

2010

VCE Further Mathematics Trial Examination 2

Suggested Solutions

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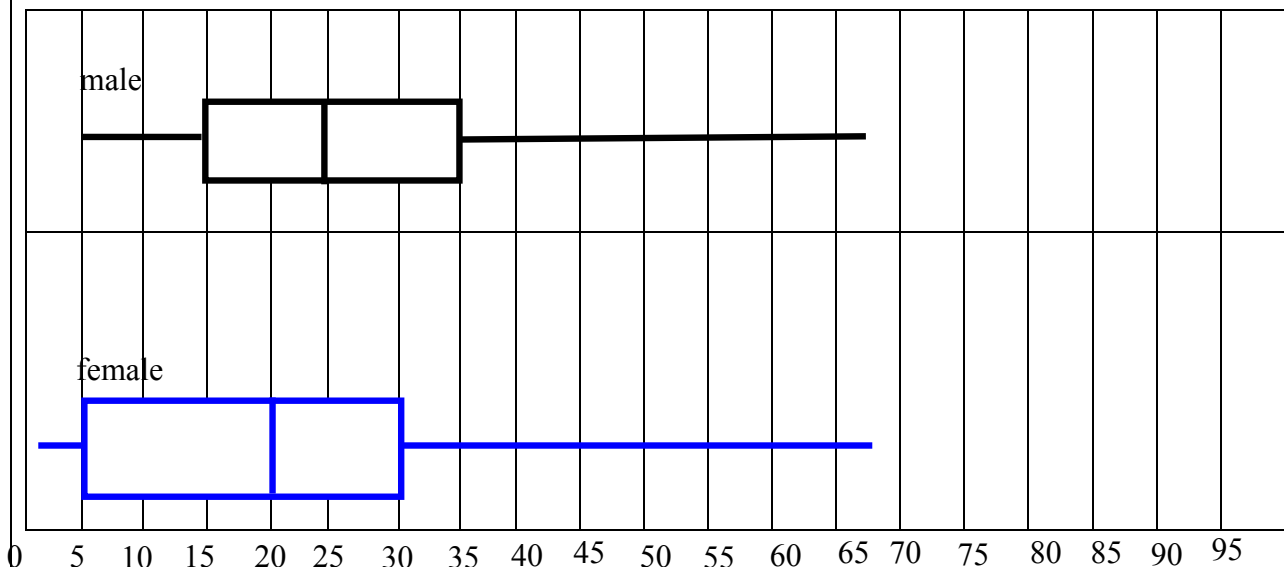
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Core**Question 1****a.**Using TI-83 calculator press stat edit and enter the data for females in L_1 Press stat calc 1-Var stats enter L_1 enter. This gives $\bar{x} = 21.5$

(1 mark)

b. The median, lower and upper quartile, maximum and minimum values can be obtained from the same screen as **1 a.**

1 mark for correct upper and lower quartile

1 mark for correct median and whiskers

(2 marks)

c.An outlier is a value that is greater than $1.5 \times IQR + Q_3$ For female $IQR = 25$ and $Q_3 = 30$

$$1.5 \times IQR + Q_3 = 1.5 \times 25 + 30 = 67.5$$

66 < 67.5 so 66 is not an outlier for females.

For male $IQR = 20$ and $Q_3 = 35$

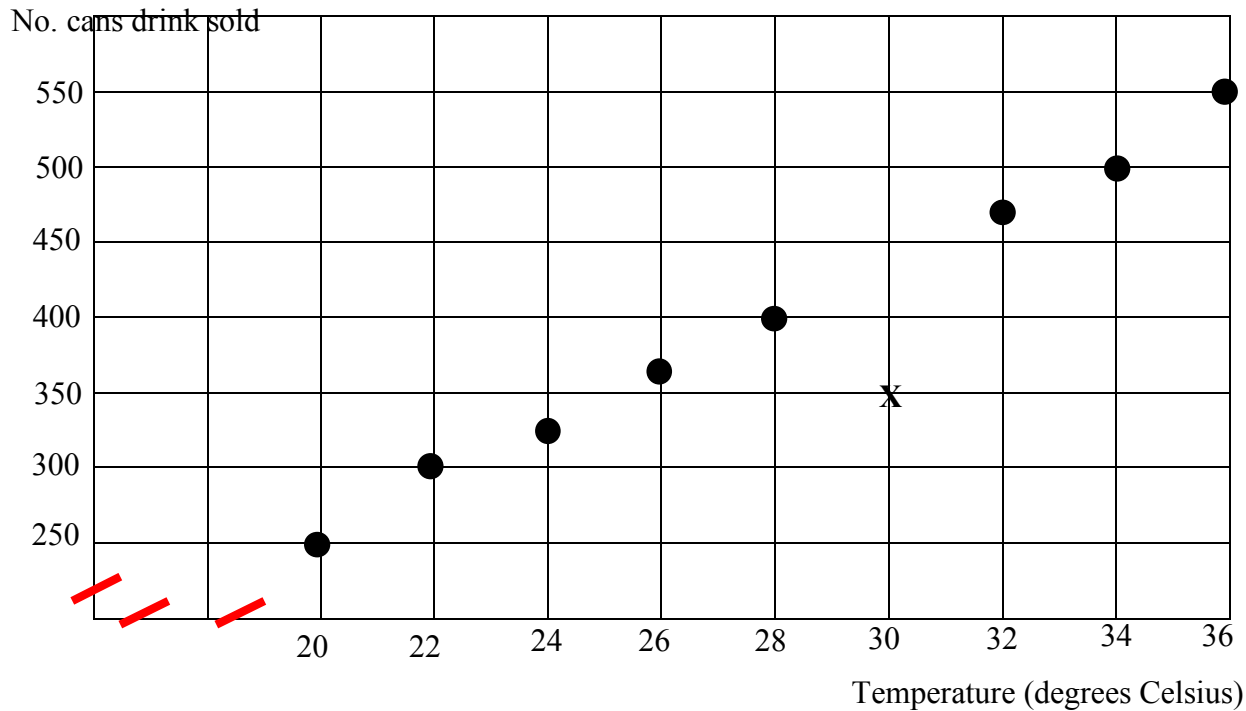
$$1.5 \times IQR + Q_3 = 1.5 \times 20 + 35 = 65$$

66 > 65 so 66 is an outlier for males.

1 mark for using definition of an outlier

1 mark for correct answer

(2 marks)

Core**Question 2****a.**

(1 mark) for correct point

b.

On calculator press stat edit and enter temperature in L_1 and number of drinks sold in L_2 . Go to stat calc LinReg ($ax + b$) $L_1 L_2$. This gives $r^2 = 91.6\%$, to one decimal place

91.6% of the variation in the number of drinks sold can be explained by the variation in the temperature.

1 mark for each of the three answers.

(3 marks)

c.

Using the same screen as for b. $a = 17.25$ and $b = -93$

So equation is

$$\text{Number of drinks sold} = -93 + 17.25 \times \text{temperature}$$

1 mark for each of the two values.

(2 marks)

Core**Question 2 (continued)****d.**

$$440 = -93 + 17.25 \times t$$

$$533 = 17.25 \times t$$

$$t = \frac{533}{17.25} = 30.9^{\circ}C$$

(1 mark)

Question 3**a.**

$$\frac{1}{26} = 0.04$$

(1 mark)

b.

On calculator press stat edit. Put the temperature values in L₁, and the number of hot dogs sold in L₂

In L₃ put $L_3 = \frac{1}{L_1}$

Go to stat calc LinReg($ax + b$) L₃,L₂

This gives $a = 2290$ and $b = 15$

$$\text{Number of hot dogs sold} = 2290 \times \frac{1}{\text{temperature}} + 15$$

(1 mark)

c.

$$\text{Number of hot dogs sold} = 2290 \times \frac{1}{15} + 15 = 168$$

(1 mark)

Module 1 Number patterns and applications

Question 1

a(i) $25 + 12 + 12 = 49$ (1 mark)	a(ii) This is an arithmetic sequence with $a = 25$ and $d = 12$ $t_n = a + (n - 1)d$ $t_n = 25 + (n - 1)12$ $t_n = 25 + 12n - 12$ $t_n = 13 + 12n$ (1 mark)
b. This is an arithmetic sequence $S_n = \frac{n}{2}[2a + (n - 1)d]$ $S_{16} = \frac{16}{2}[2 \times 25 + 15 \times 12]$ $S_{16} = 8 \times (50 + 180) = 1840 \text{ people.}$ (1 mark)	

Module 1 Number patterns and applications**Question 2**

a. i. $5 \times (1 + 0.4) = 7$ (1 mark)	a. ii. This is a geometric sequence with $a = 5$ and $r = 1.4$ $t_n = ar^{n-1}$ $t_n = 5(1.4)^{n-1}$ (1 mark)
b. This is a geometric sequence with $a = 5$, $n=10$ and $r = 1.4$ $S_n = \frac{a(r^n - 1)}{r - 1}$ $S_{10} = \frac{5(1.4^{10} - 1)}{1.4 - 1} = 349$ (1 mark)	
c. Enter two equations into calculator in sequence mode. Press $y =$ $nMin = 1$ $\mu(n) = 2n + 13$ $\mu(nMin) = 25$ $v(n) = 5(1.4)\Lambda(n - 1)$ $v(nMin) = 5$ Press 2nd table When $n = 11, \mu(n) > v(n)$ But when $n = 12, v(n) > \mu(n)$ Hence it will take 12 days. (1 mark)	

Module 1 Number patterns and applications**Question 2 (continued)****d.**

$$\text{Number infected} = 25 + 40 + 1.2 \times 40 + (1.2)^2 \times 40 + \dots$$

$$(1.2)^{18} \times 40$$

Number infected = 65 + sum of geometric sequence

where $a = 1.2 \times 40$, $r = 1.2$ and $n = 18$

$$\text{Number infected} = 65 + \frac{1.2 \times 40(1.2^{18} - 1)}{1.2 - 1}$$

$$\text{Number infected} = 65 + 6150 = 6215$$

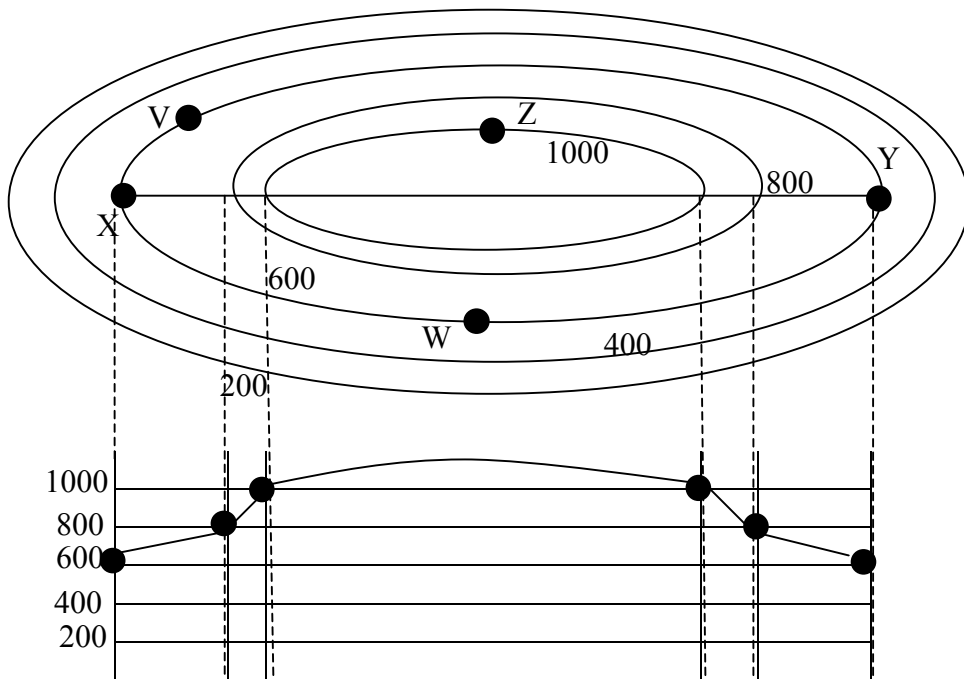
(1 mark)

Question 3

a. i. In one lot of 20 minutes, Number bacteria = double 4 = 8 (1 mark)	a. ii. 3 lots of 20 minutes in 1 hour. 4,8,16,32 32 bacteria after one hour. (1 mark)
b. 32 is 8 times greater than 4, so 8 times. (1 mark)	c. $B_t = 8B_{t-1} \quad B_0 = 4$ (1 mark)

Module 1 Number patterns and applications**Question 3 (continued)**

d. Use calculator in sequence mode Press $y =$ $nMin = 1$ $\mu(n) = 8 \times \mu(n - 1)$ $\mu(nMin) = 4$ Press 2nd table and go to $n = 6$. This is 5 hours later. $\mu(n) = 131,072$ (1 mark)	e. Using the same computer table as in d, the answer can be seen to be 4 hours. (1 mark)
f. 3600 killed in 1 hour. From table for d, when time is 4 hours, number bacteria = 16384 Number of hours to eliminate = $\frac{16384}{3600} = 4.6$ hours to 1 d.p. (1 mark)	

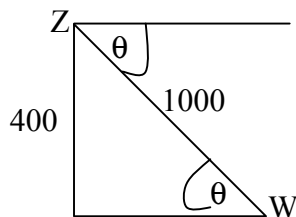
Module 2 Geometry and trigonometry**Question 1****a.**

(1 mark)

b.

W and Y are on the same contour line so there is no gradient. Gradient = 0

(1 mark)

c.

$$\sin \theta = \frac{400}{1000}$$

$$\theta = \sin^{-1}\left(\frac{400}{1000}\right) = 24^{\circ} \text{ to the nearest degree.}$$

(1 mark)

Module 2 Geometry and trigonometry**Question 1 (continued)****d.**

$$\text{Gradient} = \tan \theta = \tan 23.57817848 = 0.44$$

(1 mark)

Question 2**a.**

$$\angle X VW = 20^\circ$$

$$\angle VWX = 70^\circ \text{ (angles in triangle sum to } 180^\circ \text{)}$$

$$\angle VWY = 180 - 70 = 110^\circ$$

(angles in straight line add to 180°)

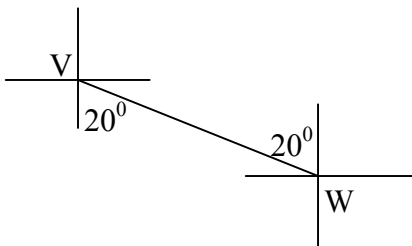
(1 mark)

b.Since triangle VWY is isosceles

$$\angle WVY = \angle WYV$$

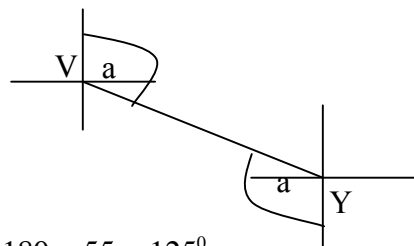
$$\therefore \angle WVY = \frac{1}{2}(180 - 110) = 35^\circ$$

(1 mark)

c.

$$\text{Bearing of V from W} = 360 - 20 = 340^\circ$$

(1 mark)

d.

$$a = 180 - 55 = 125^\circ$$

(1 mark)

Module 3 Graphs and relations**Question 3**

a. $\frac{1}{2} \times 18 = 9 \text{ m}$ (1 mark)	b. $\text{Surface area} = \frac{1}{2}(2\pi r^2 + 2\pi rh)$ $\text{Surface area} = \frac{1}{2}(2\pi \times 9^2 + 2\pi \times 9 \times 22)$ $\text{Surface area} = 876.5 \text{ m}^2$ (1 mark)
c. $V = \text{Volume of cuboid} + \text{volume of half cylinder}$ $V = 18 \times 22 \times 12 + \frac{1}{2} \times \pi \times 9^2 \times 22$ $V = 7551.16 \text{ m}^3$ (1 mark)	d. $\text{Heating cost/hour} = 7551.16 \times \frac{0.02}{100}$ $\text{Heating cost/hour} = \1.51 $\text{Heating cost/day} = \36.25 (1 mark)

Question 4

a. There are 8 equal angles at the centre that all add to 360°. $\text{Each angle} = \frac{360}{8} = 45^\circ$ $\angle AOB = 45^\circ$ $\triangle AOB$ is isosceles $\therefore \angle OAB = \frac{1}{2}(180 - 45) = 67.5^\circ$ (1 mark)	b. $\frac{AO}{\sin 67.5^\circ} = \frac{20}{\sin 45^\circ}$ $AO = \frac{20}{\sin 45^\circ} \times \sin 67.5^\circ = 26.13 \text{ m}$ (1 mark)
c. Area of Piazza = $8 \times \text{Area of triangle } AOB$ $\text{Area of Piazza} = 8 \times \frac{1}{2} \times 26.13^2 \times \sin 45^\circ$ $\text{Area of Piazza} = 1931.18 \text{ m}^2$ (1 mark)	

Module 3 Graphs and relations**Question 1**

a. i. b. $65 \times 10 = \$650$ (1 mark)	a. ii. $65 \times x = \$65x$ (1 mark)
b. $C = \$(20x + 37,890)$ (1 mark)	c. $65x = 20x + 37,890$ $45x = 37,890$ $x = 842$ (1 mark)
d. $37890 - 3789 = \$34,101$ (1 mark)	e. $SP = 65 \times 30000 = \$1,950,000$ $CP = 40 \times 30000 + 3789 = \$1,203,789$ $CP < SP$ $\therefore$ Profit $\text{Profit} = 1,950,000 - 1,203,789$ $\text{Profit} = \$746211$ Profit (1 mark) Actual profit (1 mark)

Module 3 Graphs and relations**Question 2****a.**

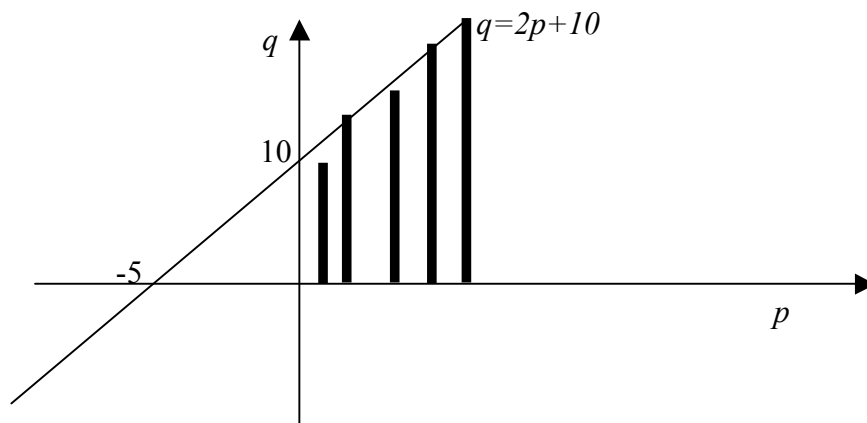
$$p + q \leq 100$$

(1 mark)

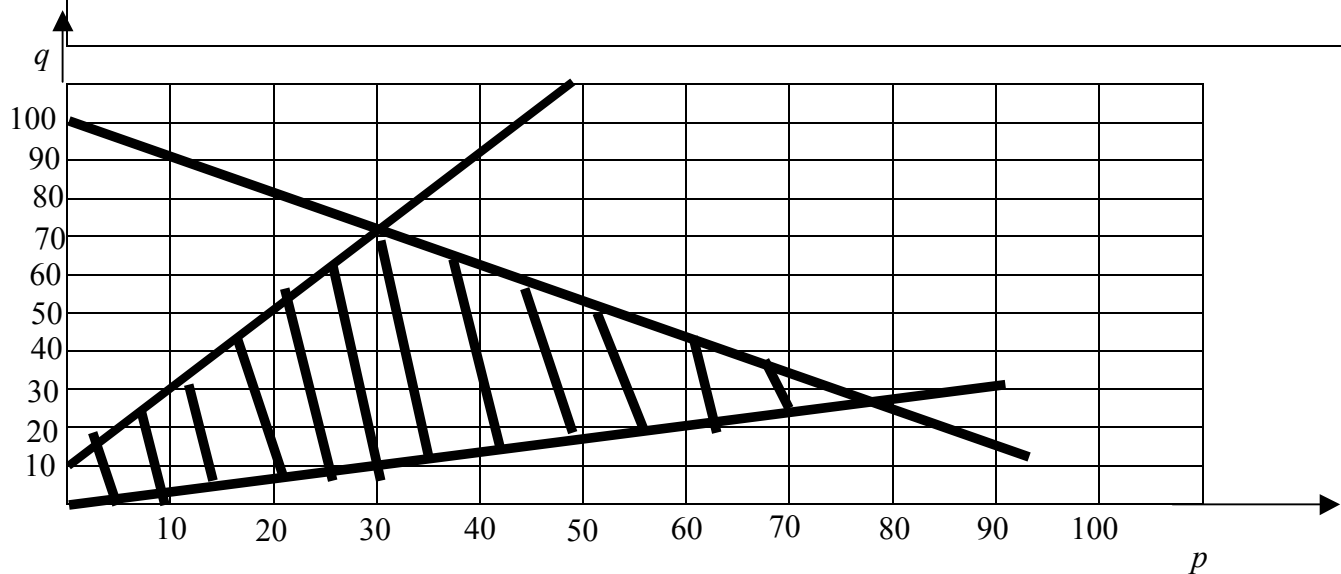
b.

$$p \leq 3q$$

(1 mark)

c.

(1 mark)



1 mark for each of the two lines and 1 mark for correct shading

(3 marks)

Module 3 Graphs and relations**Question 2 (continued)****e.**

$$\text{Profit} = 30p + 21q$$

(1 mark)

f.

The corner points are (0,0), (0,10), (30,70) and (75,25)

Profit at (0,0) = \$0

Profit at (0,10) = $21 \times 10 = \$210$

Profit at (30,70) = $30 \times 30 + 21 \times 70 = \2370

Profit at (75,25) = $30 \times 75 + 21 \times 25 = \2775

$\therefore$ make 75 units of glasswork and 25 units of plastic.

(1 mark)

Module 4 Business-related mathematics**Question 1**

a. $\frac{48}{25} = \$1.92$ (1 mark)	b. Increase = $57 - 48 = \$9$ % increase = $\frac{9}{48} \times 100 = 18.75\%$ (1 mark)
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Question 2

a. Total earned = $280 + \frac{12}{100} \times 1300 = \436 (1 mark)	b. Commission = $464.50 - 150 = 314.50$ Let percentage be $x\%$ $\frac{x}{100} \times 1850 = 314.50$ $x = 314.50 \div 1850 \times 100 = 17\%$ (1 mark)
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Question 3

a. Stamp duty = $60 + \frac{4}{100} (45000 - 3000) = \1740 (1 mark)	b. i. There are 60 months in 5 years. Instalments paid = $950 \times 60 = \$57,000$ (1 mark)
b. ii. Total amount paid = $57,000 + 5000 = \$62,000$ (1 mark)	b. iii. Interest = $57,000 - 45,000 = \$12,000$ (1 mark)
b. iv. $R = \frac{100I}{Pt} = \frac{100 \times 12000}{45000 \times 5} = 5.3\%$ (1 mark)	

Module 4 Business-related mathematics**Question 4**

a. $\frac{6}{12} = 0.5\%$ (1 mark)	b. $\frac{0.5}{100} \times 50000 = \250 (1 mark)
c. $50000(1.005)^{60} = \$67442.51$ (1 mark)	d. Press Apps Finance Enter TVM Solver Enter $N = 60$ $I = 6$ $PV = 50000$ $PMT =$ $FV = 0$ $P / Y = 12$ $C / Y = 12$ End Put cursor on PMT and press alpha solve. $PMT = \\$966.64$ (1 mark)

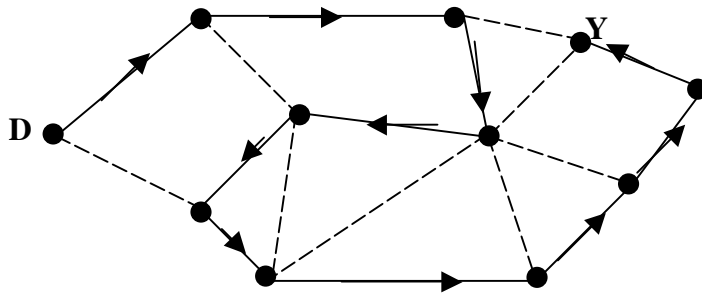
Question 5

a. Amount depreciation = $45000 - 18800$ Amount depreciation = 26200 Cost/km = $\frac{26200}{42000} = 0.62 = 62 \text{ cents.}$ (1 mark)	b. Depreciation = $14000 \times \frac{62}{100} = 8733.33$ Value after two years = $18,800 - 8733.33$ Value after two years = \$10,066.67 (1 mark)
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Module 5 Networks and decision mathematics

Question 1

a. i.



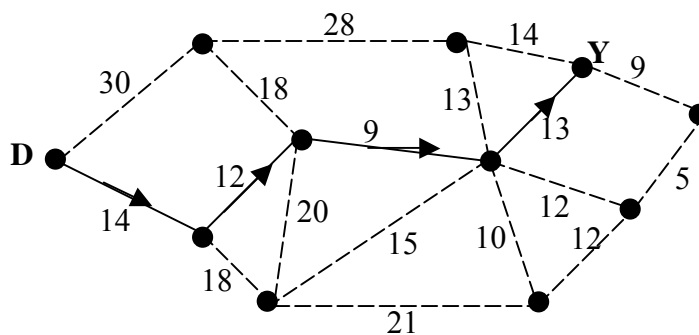
(1 mark)

a. ii.

This is a Hamiltonian path.

(1 mark)

b. i.

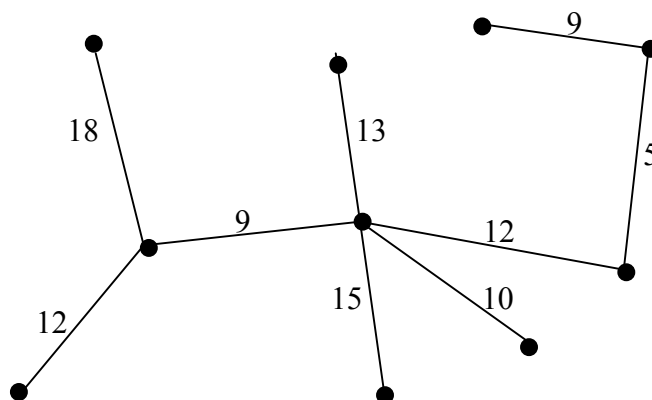


(1 mark)

b. ii.

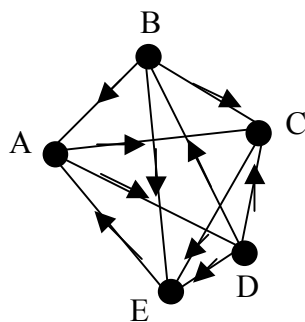
$$14 + 12 + 9 + 13 = 48 \text{ km.}$$

(1 mark)

Module 5 Networks and decision mathematics**Question 1 (continued)****c.**

$$18 + 12 + 9 + 15 + 10 + 12 + 5 + 13 + 9 = 103 \text{ km.}$$

(1 mark)

Question 2**a.**

(1 mark)

b.

$$D^1 = \begin{matrix} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

(1 mark)

c.

Yacht	One – Step Dominance
<i>Alpha Star</i>	2
<i>Battering Ram</i>	3
<i>Challenger</i>	1
<i>Defending Champ</i>	3
<i>Earnest Endeavour</i>	1

(1 mark)

d.*Battering Ram and Defending Champ*

(1 mark)

Module 5 Networks and decision mathematics**Question 2 (continued)****e.**

$$D^2 = \begin{matrix} & A & B & C & D & E \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 2 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

(1 mark)

f.

Yacht	Two – Step Dominance
<i>Alpha Star</i>	4
<i>Battering Ram</i>	4
<i>Challenger</i>	1
<i>Defending Champ</i>	5
<i>Earnest Endeavour</i>	2

(1 mark) for any two correct.
(2 marks) for all five correct**g.** $D^1 + D^2$ gives

$A = 2 + 4 = 6$

$B = 3 + 4 = 7$

$C = 1 + 1 = 2$

$D = 3 + 5 = 8$

$E = 1 + 2 = 3$

Place	Yacht
1	<i>Defending Champ</i>
2	<i>Battering Ram</i>
3	<i>Alpha Star</i>
4	<i>Earnest Endeavour</i>
5	<i>Challenger</i>

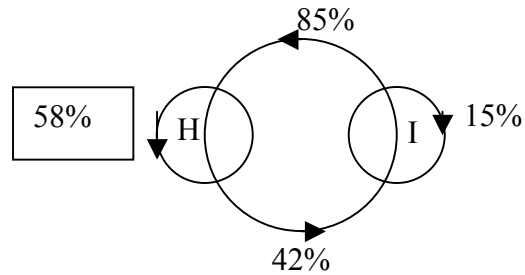
(1 mark) for 2 correct
(2 marks) for all correct.**h.**

If B defeated A, then A defeated C, D and E. This means that E defeated B, C and D.
Hence, D defeated B and C. So C defeated B i.e. *Battering Ram*.

(1 mark)

Module 6 Matrices**Question 1**

a. $A = \begin{bmatrix} 8 & 2 \\ 3 & 12 \end{bmatrix}$ (1 mark)	b. i. $\begin{bmatrix} 8 & 2 \\ 3 & 12 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 5 & 1 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ (1 mark)
b. ii. $\begin{bmatrix} 20 & 8 \\ 13 & 25 \end{bmatrix}$ (1 mark)	c. 8 is the number in the first row, second column. It represents the number of behinds obtained by Hawood. (1 mark)
d. $\begin{bmatrix} 6 \\ 1 \end{bmatrix}$ (1 mark)	e. $\begin{bmatrix} 20 & 8 \\ 13 & 25 \end{bmatrix} \times \begin{bmatrix} 6 \\ 1 \end{bmatrix}$ (1 mark) for correct matrices (1 mark) for multiplication.
f. i. $\begin{bmatrix} 120 + 8 \\ 78 + 25 \end{bmatrix} = \begin{bmatrix} 128 \\ 103 \end{bmatrix}$ (1 mark)	f. ii. 2×1 (1 mark)

Module 6 Matrices**Question 2****a. i.**

(1 mark)

a. ii.

This Week

 $H \quad I$

$$\begin{bmatrix} 0.58 & 0.85 \\ 0.42 & 0.15 \end{bmatrix} \begin{matrix} H \\ I \end{matrix} \text{ Next Week}$$

(1 mark)

b.

$$S_0 = \begin{bmatrix} 24 \\ 10 \end{bmatrix}$$

(1 mark)

c.

$$S_2 = T^2 S_0$$

$$S_2 = \begin{bmatrix} 0.58 & 0.85 \\ 0.42 & 0.15 \end{bmatrix}^2 \begin{bmatrix} 24 \\ 10 \end{bmatrix} = \begin{bmatrix} 22.8466 \\ 11.1534 \end{bmatrix}$$

Expect 11 to be injured.

(1 mark)

d.

$$S_{100} = T^{100} S_0$$

$$S_{100} = \begin{bmatrix} 22.76 \\ 11.24 \end{bmatrix}$$

Expect about 23 players to be healthy in the long term.

(1 mark)

Module 6 Matrices**Question 3**

Let number of people who bought beef, chicken, vegetable and curry pies last week be x , $3x$, x and x , respectively.

$$\begin{bmatrix} 0.6 & 0.1 & 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} x \\ 3x \\ x \\ x \end{bmatrix} = [0.6x + 0.3x + 0.1x + 0.2x]$$

$$= [1.2x]$$

$$1.2x = 12,000$$

$$x = 10,000$$

$$6x = 60,000$$

(1 mark)

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