

The Mathematical Association of Victoria
Trial Examination 2011
Further Mathematics Exam 2--SOLUTIONS

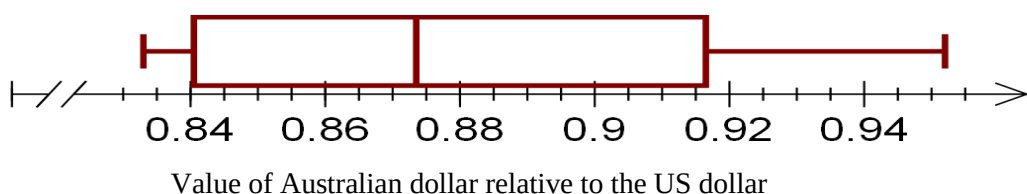
SECTION A: Core--Data analysis

Question 1

- a. The median is the middle value when the data is ordered from smallest to largest.
 There are 12 data points, so the median will be the $\frac{12+1}{2} = 6.5^{\text{th}}$ score.
 Taking the mean of the 6th and 7th scores give $\frac{0.886 + 0.861}{2} = 0.874$ **A1**
- b. $\frac{5}{12} \times 100\% = 42\%$ **A1**
- c. Range = highest value – lowest value
 $= 0.952 - 0.833$
 $= 0.119$ **A1**

Question 2

- a i. US dollar: min = 0.833, $Q_1 = 0.841$, median = 0.874, $Q_3 = 0.917$, max = 0.952 **A1**
- a ii. Outlier if less than $Q_1 - 1.5 \text{ IQR}$, or greater than $Q_3 + 1.5 \text{ IQR}$, i.e. < 0.727 or > 1.031
 There are no outliers present in the data

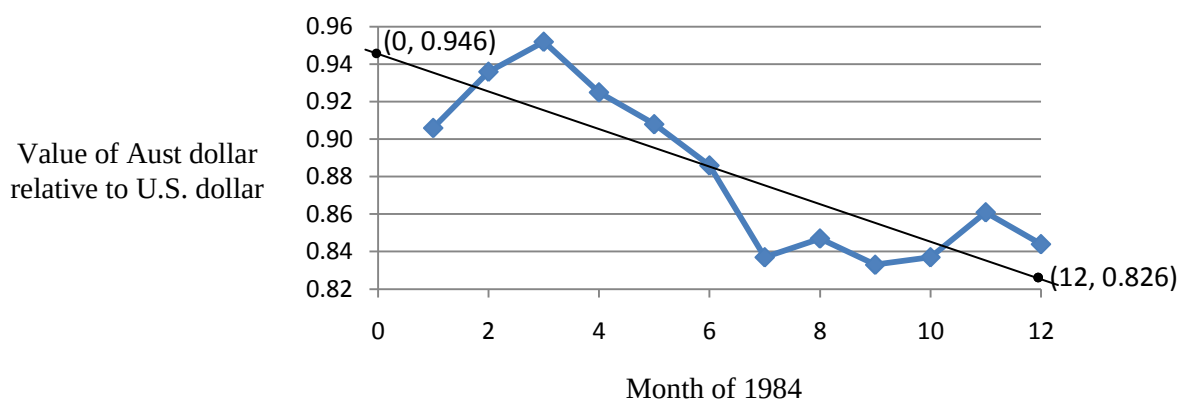


A1

- b. The boxplot of the Australian dollar relative to the Japanese yen has a slight negative skew with an outlier at 219.70. **A1**
 (Accept approximately symmetrical with an outlier at 219.70)

Question 3

- a. The regression line drawn on the time series plot should pass through the points (0, 0.946) and (12, 0.826). Any two correct points on the line are acceptable.

**A2**

- b. In November 1984 the value of the Aust dollar relative to the U.S. dollar was 86 cents.

$$\begin{aligned}
 \text{Residual} &= \text{actual value} - \text{predicted value} \\
 &= 0.861 - [0.946 - 0.01 \times 11] \\
 &= 0.861 - 0.836 \\
 &= 0.025
 \end{aligned}$$

A1

- c. Month for July 1985: $12 + 7 = 19$

$$\begin{aligned}
 \text{Aust dollar value} &= 0.946 - 0.01 \times 19 \\
 &= 0.756
 \end{aligned}$$

A1**A1**

- d. The linear model assumes the value of the Australian dollar relative to the U.S. dollar will continue to fall at a constant rate. Extrapolating outside the data set is not appropriate. Predicting long term values with this model will be very unreliable.

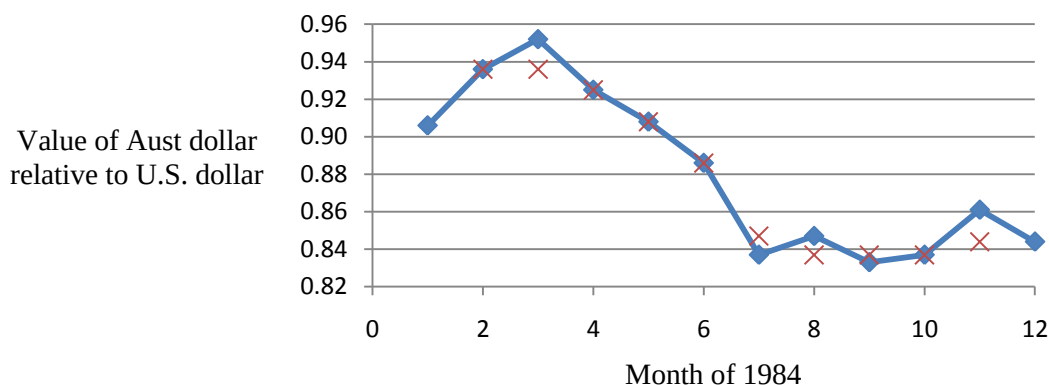
A1

- e. $r = -0.834$

The coefficient of determination $r^2 = 0.695$ (accept $r^2 = (-0.834)^2 = 0.696$)
 69.5% (or 69.6%) of the variation in the value of the Australian dollar can be explained by the variation in the month of the year.

A1

- f. Three median smoothing shown with cross x in graph below

A2

Module 1: Number patterns**Question 1**

a. Arithmetic sequence: $a = 11, d = 2, t_n = a + (n-1)d$
 $t_{12} = 11 + 11(2) = 33$ **A1**

b. Arithmetic series: $S_n = \frac{n}{2}[2a + (n-1)d]$
 $S_{15} = \frac{15}{2}[22 + (14)2] = 375$ **A1**

c i. $t_n = 55 \quad 11 + (n-1)2 = 55$
 $2(n-1) = 44$
 $n = 23$ **A1**

c ii. $S_{23} = \frac{23}{2}(11 + 55)$ OR $S_{23} = \frac{23}{2}[22 + (23-1)2]$
 $= 759$ **A1**

Question 2

a. $r = \frac{2.2}{2} = 1.1$ **A1**

b. $t_7 = 2 \times 1.1^6 = 3.5$ metres **A1**

c. Geometric series: $S_n = \frac{a(r^n - 1)}{r - 1}$
 $\frac{2(1.1^n - 1)}{1.1 - 1} = 150$ **M1**
 $2(1.1^n - 1) = 15$
 $1.1^n - 1 = 7.5$
 $1.1^n = 8.5$
 $n = 22.45$ using the calculator

There are 22 trees planted in a row that is 150 metres long. **A1**

Question 3**a.**

t_0	10
t_1	$1.1 \times 10 + 5 = 16$
t_2	$1.1 \times 16 + 5 = 22.6$
t_3	29.86
t_4	37.85
t_5	46.63
t_6	56.29
t_7	66.92
t_8	78.62
t_9	91.48

$t_5 = 46.63 \text{ kg}$

A1

$$\text{b. } \frac{t_1}{t_0} = \frac{16}{10} = 1.6 \quad \frac{t_2}{t_1} = \frac{22.6}{16} = 1.41$$

M1

Since $\frac{t_1}{t_0} = 1.6 \neq 1.41 = \frac{t_2}{t_1}$ then sequence is not geometric (values must be shown)

$$\text{c. } \text{From table } t_8 = 78.62 \text{ and } t_9 = 91.48$$

Nine years after ‘first yield’ the average yield will exceed 80 kg per tree.

A1

$$\text{d. } t_6 - t_5 = 56.29 - 46.63 = 9.66 \text{ kg}$$

A1

$$\text{e. } t_n = 5 \times 1.3^n$$

A1

f. The Corregiola olive trees starts with a higher yield, but the yield of the Paragon olive trees increases at a faster rate.

The greater increase in the yield of the Paragon olive trees is due to the coefficient of p_n being 1.3, whereas for the Corregiola olive trees the coefficient of t_n is 1.1.

A1

g. Paragon yield exceeds Corregiola yield after 14 years.

A1

Years after first yield (n)	Corregiola $t_{n+1} = 1.1t_n + 5$	Paragon $p_{n+1} = 1.3p_n$
0	10	5
1	16	6.5
2	22.6	8.45
3	29.86	10.99
4	37.85	14.28
5	46.63	18.56
6	56.29	24.13
7	66.92	31.37
8	78.62	40.79
9	91.48	53.02
10	105.62	68.93
11	121.18	89.61
12	138.3	116.49
13	157.13	151.43
14	177.85	196.87

Module 2 Geometry and Trigonometry

Question 1

a. 10°

A1

b. $\sin(10^\circ) = \frac{2000}{PA}$

$$PA = \frac{2000}{\sin(10^\circ)} = 11\,517.54$$

$$PA = 11\,518 \text{ metres}$$

A1

c. i. Horizontal distance from A to P = $\frac{2000}{\tan(10^\circ)} = 11\,342.56$ metres

$$AM = 11\,342.56 - 3000 = 8343 \text{ m}$$

M1

c. ii. $\tan \theta = \frac{2000}{8342.56}$

$$\theta = 13.5^\circ$$

A1

Question 2

a. i. Using the cosine rule: $\cos(\angle AOB) = \frac{OA^2 + OB^2 - AB^2}{2 \times OA \times OB}$

$$\angle AOB = \cos^{-1}\left(\frac{32^2 + 45^2 - 67^2}{2 \times 45 \times 32}\right)$$

M1

$$\angle AOB = 120^\circ$$

a. ii. Area = $\frac{1}{2} \times OA \times OB \sin(\angle AOB)$

$$\text{Area} = \frac{1}{2} \times 32 \times 45 \times \sin(120^\circ)$$

$$\text{Area} = 623.5 \text{ km}^2$$

A1

b. Bearing of B from O = $360^\circ - 130^\circ = 230^\circ$

A1

c. Finding $\angle OAB$

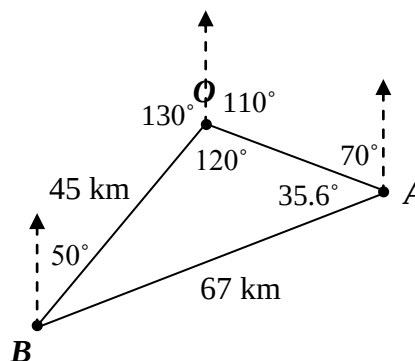
Using the sine rule:

$$\frac{OB}{\sin(\angle OAB)} = \frac{AB}{\sin(\angle AOB)}$$

$$\frac{45}{\sin(\angle OAB)} = \frac{67}{\sin(120^\circ)}$$

$$\angle OAB = 35.6^\circ$$

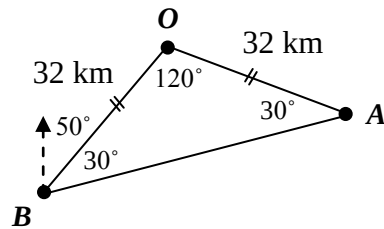
A1



$$\text{Bearing of B from A} = 360^\circ - (35.6^\circ + 70^\circ) = 254.4^\circ$$

M1

d i. $AB^2 = OA^2 + OB^2 - 2 \times OA \times OB \cos(\angle AOB)$
 $AB^2 = 32^2 + 32^2 - 2 \times 32 \times 32 \cos(120^\circ)$
 $AB = \sqrt{3072}$
 $AB = 55.4$ metres

**A1**

d. ii. $OA = OB = 32$ km
Triangle OAB is isosceles
 $\angle OBA = 30^\circ$
Bearing of A from B is $(50^\circ + 30^\circ) = 80^\circ$

A1**Question 3**

a. Radius of oil barrel = 50 cm

Cylinder: $V = \pi r^2 h$

$V = \pi \times 50^2 \times 85$

M1

$V = 667588.4 \text{ cm}^3$

$V = 667.5884$ litres

The cylindrical oil barrel contains 668 litres of oil when full

A1

- b.** Containers X and Y are similar in shape, therefore the corresponding length dimensions are in the same ratio.

Volume container $Y = 3.375 \times$ Volume container X

$\Rightarrow$ Length container $Y = \sqrt[3]{3.375} \times$ Length container X

M1

Length container $Y = 1.5 \times$ Length container X

$\Rightarrow$ Surface Area container $Y = 1.5^2 \times$ Surface Area container X

$13.95 = 1.5^2 \times$ Surface Area container X

$\Rightarrow$ Surface Area container $X = \frac{13.95}{1.5^2} = 6.2 \text{ m}^2$

A1

Module 3: Graphs and relations

Question 1

a. $0 \leq x \leq 15$

$0 \leq y \leq 8$

A1

Accept $x \leq 15$ and $y \leq 8$

b. Both lines correctly labelled with the equation

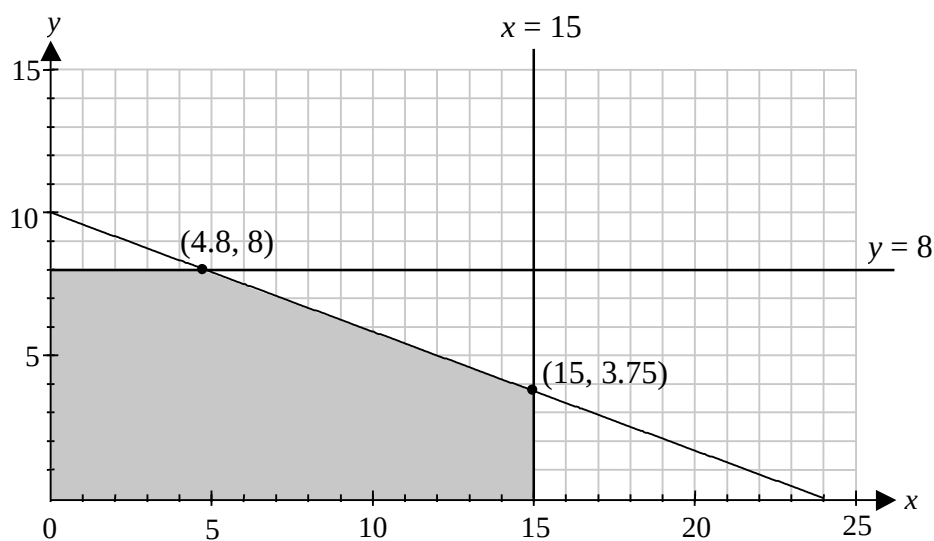
A1

c. Line $5x + 12y = 120$ drawn with intercepts $(0, 10)$ and $(24, 0)$
or, the points of intersection $(4.8, 8)$ and $(15, 3.75)$ marked

A1

Feasible region shaded

A1



d. Objective to maximize $= 4x + 10y$

Point	Objective $= 4x + 10y$	Number of people
$(4.8, 8)$	$4 \times 4.8 + 10 \times 8 = 99.2$	99
$(15, 3.75)$	$4 \times 15 + 10 \times 3.75 = 97.5$	97
$(0, 8)$	$4 \times 0 + 10 \times 8 = 80$	80
$(15, 0)$	$4 \times 15 + 10 \times 0 = 60$	60

M1

The greatest number of people Millie can assist is 99.

A1

This will occur with 4.8 small tables and 8 large tables as shown by the calculations above.
It assumes she has a fraction of a table which would be possible in a restaurant.

Question 2

$$5 = \frac{k}{120} \quad \Rightarrow \quad k = 600 \quad \text{A1}$$

Manager's equation is $n = \frac{600}{t}$

When $t = 75$, $n = \frac{600}{75} = 8$ staff A1

Question 3

a. $C = 2000 + 35x$ A1

b. The gradient of the line represents the cost to the restaurant per person. The gradient is \$35.
If 10 extra people attend the additional cost to the restaurant will be $10 \times 35 = \$350$ A1

c. Revenue = $75x$ A1

Breakeven point occurs when Revenue = Costs

$$75x = 2000 + 35x$$

$$40x = 2000$$

$$x = 50$$

Fifty people would need to attend the function for the restaurant to breakeven. A1

d. Profit = Revenue – Costs
 $P = 75x - (2000 + 35x)$
 $3480 = 40x - 2000$
 $x = \frac{5480}{40} = 137$ people A1

e. Let the m be the amount charged per person to make a profit of at least \$4000
 $P = mx - (2000 + 35x)$
 When $x = 125$
 $P = 125m - (2000 + 35 \times 125)$
 $P = 125m - 6375$
 $125m - 6375 \geq 4000$ M1
 $125m \geq 10375$
 $m \geq 83$
 The lowest charge is \$83 per person for profit of at least \$4000 A1

Module 4 Business related mathematics**Question 1**

- a. Value = $490\,000 \times (1.06)^3 = 583\,597.84$
Value = \$583 600 A1
- b. $700\,000 = 490\,000 \times (1.06)^n$ M1
 $n = 6.1$ years A1
- c. Capital Gains Tax = $0.45 \times \$150\,000 = \$67\,500$ A1

Question 2

- a i. Finance Solver
N = 60
I(%) = 10.4
PV = -35 000
Pmt = ?
FV = 0
PpY = 12
CpY = 12
Monthly Payment is \$750.55 A1
- a ii. Total Interest = $750.55 \times 60 - 35\,000$
Total Interest = \$10 033 M1
- b. If the monthly payment is \$800 per month for two years.
Finance Solver
N = 24
I(%) = 10.4
PV = -35 000
Pmt = 800
FV = ?
PpY = 12
CpY = 12
Amount outstanding after two years is \$21 812.91 A1
- If the monthly payment is reduced to \$600 for another three years
Finance Solver
N = 36
I(%) = 10.4
PV = -21 812.91
Pmt = 600
FV = ?
PpY = 12
CpY = 12
Amount outstanding at the end of five years is \$4537.66
\$4538 correct to the nearest dollar A1

Question 3

In Flat Rate Depreciation, the annual depreciation is constant.

This will be a percentage of the initial value of the item.

A1

In Reducing Balance Depreciation, the annual depreciation is a percentage of the current value of the item. The depreciation decreases from one year to the next.

A1**Question 4**

a. Annual flat rate depreciation = 15% of \$20 000 = \$3000

A1

b. Depreciation over four years = $3 \times \$3000 = \9000

A1

c. Value of tractor after six years = $\$20\,000 - 6 \times \$3000 = \$2000$

A1**Question 5**

Bank statement for October

Date	Transaction	Debit	Credit	Balance
01 October	OPENING BALANCE			\$1677.76
07 October	Cheque		\$520.00	\$2197.76
15 October	EFT		\$1359.05	\$3556.81
24 October	Automatic payment	\$1506.93		\$2049.88
31 October	CLOSING BALANCE			\$ 2049.88

Work backwards to find missing balances

Minimum Monthly balance is \$1677.76

A1

$$\text{Interest} = \frac{1}{12} \times \frac{3.75}{100} \times 1677.76$$

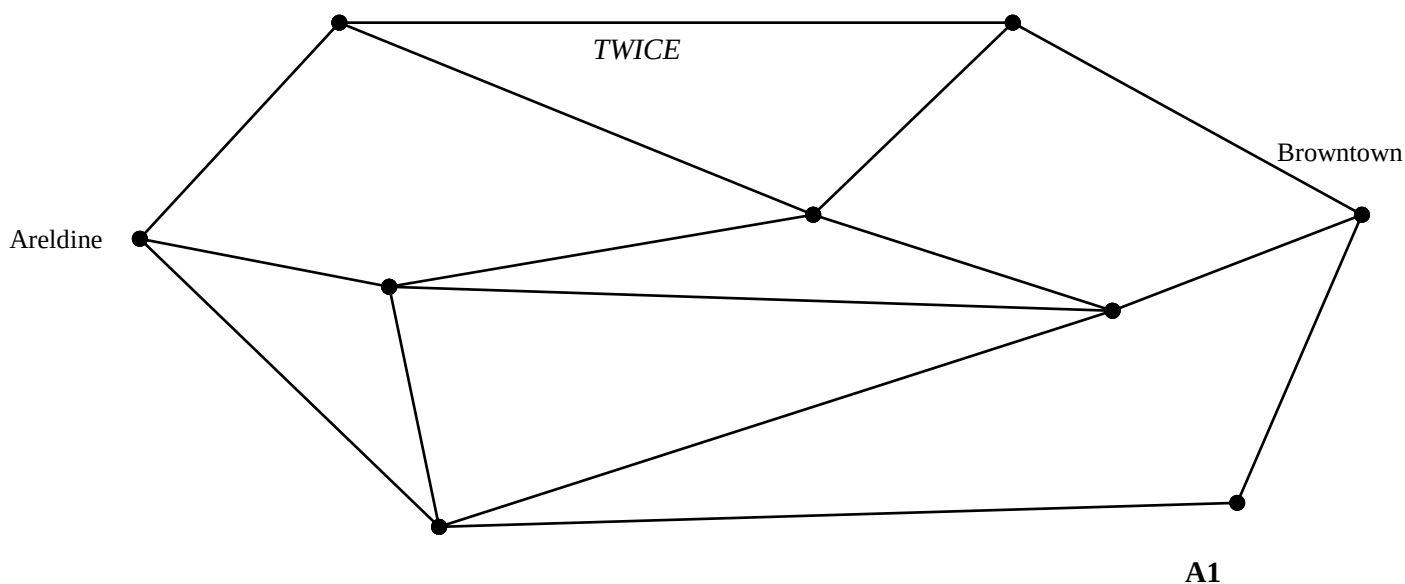
$$\text{Interest} = \$5.24$$

A1

Module 5: Networks and decision mathematics

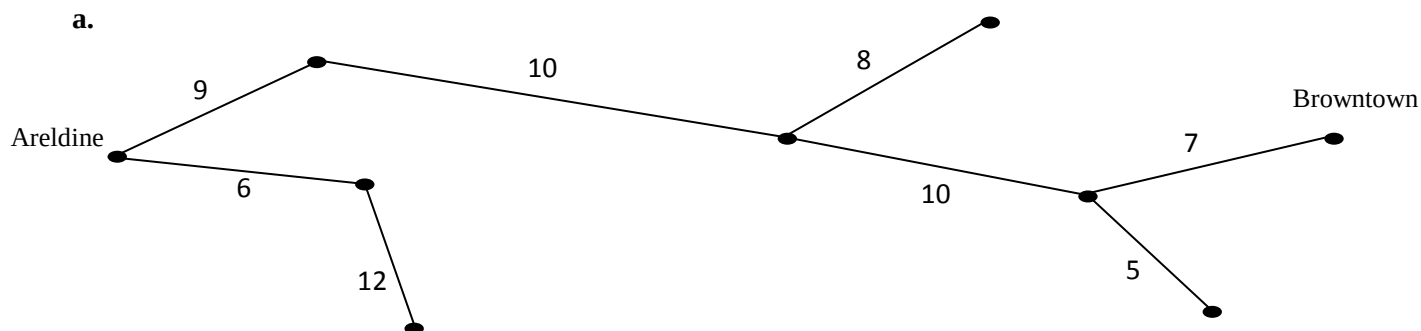
Question 1

- | | | |
|-----------|--|-----------|
| a. | Path | A1 |
| b. | Euler path | A1 |
| c. | There are four vertices of odd degree.
An Euler Path can have at most two vertices of odd degree. | A1 |
| d. | | |



Question 2

- a.**

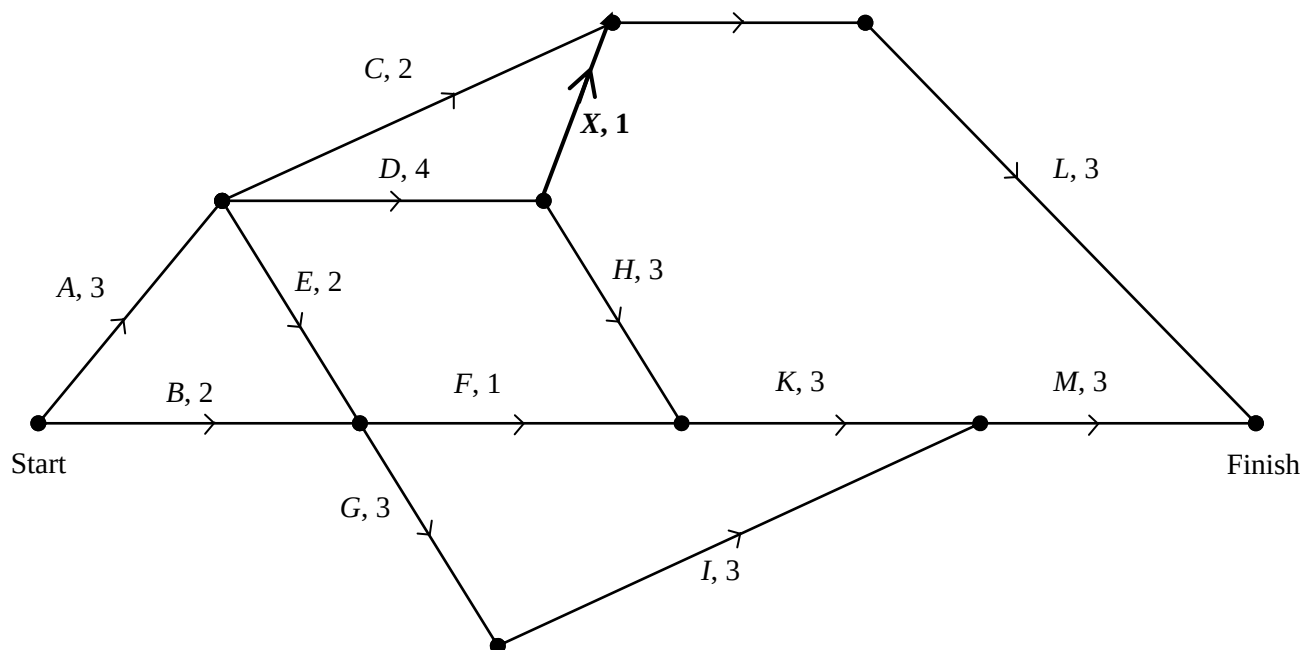


- b.** $12+6+9+10+8+10+7+5=67$ **A1**
Minimum cost is \$67 000

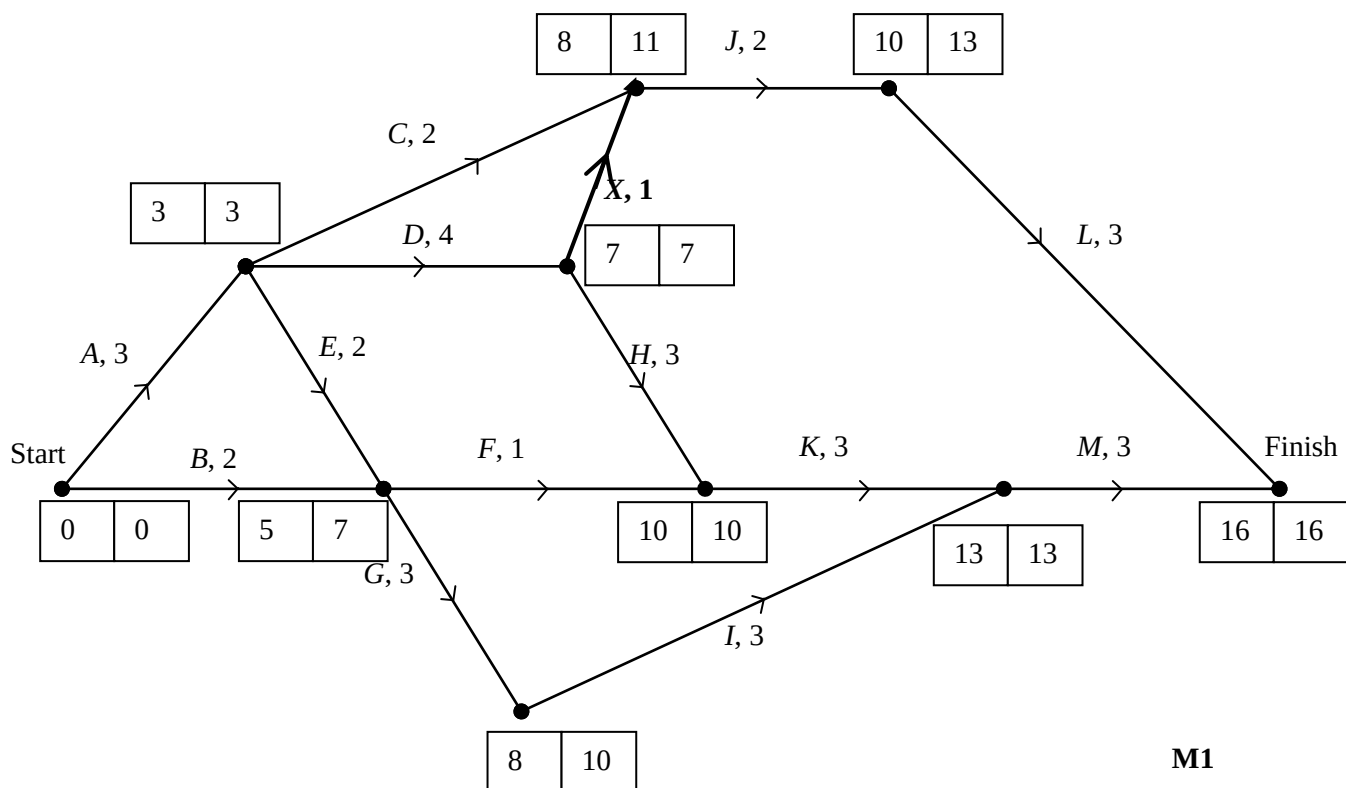
Question 3

a. i. The predecessor of task *E* is task *A*. **A1**

a. ii. Activity *X* drawn correctly on network **A1**



b.



M1

Task	Time to complete task (Days)	Earliest Start Time (Days)	Latest Start Time (Days)
<i>G</i>	3	5	7
<i>L</i>	3	10	13
<i>M</i>	3	13	13
<i>X</i>	1	7	10

A1

- c. Critical path is *ADHKM* **A1**
- d. Earliest completion time: $3 + 4 + 3 + 3 + 3 = 16$ **A1**
- e. Float = latest start time – earliest start time
For activity *F*: $9 - 5 = 4$ **A1**

Question 4

- a. There is little point in reducing the completion time of task *J* because it is not on the critical path either before or after reducing tasks *H* and *K*.
By either reducing *H* by 1 day and *K* by 1 day, or, *K* by 2 days the minimum completion time for the project will be 14 days.
However, it costs more per day to reduce task *K* than task *H*.
The minimum cost will be \$3000 **A1**
- b. This minimum cost occurs when *H* is reduced by 1 day and *K* is reduced by 1 day.
The completion time of task *J* will not be reduced. **A1**

Module 6: Matrices**Question 1**

a. $\begin{bmatrix} 3.95 \\ 6.50 \\ 10.00 \end{bmatrix} \begin{matrix} B \\ R \\ P \end{matrix}$ A1

b. $\begin{bmatrix} 3 & 1 & 5 \end{bmatrix}$ A1

c. i. $\begin{bmatrix} 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} 3.95 \\ 6.50 \\ 10.00 \end{bmatrix}$ M1

c. ii. $[68.35]$ Must be written as matrix A1

Question 2

- a. Joan bought the most punnets of seedlings.
In total she bought 7 punnets of seedlings (3 bean and 4 lettuce) A1

- b. The element in the second row and third column of the matrix is zero.
This means Doris bought no punnets of lettuce seedlings. A1

- c. i.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 0 \\ 3 & 0 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} 20 & -4 & -5 \\ -4 & 1 & 1 \\ -15 & 3 & 4 \end{bmatrix}$$

The missing element is 20.

A1

- c. ii. Solving the matrix equation to find the cost of one punnet of each type of seedling.

$$\begin{bmatrix} b \\ t \\ l \end{bmatrix} = \begin{bmatrix} 20 & -4 & -5 \\ -4 & 1 & 1 \\ -15 & 3 & 4 \end{bmatrix} \begin{bmatrix} 16.95 \\ 37.85 \\ 36.45 \end{bmatrix} = \begin{bmatrix} 5.35 \\ 6.50 \\ 5.10 \end{bmatrix}$$
 A1

Kate's seedling purchase can be written as $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$

The total cost will be $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 5.35 \\ 6.50 \\ 5.10 \end{bmatrix} = [34.15]$ A1

Kate pays \$34.15 for her seedlings.

Question 3**a.** 20%**A1**

$$\mathbf{b.} \quad \begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.4 & 0.5 & 0.7 \\ 0.3 & 0.1 & 0.1 \end{bmatrix}^3 \begin{bmatrix} 0 \\ 0 \\ 1000 \end{bmatrix} = \begin{bmatrix} 336 \\ 492 \\ 172 \end{bmatrix} \begin{matrix} O \\ I \\ N \end{matrix}$$

336 of these gardeners will prefer to use organic fertilizer in three year's time.

A1

$$\mathbf{c.} \quad \begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.4 & 0.5 & 0.7 \\ 0.3 & 0.1 & 0.1 \end{bmatrix}^{20} \begin{bmatrix} 0 \\ 0 \\ 1000 \end{bmatrix} = \begin{bmatrix} 333.\dot{3} \\ 500 \\ 166.\dot{6} \end{bmatrix} \begin{matrix} O \\ I \\ N \end{matrix} \quad \text{and}$$

$$\begin{bmatrix} 0.3 & 0.4 & 0.2 \\ 0.4 & 0.5 & 0.7 \\ 0.3 & 0.1 & 0.1 \end{bmatrix}^{21} \begin{bmatrix} 0 \\ 0 \\ 1000 \end{bmatrix} = \begin{bmatrix} 333.\dot{3} \\ 500 \\ 166.\dot{6} \end{bmatrix} \begin{matrix} O \\ I \\ N \end{matrix}$$

$$\text{The steady state matrix is } \begin{bmatrix} 333.\dot{3} \\ 500 \\ 166.\dot{6} \end{bmatrix} \begin{matrix} O \\ I \\ N \end{matrix}$$

A1**d.** Half**A1****Question 4**

$$T = \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.5 \end{bmatrix}$$

$$D_2 = TD_1 + \begin{bmatrix} 4 \\ 4 \end{bmatrix} \Rightarrow D_2 = \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.5 \end{bmatrix} \times \begin{bmatrix} 10 \\ 20 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 20 \\ 18 \end{bmatrix} \begin{matrix} H \\ L \end{matrix}$$

A1

$$D_3 = TD_2 + \begin{bmatrix} 4 \\ 4 \end{bmatrix} \Rightarrow D_3 = \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.5 \end{bmatrix} \times \begin{bmatrix} 20 \\ 18 \end{bmatrix} + \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 25 \\ 21 \end{bmatrix} \begin{matrix} H \\ L \end{matrix}$$

On the third weekend 25 people will attend the Horticulture demonstration

A1**END OF SOLUTIONS**