

2010 Further Mathematics Trial Exam 2 Solutions

Free download from www.itute.com © Copyright 2010 itute.com

Core – Data analysis

Q1a

Subject	Number
English	10
Literature	2
Chinese	4
Indonesian	2
Legal	3
Accounting	3
Music	2
Psychology	2
Biology	3
Chemistry	4
Physics	4
Further Maths	4
Specialist Maths	2
Math Methods	6

Q1b Students who have chosen Chemistry or Math Methods or both are B, C, D, E, G and I, i.e. 6 out of 10 students.

$$\frac{6}{10} \times 100\% = 60\%$$

Q1c If 'Number' stands for the number of times that a subject is chosen, then it is a frequency distribution table. If it stands for the number of students choosing a subject, then it is not.

Q2a 50 is the lower quartile (25th percentile) for Mathematics.
 $\therefore 75\%$ of 24 = 18 students pass the test.

Q2b 6 students fail English and 6 students fail Mathematics. It is possible that the same 6 students fail both subjects.
 $\therefore$ maximum 6 students failing both subjects.

Q2c English 25%

Q2d For Mathematics the marks are negatively skewed, $\therefore$ the mean is on the left of the median. For Science the marks are positively skewed and the mean is on the right of the median.
 $\therefore$ Mathematics and Science have approximately the same class average.

Q3a

2	3	4	5	6	7
675	710	805	950	900	850
662	730	822	885	900	902

$$\frac{675 + 710 + 805}{3} = 730, \quad \frac{950 + 900 + 850}{3} = 900$$

Q3b The 3-year moving average increases with time, i.e. shows an uptrend.

$$Q3c \text{ Number of workers} = 40 \times \text{year} + 617$$

$$Q3d \text{ Number of workers} = 40 \times 11 + 617 = 1057$$

Q4a Lower quartile = 45, upper quartile = 65
 $\therefore$ interquartile range = $65 - 45 = 20$ (20%)

Q4b

t	0	0	0.5	1	1.5	1.5	1.5	2	2.5
R	85	95	90	65	45	60	65	70	50
$\log_{10} R$	1.93	1.98	1.95	1.81	1.65	1.78	1.81	1.85	1.70

t	2.5	3	3	3.5	4	4.5	4.5	5	5.5
R	55	40	50	35	50	40	45	40	45
$\log_{10} R$	1.74	1.60	1.70	1.54	1.70	1.60	1.65	1.60	1.74

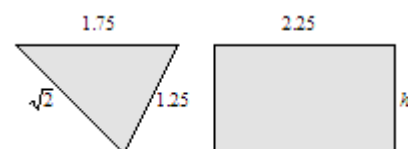
$$\log_{10} R = -0.058t + 1.889$$

$$Q4c \quad r^2 = 0.588 \approx 59\%$$

1

Module 2: Geometry and trigonometry

Q1a Split the cross-section:



Triangle:

$$s = \frac{\sqrt{2} + 1.75 + 1.25}{2} = 2.207$$

$$\text{Area} = \sqrt{s(s - \sqrt{2})(s - 1.75)(s - 1.25)} = 0.875$$

$$\text{Also, area} = \frac{1}{2} \times \text{base} \times \text{height}, \quad \frac{1}{2} \times 1.75 \times h = 0.875, \quad h = 1$$

Rectangle:

$$\text{Area} = 2.25 \times 1 = 2.25$$

$$\text{Total} = 0.875 + 2.25 = 3.125 \text{ m}^2$$

$$Q1b \text{ Volume} = 3.125 \times 15 = 46.875 \text{ m}^3$$

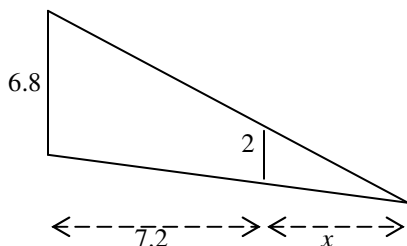
$$Q1c \text{ Dimension scale factor} = \frac{6}{4} = 1.5$$

$$\therefore \text{cross-sectional area} = 1.5^2 \times 3.125 = 7.03125 \text{ m}^2$$

$$\text{Volume} = 7.03125 \times 15 \approx 105.469 \text{ m}^3$$

$$\text{Q2a } \tan \theta = \frac{6.8-5.6}{7.2}, \theta = \tan^{-1}\left(\frac{6.8-5.6}{7.2}\right) = 9.46^\circ$$

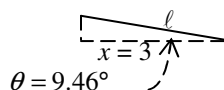
Q2b



Let x be the horizontal distance of the end of the shadow from the pole.

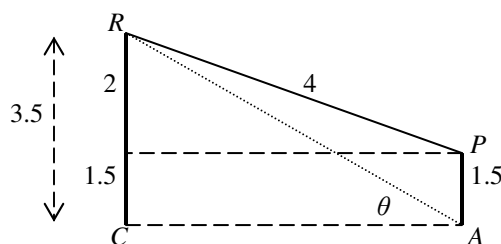
$$\frac{x}{7.2+x} = \frac{2}{6.8}, \therefore x = 3 \text{ m}$$

Q2c Let ℓ be the length of the shadow.



$$\frac{3}{\ell} = \cos 9.46^\circ, \therefore \ell = 3.04 \text{ m}$$

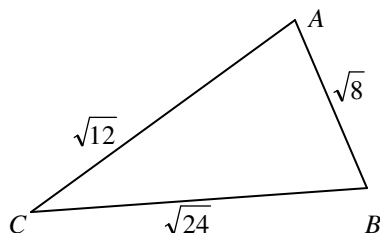
Q3a



$$\overline{CA} = \sqrt{4^2 - 2^2} = \sqrt{12}$$

$$\text{Angle of elevation: } \tan \theta = \frac{3.5}{\sqrt{12}}, \theta = \tan^{-1}\left(\frac{3.5}{\sqrt{12}}\right) = 45.3^\circ$$

Q3b



Using the same method in finding $\overline{CA}$, $\overline{AB} = \sqrt{8}$ and $\overline{BC} = \sqrt{24}$.

$$\text{The cosine rule: } \cos \angle BAC = \frac{(\sqrt{12})^2 + (\sqrt{8})^2 - (\sqrt{24})^2}{2\sqrt{12}\sqrt{8}} = \frac{-1}{\sqrt{24}}$$

$$\therefore \angle BAC = \cos^{-1}\left(\frac{-1}{\sqrt{24}}\right) = 101.8^\circ$$

Q3c $\overline{PQ} > \overline{AB}$, $\overline{QR} > \overline{BC}$ and $\overline{RP} > \overline{CA}$

$\therefore$ area of $\triangle PQR >$ area of $\triangle ABC$.

Module 3: Graphs and relations

Q1a Read from graph, cost = \$1.80

Q1b Sending the documents in two letters, 500 g and 100 g.
Cost = $3.00 + 1.20 = \$4.20$

Q1c

$$C = \begin{cases} 1.20, & 0 < w \leq 125 \\ 1.80, & 125 < w \leq 250 \\ 3.00, & 250 < w \leq 500 \end{cases}$$

Q2a 12 hours 25 minutes after 1:30 am, i.e. 1:55 pm.

Q2b Read from graph: About 8.2 hours in the first period and 8.8 hours in the second period. Total = $8.2 + 8.8 = 17$ hours.

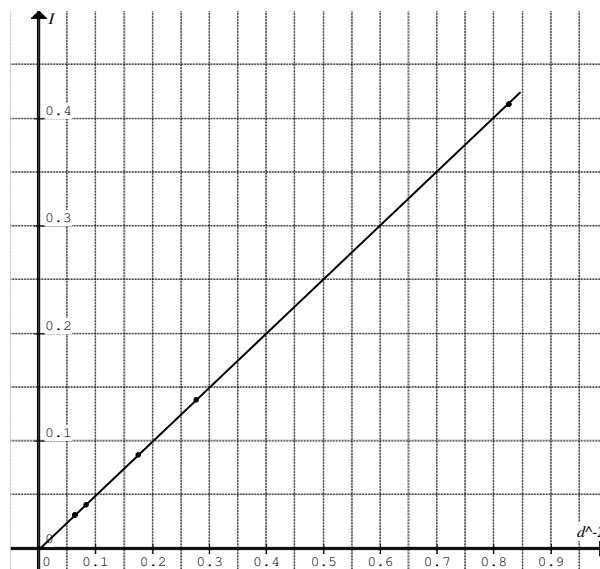
Q2c The steepest negative gradient of the graph is at $t = 15.5$ hours. Draw a tangent to the graph at $t = 15.5$.

$$\text{Gradient of the tangent} = \frac{7.5-3.5}{3} \approx 1.3$$

Fastest rate of decrease ≈ 1.3 metres per hour.

Q3a

d	1.1	1.9	2.4	3.5	4.0
d^2	0.826	0.277	0.174	0.082	0.063
I	0.413	0.138	0.087	0.041	0.031



Q3b I against d^{-2} is a straight line through the origin.

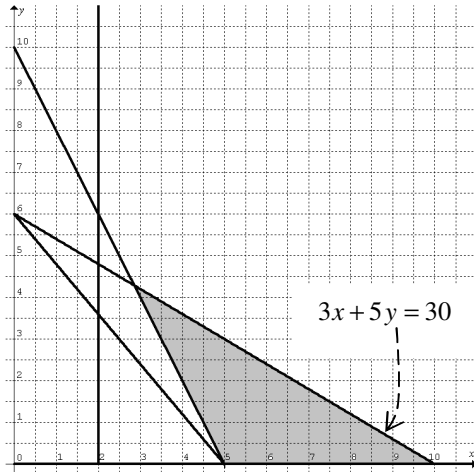
$$\therefore I \propto d^{-2}$$

The constant of proportionality is the gradient of the line

$$= \frac{0.413}{0.826} = 0.5$$

$$\therefore I = 0.5d^{-2}$$

Q4a



Q4b The whole number values of x and y that maximise $F = 3x + 5y$ are on the line segment $3x + 5y = 30$ in the feasible region.

$x = 5$ and $y = 3$ or $x = 10$ and $y = 0$

Module 4: Business-related mathematics

Q1a Discount = $18.95 - 13.00 = 5.95$

$$\% \text{ discount} = \frac{5.95}{18.95} \times 100\% \approx 31\%$$

Q1b Let $\$x$ be the discounted price without GST.

$\therefore$ discounted price including 10% GST = $1.1x = 13.00$

$\therefore x = 11.82$

GST = $13.00 - 11.82 = \$1.18$

Q2a Total amount = $10\% \times 1200 + 63 \times 24 = \1632

Interest = $1632 - 1200 = \$432$

Amount borrowed = $1200 - 10\% \times 1200 = \1080

$$\text{Flat rate of interest} = \frac{432}{1080 \times 2} = 0.20 = 20\% \text{ pa}$$

$$\text{Q2b Effective interest rate} = \frac{2n}{n+1} \times \text{flat rate}$$

$$= \frac{2 \times 24}{24+1} \times 20\% = 38.4\%$$

Q2c Total amount = $10\% \times 1200 + 29 \times 52 = \1628

Interest = $1628 - 1200 = \$428$

Principal = $\$1080$

$$\text{Flat rate of interest} = \frac{428}{1080 \times 2} = 0.198148 = 19.8148\% \text{ pa}$$

$$\text{Effective interest rate} = \frac{2 \times 52}{52+1} \times 19.8148\% = 38.8819\%$$

Extra interest paid effectively after 2 years

$$= 1080 \times 38.8819\% \times 2 - 1080 \times 38.4\% \times 2 = \$10.41$$

Q3a Brisbane has the highest CPI. $\therefore$ highest inflation.

Q3bi There was an increase of 3.1% from June 2009 to June 2010.

$\therefore 103.1\%$ of June quarter 2009 CPI = June quarter 2010 CPI, i.e. $103.1\% \times x = 172.1$, where x is the June quarter 2009 CPI.

$$\therefore x = \frac{172.1}{103.1\%} = 166.9$$

Q3bii Let r be the quarterly constant rate.

$$R = 1 + \frac{r}{100}$$

$$100.0 \times R^{80} = 172.1, R^{80} = 1.721, R = 1.0068,$$

$$\frac{r}{100} = 0.0068, r = 0.68$$

Q4a $12000 \times 10 = \$120000$

Q4b TVM Solver:

N = 10

I% = 7.5

PV =

PMT = -12000

FV = 0

P/Y = 1

C/Y = 1

Amount of initial investment = PV = $\$82369$

Q4c The initial investment PV must generate an annual interest of $\$12000$ to cover the $\$12000$ annuity.

$$\text{PV} = \frac{12000}{7.5\%} = \$160000$$

$\therefore$ the minimum amount is $\$160000$.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors