

Trial Examination 2010

VCE Further Mathematics Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
Core	3	1	15
Section	Number of modules	Number of modules to be answered	Number of marks
Modules	6	3	45

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white-out liquid/tape.

Materials supplied

Question booklet of 25 pages with a detachable sheet of miscellaneous formulas in the centrefold.

Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.

Please ensure that you write your **name** and your **teacher's name** in the space provided on this page.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2010 VCE Further Mathematics Units 3 & 4 Written Examination 2.

Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π , surds or fractions.
Diagrams are not to scale unless specified otherwise.

	Page
Core	3
Module	
Module 1: Number patterns	7
Module 2: Geometry and trigonometry	10
Module 3: Graphs and relations	14
Module 4: Business-related mathematics	17
Module 5: Networks and decision mathematics	19
Module 6: Matrices	23

Core**Question 1**

The percentage effectiveness of Drug *A* and Drug *B* on different patients are compared in this back-to-back stem plot.

Drug <i>A</i>	stem	Drug <i>B</i>
8 4	1	0 2 4
8 5 3 3 1	2	2 2 4 7 8
4 3	3	3 5 6
1	4	4 7 9
6	5	2

Key 6/5 = 65

- a. Based upon the median values of the two data sets, which drug is more effective?

1 mark

- b. What is the total number of patients tested using these drugs?

1 mark

Question 2

According to the Australian Government Department of Immigration and Citizenship, the figures for settler arrivals over the period 2002 to 2009 are shown in the following table.

Year	Settler arrivals
2002–2003	93 900
2003–2004	111 600
2004–2005	123 400
2005–2006	131 600
2006–2007	140 100
2007–2008	149 400
2008–2009	158 021

- a. Draw a time series for the number of settler arrivals.

2 marks

- b. Calculate the four-point moving average centered on the data for 2005 to 2006.

2 marks

Question 3

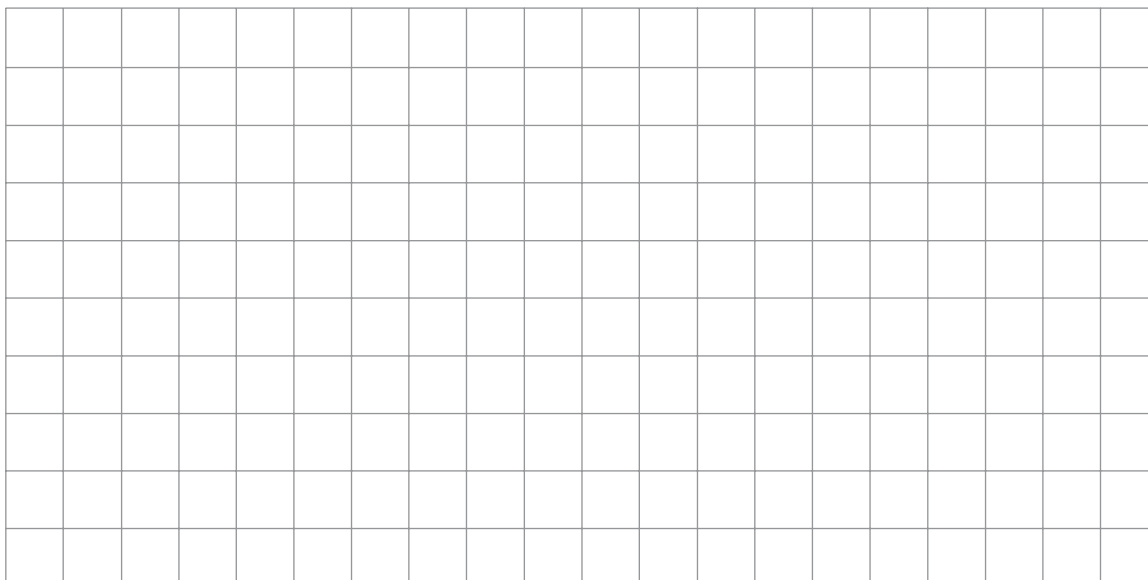
Six friends make a pact and challenge each other to do their best on their upcoming Further Maths Examination. In the final month before the exam, they spend different amounts of time studying and achieve varying results.

Name	Alan	Beatrice	Cassie	Darvesh	Elim	Francesca
Hours of study	25	18	4	12	35	40
Further Maths result %	70	50	70	80	95	90

- a. Calculate the mean and standard deviation of the number of hours studied (to 1 decimal place).

2 marks

- b. Draw a scatterplot of the data below, labelling the axes.



2 marks

- c. Calculate the value of Pearson's product-moment correlation coefficient, r (to 2 decimal places).

1 mark

- d. Complete the following sentence.

_____ % of the variation in the Further Maths results is due to a variation in
_____.

1 mark

e. Use this estimate of the regression equation:

Further Maths result % = $58 + 0.7 \times \text{hours of study}$

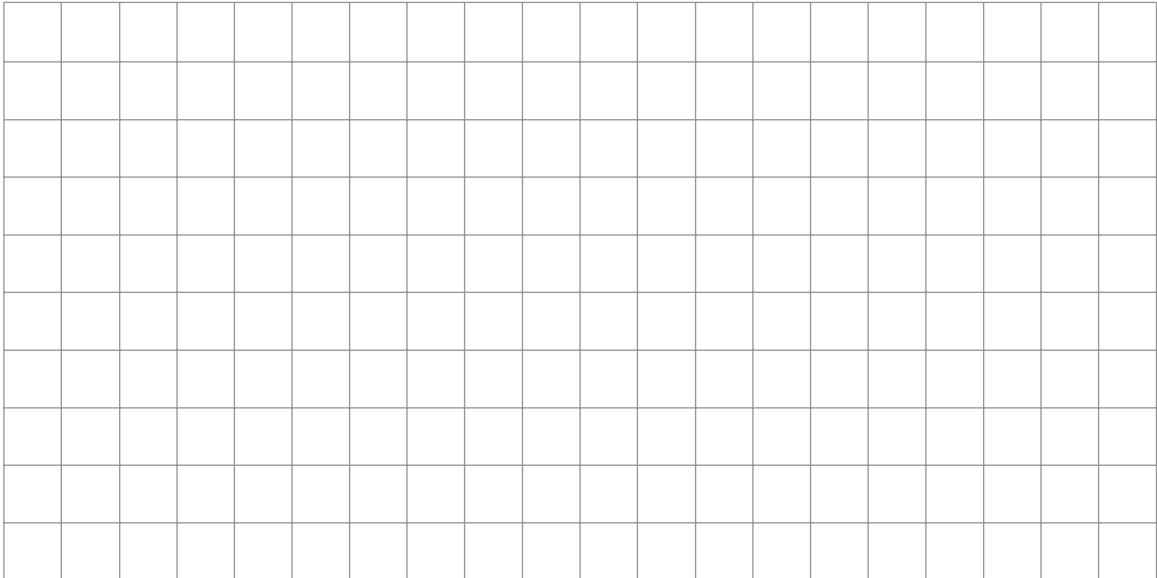
to complete the following sections.

i. Complete the table of residuals.

Hours of study	25	18	4	12	35	40
Actual Maths %	70	50	70	80	95	90
Predicted Maths %						
Residual						

2 marks

ii. Draw a residual plot below.



1 mark

Total 15 marks

END OF CORE

MODULES**Module 1: Number patterns****Question 1**

Grace and Anton have started up a tourism and accommodation business. The business keeps a record of the number of guests that they have attracted each month since they commenced operation. They notice that the guest number increases by the same amount each month. The results are shown in the table below.

Month (2010)	June	July	August	September
Guests	146	164	182	200

- a. Assuming that current trends continue, write an equation for G_n , the number of customers per month in terms of month number n . Let $n = 1$ represent June 2010.

2 marks

- b. Determine the number of customers that the business would be expected to attract in August 2011.

1 mark

- c. Grace knows that the business can only accommodate 500 guests.
During which year and month will this figure be reached?

2 marks

Question 2

When setting up the business, Anton offers discounts to regular customers of 27% off monthly prices during 2010. As their business becomes successful however, these discounts are phased out so that the discount is reduced to 18% in 2011 and 12% in 2012.

- a. Write an expression for the percentage discount in the n th year of operation (taking 2010 as the first).

1 mark

- b. If this forms a regular sequence, determine the percentage discount in 2014. Give an exact answer.

1 mark

- c. State the year for which the percentage discount is first below 3%.

2 marks

- d. Grace wants to determine the effect of these discounts. She decides to add together all of the annual discounts to form the total discount. For example, the total discount after 3 years would be $27 + 18 + 12 = 57\%$.

Find the total discount after 12 years (to 1 decimal place).

2 marks

- e. Grace believes that there is a limit to the total discount, and that the total cannot exceed this number no matter how many years of discounts are added.

What is the limit?

1 mark

Question 3

Grace notices that the cost of guided tours to Zamboanga in any year is affected by the prices in the two years preceding. She determines that the price in a certain year is always the mean of that in the two years preceding plus \$40 extra.

- a. Determine a difference equation for the price in year $n + 2$, P_{n+2} .

1 mark

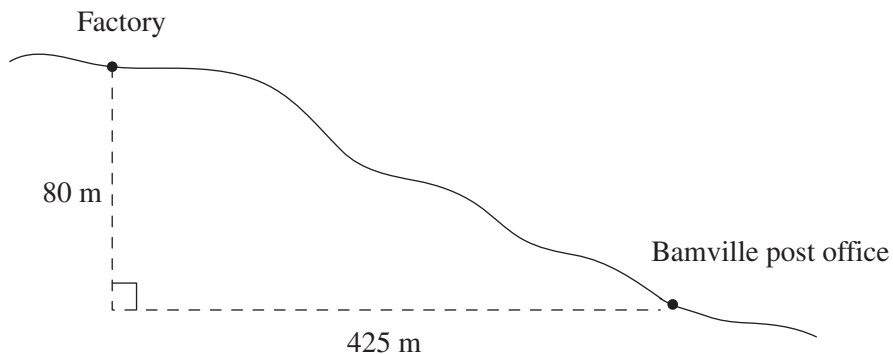
- b. Tour costs were \$1230 in 2008 and \$1320 in 2009.

What would a tour cost in 2012? Answer to the nearest cent.

2 marks

Total 15 marks

END OF MODULE 1

Module 2: Geometry and trigonometry**Question 1**

A factory is situated on a hill on the outskirts of Bamville.

The horizontal distance between the factory and the Bamville post office is 425 metres.

The vertical distance between the factory and the Bamville post office is 80 metres.

- a. Calculate the angle of elevation of the factory from the post office. Write your answer correct to the nearest degree.

1 mark

- b. Calculate the angle of depression of the post office from the factory. Write your answer correct to the nearest degree.

1 mark

Question 2

The factory produces tennis balls which have a diameter of 84 mm.

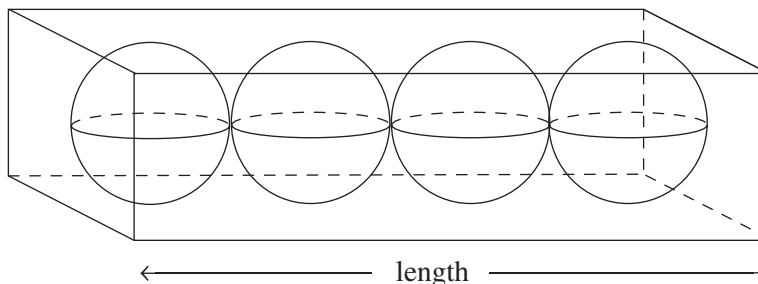
- a. Calculate the circumference of the largest cross-section of each tennis ball. Write your answer correct to the nearest millimetre.

1 mark

The factory also produces golf balls. Each golf ball has a circumference of 88 mm.

- b.** Show that the radius of each golf ball is 14 mm when written correct to the nearest mm.

1 mark



The golf balls are packed neatly into a box in the shape of a rectangular prism. The box is designed so that four golf balls are placed tightly in a row as shown in the diagram above.

- c.** Using 14 mm as the radius of each golf ball, calculate the length of the box.

1 mark

- d.** Use your answer to part **c** to calculate the volume of the box. Write your answer correct to the nearest cm^3 .

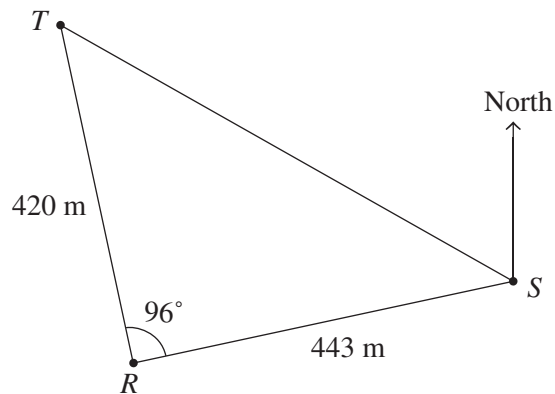
2 marks

- e.** A box containing four golf balls is made up of the golf balls and the air surrounding them. Calculate the volume of air in the box containing four golf balls. Write your answer correct to the nearest cm^3 .

2 marks

Question 3

The boundaries surrounding the Bamville golf course form a triangle shown in the diagram below.



A map of the golf course is created with a scale of 1 : 3000.

- a. Calculate the distance on the map that represents the boundary RT .

1 mark

- b. Use the cosine rule to show that boundary ST is 642 metres (correct to the nearest metre).

1 mark

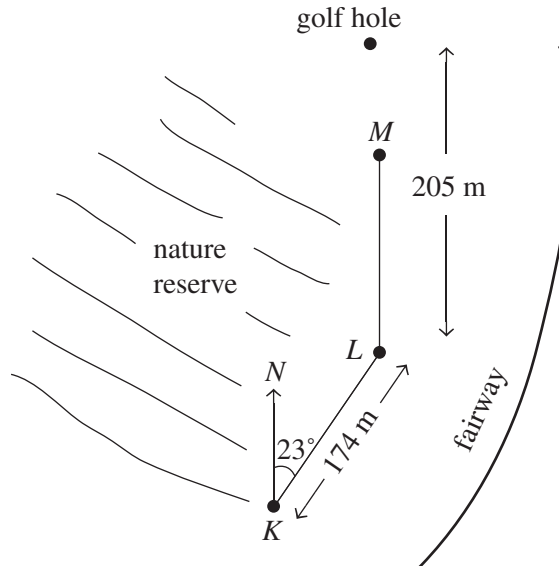
- c. Use the value of 642 metres for boundary ST to determine the size of angle RTS . Write your answer in degrees correct to the nearest degree.

1 mark

Question 4

One hole of the Bamville golf course requires each player to navigate around a fenced-off nature reserve KLM .

- the distance KL is 174 metres on a bearing of 23°
- the distance from point L to the hole is 205 metres
- the hole is due North of point L



Calculate the bearing of point K from the hole. Write your answer correct to 1 decimal place.

3 marks
Total 15 marks

END OF MODULE 2

Module 3: Graphs and relations**Question 1**

Danilyn operates a swimming school. She has two employees, Trevor and Jane. Trevor can take 12 junior swimmers in a class while Jane can take 16. Trevor must take at least one more junior class than Jane. There are 384 junior swimmers needing lessons. The total number of classes cannot exceed 30. Neither employee is available for more than 20 lessons per week. All lessons have a duration of one hour.

Let x represent the number of classes that Trevor takes per week, and let y represent the number of classes that Jane takes.

- a. Write a full set of constraints to fit these conditions.

2 marks

Due to a change of staff conditions, these constraints must be altered to the following:

$$10x + 20y \geq 400$$

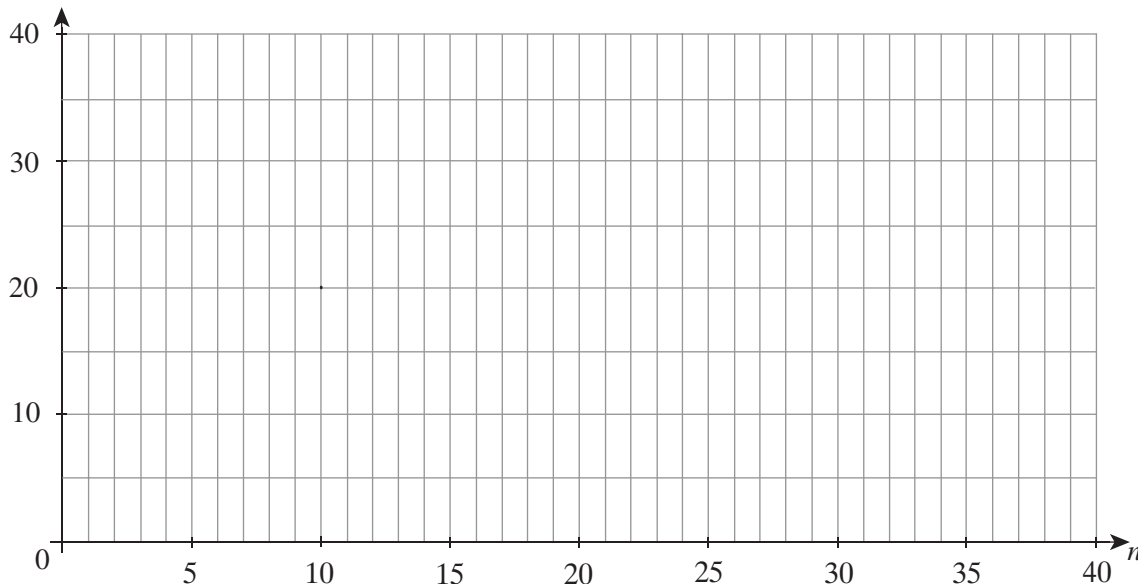
$$y \leq 2x$$

$$x + y \leq 30$$

$$x \geq 9$$

$$y \geq 0$$

- b. Sketch these constraints on the grid shown below.



3 marks

Trevor is paid \$20 per hour while Jane is paid \$24 per hour.

- c. Write an equation for C , the total weekly wages.

1 mark

- d. Mark all key points on the graph with labels A to D and determine the objective (wage cost) value at each of these points.

2 marks

- e. Hence determine the optimal (cheapest) means by which Danilyn can teach all the required junior classes according to the conditions outlined above. Specify the resulting total weekly wages bill.

1 mark

Question 2

Danilyn is considering the profitability of the pool in which the classes are conducted. She pays \$1000 per week for the use of the pool and there are usage costs of \$15 per hour for such requirements as electricity and wages. When in use however, the pool makes \$50 per hour in revenue.

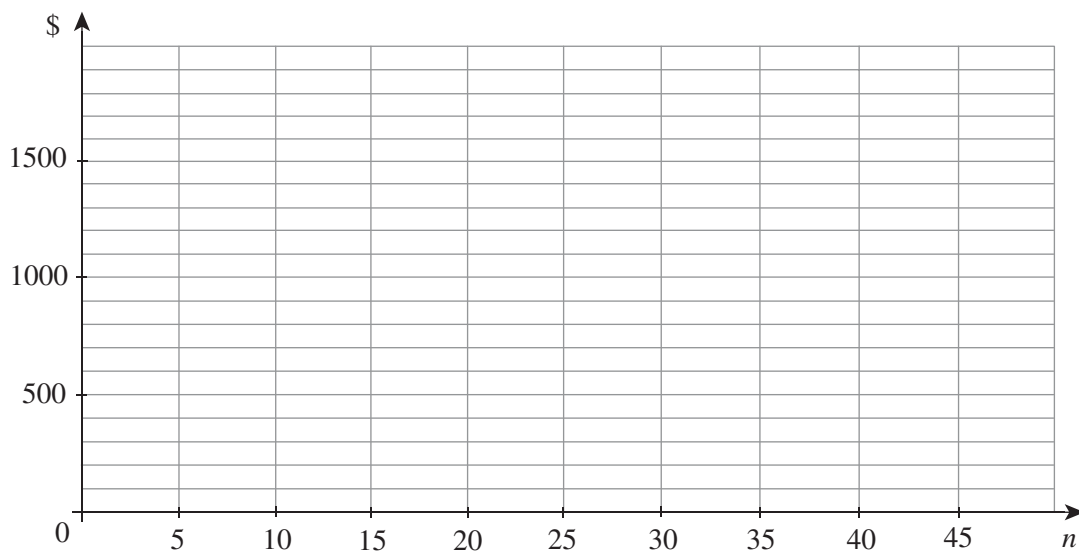
- a. Write an equation for total cost, C , of n hours of operation per week.

1 mark

- b. Write an equation for revenue, R , of n hours of operation per week.

1 mark

- c. Sketch both C and R on the axes below and label the break even point.



2 marks

- d. Determine algebraically the minimum number of hours that the pool must be booked each week in order to make a profit. The pool must be booked for a whole number of hours.

2 marks

Total 15 marks

END OF MODULE 3

Module 4: Business-related mathematics**Question 1**

Jacko manufactures and supplies, direct to the public, treated pine sleepers for use in building retaining walls.

- a. The logs cost him \$5 each and the treatment costs a further \$2.50 each. He sells them for \$13.90 each. What is his percentage profit (to 1 decimal place)?

1 mark

- b. The machine Jacko uses to treat the pine logs costs him \$180 000 to purchase. He took out a reducing balance business loan for \$170 000. The interest rate is 9.2% per annum and is added quarterly. He wishes to pay the loan out over ten years. Calculate his quarterly repayment.

2 marks

- c. The machine will be depreciated over a ten year period. Find the scrap value after ten years if the machine is depreciated at
- i. 12% per annum reducing value.

- ii. 8% per annum flat rate.

2 + 2 = 4 marks

Question 2

George is buying a house and needs to borrow \$210 000 to make his purchase.

He is offered a housing loan at 8.4% per annum compounding monthly over 20 years.

- a.** If he makes monthly repayments of \$1900, how much will he still owe after 10 years?

1 mark

- b.** If he makes fortnightly repayments of \$950, how much will he still owe after 10 years?

1 mark

- c.** Explain why the answers to parts **a** and **b** are different.

1 mark

Question 3

An antique ring was purchased as an investment for \$32 000 and sold three years later for \$41 500. Over this period the average rate of inflation was 2.3% per annum.

- a.** Calculate the profit (or loss) after allowing for inflation.

2 marks

- b.** The original owner of the ring (the person who purchased it for \$32 000) entered into a hire purchase agreement with a 20% deposit and 60 monthly payments of \$600.

- i.** Calculate the flat rate of interest.

2 marks

- ii.** Calculate the effective rate of interest.

1 mark

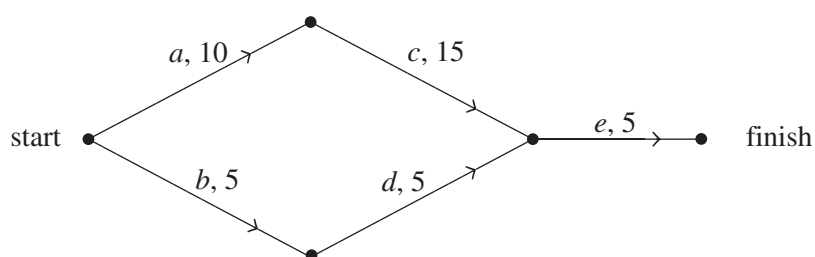
END OF MODULE 4

Module 5: Networks and decision mathematics**Question 1**

Carly is the manager of Cuppa café. Every morning she completes various activities prior to opening the café. The activities, their immediate predecessor(s) and their duration, in minutes, are shown in the table below.

Activity	Immediate predecessor(s)	Duration (minutes)
<i>a</i>	–	10
<i>b</i>	–	5
<i>c</i>	<i>a</i>	15
<i>d</i>	<i>b</i>	5
<i>e</i>	<i>c, d</i>	5

The network graph that describes this project is shown below.



A new activity, activity *f*, is required to complete the project.

Activity	Immediate predecessor	Duration (minutes)
<i>f</i>	<i>b</i>	10

- a. Draw and label activity *f* on the network graph above.

1 mark

- b. Determine the minimum time required by Carly to complete all activities prior to opening the café.

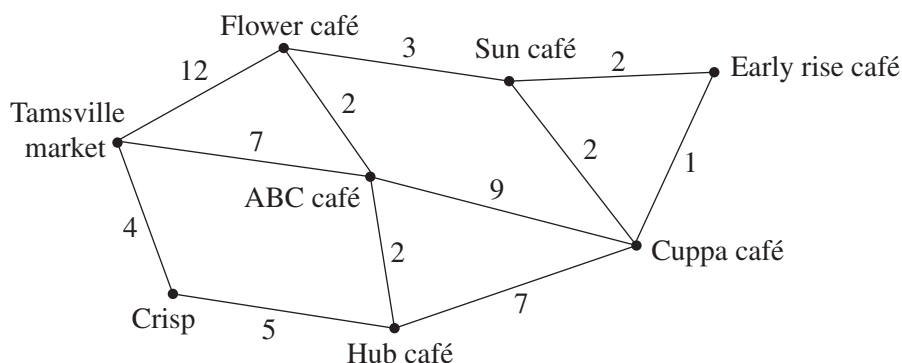
1 mark

- c. Name one activity that is **not** on the critical path.

1 mark

Question 2

Ken is a delivery driver who supplies cafés each morning. His day begins at Tamsville market. The distances between the market and various cafés, in kilometres, are shown in the network graph below.



- a. Determine the length of the shortest path between Tamsville market and Cuppa café.

1 mark

- b. Ken intends to begin at Tamsville market, visit every café once only and finish the journey at Cuppa café.

- i. What term describes this type of journey?

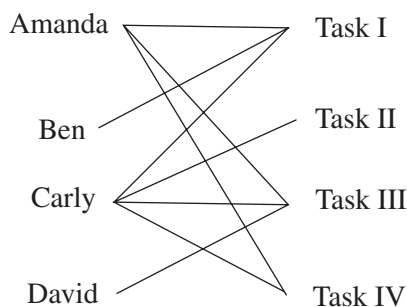
- ii. Which café must Ken visit first after leaving Tamsville market?

1 + 1 = 2 marks

Question 3

Carly employs Amanda, Ben and David to work in the café.

Various tasks require completion throughout the day according to the bi-partite graph below.



According to the bi-partite graph:

Amanda must be allocated to Task

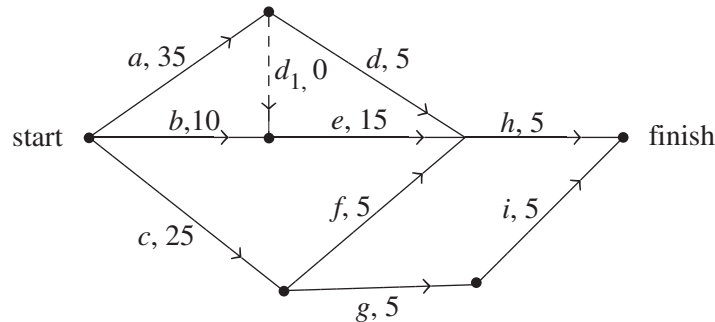
Carly must be allocated to Task

2 marks

Question 4

Various activities are performed each day by Carly, Amanda, Ben and David, such as preparing sandwiches, foccacias, salads, coffee etc.

Their activities, their immediate predecessor(s) and their duration, in minutes, are shown in the network graph below.



- a. Determine the earliest start time for activity *h*.

1 mark

- b. Determine the latest start time for activity *f*.

1 mark

- c. List the activities on the critical path.

1 mark

- d. Using your answer to part c above, determine the project completion time.

1 mark

Carly is able to reduce the completion time of one activity in this project. She is prepared to pay extra to reduce this activity. It will cost Carly \$11 per minute to reduce the activity.

Individually reducing the duration of some activities will have no effect on the project completion time.

Other activities may be reduced and will result in a reduced project completion time, but will **not** result in the minimum project completion time.

- e. Determine the activity that, if reduced, will result in the **minimum** possible project completion time.

1 mark

- f.** How much money should Carly spend reducing the chosen activity? Justify your answer.

2 marks

Total 15 marks

END OF MODULE 5

Module 6: Matrices**Question 1**

Flynow is a flight booking system that offers discount flights for members. They offer an Asian discount sale. Members are permitted a return flight to each of four destinations. The standard prices to Hong Kong, Manila, Shanghai and Bangkok are as shown in the table below for Economy, Business and First Class passengers.

	Economy	Business	First Class
Hong Kong	1650	2170	2980
Manila	1230	1610	2130
Bangkok	1710	2430	2750
Shanghai	1710	2160	3040

- a. Produce a 3×4 matrix, A , to include this data (one column for each destination).

1 mark

- b. Matrix $B = \begin{bmatrix} 0.92 & 1.00 & 0.91 \\ 0.90 & 0.98 & 0.96 \\ 0.95 & 0.79 & 0.92 \\ 0.92 & 0.96 & 0.98 \end{bmatrix}$ gives the discounts for each of three possible membership types,

Q , R and S (a row for each destination).

- i. Calculate the matrix product AB .

1 mark

- ii. Describe the nature of the information that matrix AB provides. Describe what each row represents and what each column represents and give an example for one matrix element.

2 marks

- iii. Would the matrix product BA be defined and meaningful? If so, explain what its physical significance would be.

2 marks

Question 2

The number of economy flight seats available each year depends on the number available from the previous year only as follows:

- The flights to Manila in any year increase by 30% but are reduced by 10% of Bangkok flights from the preceding year.
 - The flights to Hong Kong in any year increase by 20% but are reduced by 20% of Manila flights from the preceding year.
 - The flights to Bangkok in any year increase by 10% but are reduced by 10% and 20% respectively of Manila and Hong Kong flights from the preceding year.
- a. Write a transition matrix that determines the annual transition if the initial state matrix is in

the form $\begin{bmatrix} \text{Manila} \\ \text{Hong Kong} \\ \text{Bangkok} \end{bmatrix}$.

2 marks

Due to an airline amalgamation, a new transition matrix, A , is required. It is found to be

$$A = \begin{bmatrix} 1.2 & 0 & -0.1 \\ -0.1 & 1.2 & 0 \\ -0.1 & -0.2 & 1.1 \end{bmatrix}$$

Flynow want to produce a matrix B so that AB gives the new ticket prices after certain discounts are applied. The resulting matrix is

$$B = \begin{bmatrix} 0.91 & 0 & 0 \\ 0 & 0.95 & 0 \\ 0 & 0 & 0.92 \end{bmatrix}$$

- b. Calculate the matrix product AB .

1 mark

It is known that the proportions in 2009 were given by the matrix

$$S_0 = \begin{bmatrix} 0.4 \\ 0.35 \\ 0.25 \end{bmatrix}$$

- c. Assuming that the transition matrix A was also applicable for years preceding 2009, determine the proportions in 2008. Express the answer to four decimal places.

2 marks

Flynow want to determine the likely long term proportions for each of these three destinations.

- d. Find these long-term proportions to two decimal places or show that such a steady state does not exist. What can be said about the use of this transition matrix for future years?

2 marks

- e. Using the original prices of economy flights given earlier (ignoring Shanghai flights) and using these 2010 proportions, determine the mean price of economy flights to these three destinations. Matrix methods must be used.

2 marks

Total 15 marks

END OF QUESTION AND ANSWER BOOKLET