 \\ 300 \end{bmatrix}$

1 mark

c. $TS_0 = \begin{bmatrix} 206 \\ 237 \\ 357 \end{bmatrix}$

1 mark

d. $T^4S_0 = \begin{bmatrix} 216 \\ 206 \\ 378 \end{bmatrix}$

1 mark

e. $\begin{bmatrix} 221 \\ 133 \\ 447 \end{bmatrix}$

In the long term , steady state solution $X=221, Y=133, Z=447$

1 mark

Total 15 marks