Further Mathematics

2010

Trial Examination 1

Core – Data analysis
Module 1 – Number patterns
Module 5 – Networks and decision mathematics
Module 6 – Matrices

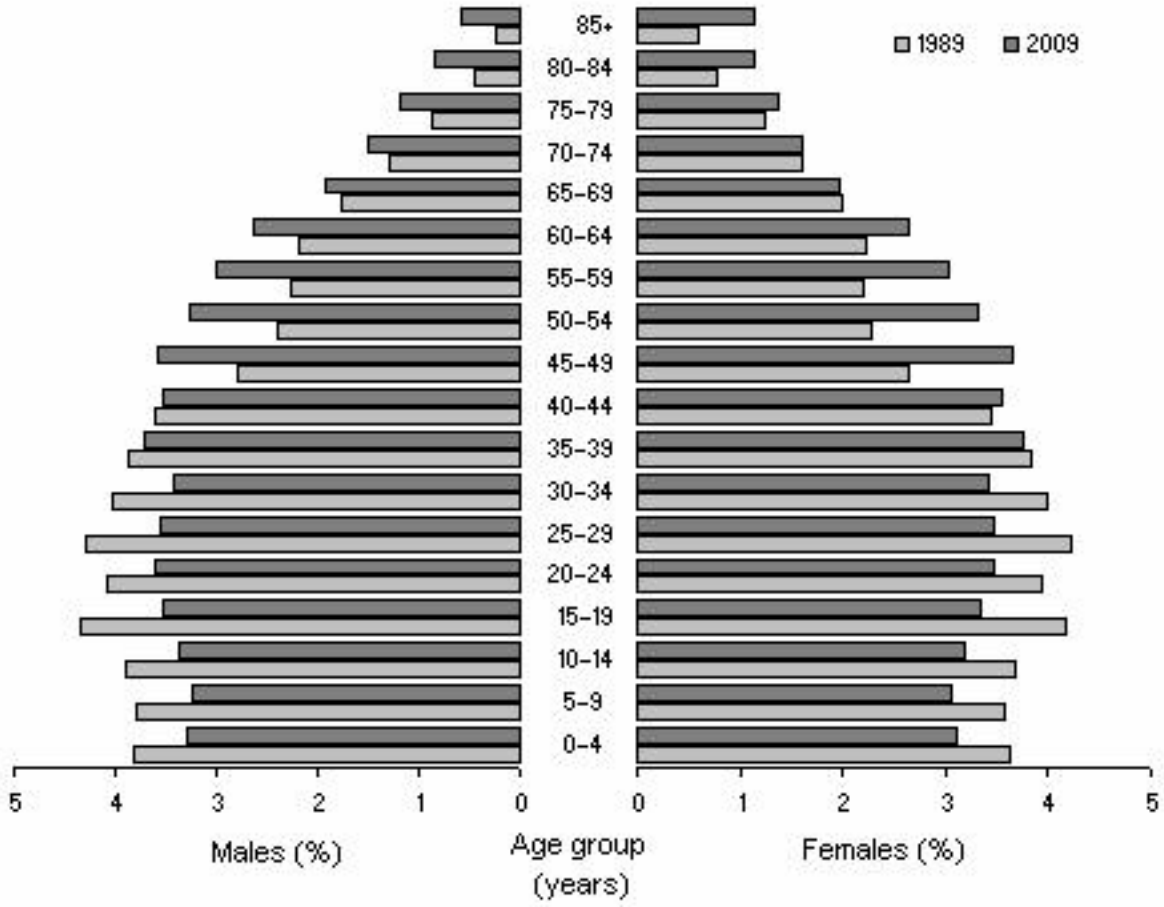
SECTION A Instructions

Answer **all** questions
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if **more than one** answer is completed for any question.

Core – Data analysis

The following information relates to Questions 1, 2 and 3

Population structure, Age and sex **-** Australia **-** 1989 and 2009



(http://www.abs.gov.au/Ausstats/abs@.nsf/mf/3201.0)

Question 1

The modal age class interval of Australians in 2009 was

A. 15 - 19

B. 25 - 29

C. 35 - 39

D. 45 - 49

E. 85^{+}

Question 2

In 2009 the proportion of Australians aged 80 or more was closest to

A. 3.5%

B. 3%

C. 2.5%

D. 2%

E. 1.8%

Question 3

Which one of the following statements is true?

A. The median age of Australians in 1989 was the same as the *average* age of Australians in 1989.

B. The median age of Australians in 2009 was the same as the *average* age of Australians in 2009.

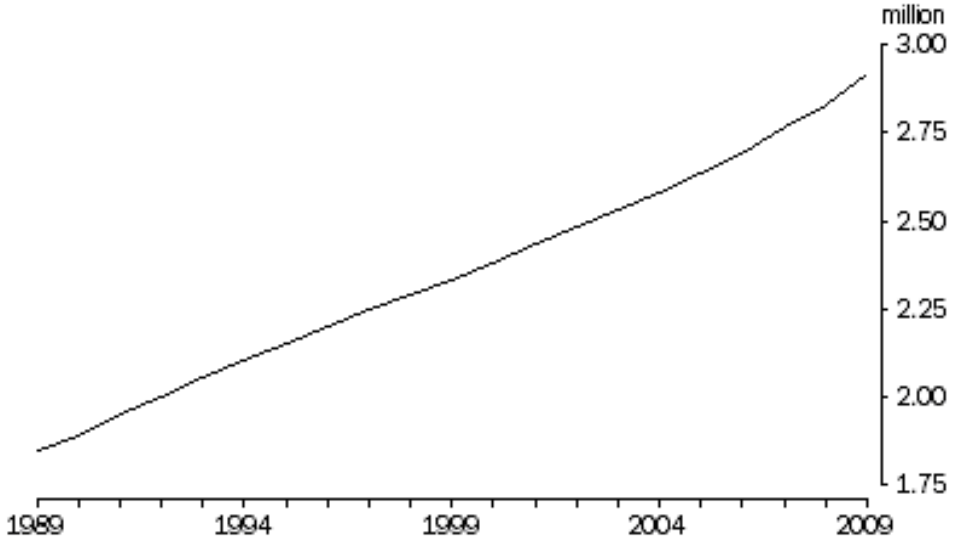
C. The median age of Australians in *1989* was the same as the median age of Australians in *2009*.

D. The median age of Australians in *1989* was greater than the median age of Australians in *2009*.

E. The median age of Australians in *1989* was less than the median age of Australians in *2009*.

Question 4

Population aged 65 years or more, Australia



(http://www.abs.gov.au/Ausstats/abs@.nsf/mf/3201.0)

Between 1989 and 2009 the average annual increase in the *number of Australians aged 65 years or more* was closest to

A. 0.05

B. 0.5

C. 50000

D. 500000

E. 2900000

The following information relates to Questions 5 and 6

Australian Bureau of Statistics

2006 Census of Population and Housing
Victoria (State)

HOURS WORKED per week

Count of employed persons aged 15 years and over

	Males	Females	Persons
None	37,525	44,970	82,495
1 - 15 hours	83,294	179,590	262,884
16 - 24 hours	57,511	147,339	204,850
25 - 34 hours	69,555	145,314	214,869
35 - 39 hours	198,182	170,736	368,918
40 hours	278,077	160,971	439,048
41 - 48 hours	169,778	82,505	252,283
49 - 60 hours	293,700	91,509	385,209
Total	**1,187,622**	**1,022,934**	**2,210,556**

Question 5

The shape of the distribution of hours worked for females is described (choose the **BEST** alternative) as

A. asymmetric

B. symmetric

C. negatively skewed

D. positively skewed

E. positively skewed with outliers

Question 6

The average number of hours worked per week by a Victorian aged 15 years or over in 2006 was closest to

A. 29

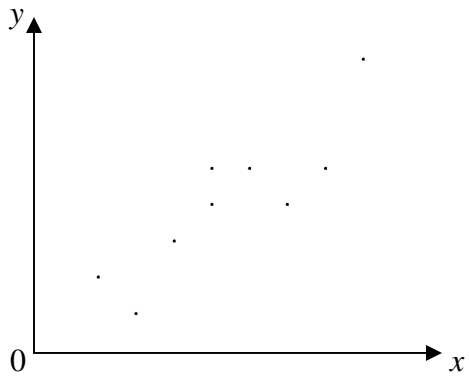
B. 31

C. 32

D. 34

E. 38

The following set of data relates to Questions 7 and 8



Question 7

The Pearson's correlation coefficient r is closest to

A. 0.2

B. 0.4

C. 0.5

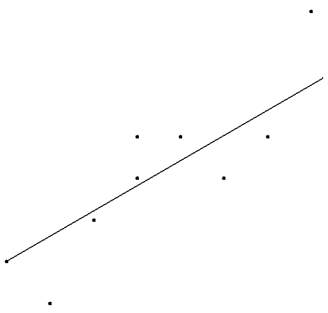
D. 0.6

E. 0.8

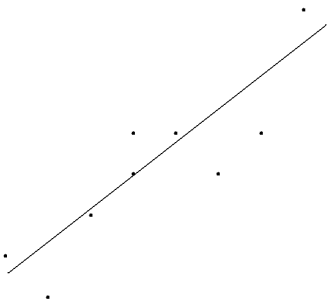
Question 8

The 3-median regression line for the set of data is closest to

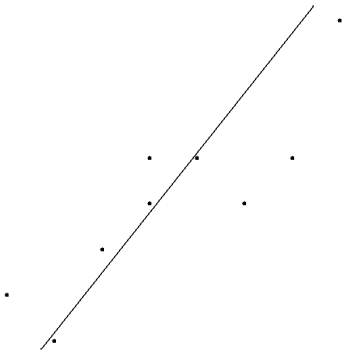
A.



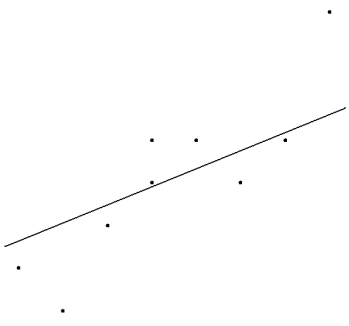
B.



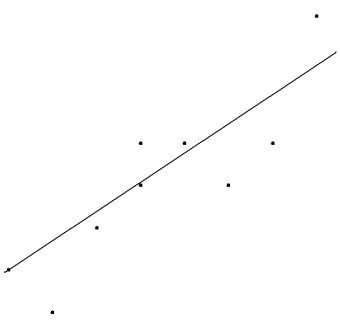
C.



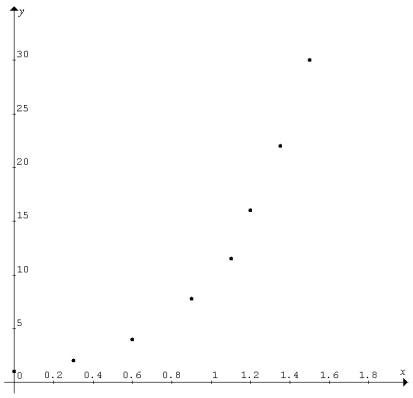
D.



E.



The following set of data relates to Questions 9 and 10



Question 9

Which one of the following graphs would be close to linear?

A. y versus $\log_{10} x$

B. $\log_{10} y$ versus x

C. y versus $\dfrac{1}{x}$

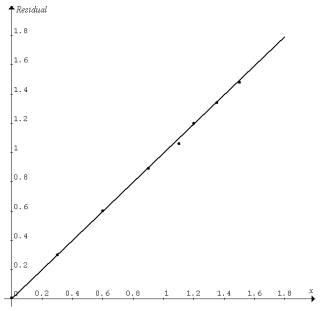
D. y^2 versus x

E. $\dfrac{1}{y}$ versus x

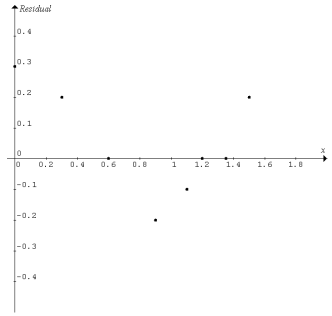
Question 10

The residual plot of the linearised data is most likely to be

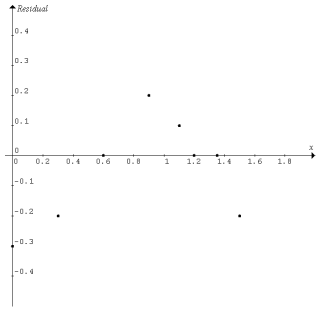
A.



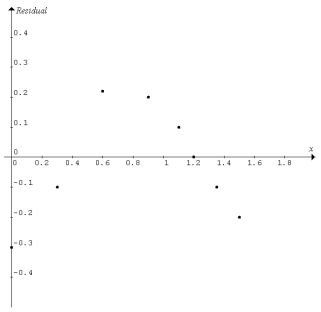
B.



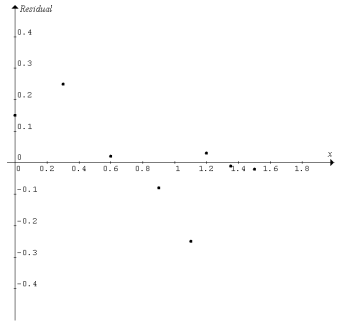
C.



D.



E.



The following information relates to Questions 11 and 12

The following table contains the actual and seasonally adjusted monthly unemployment rates in Australia.

Month 2009-2010	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun
Actual unemployment rate %	5.8	5.8	5.8	5.8	5.7	5.6	5.4	5.3	5.3	5.3	5.2	5.2
Seasonally adjusted unemployment rate %	5.8	5.8	5.7	5.8	5.6	5.5	5.2	5.3	5.4	5.4	5.1	5.1

(www.abs.gov.au)

Question 11

The seasonal index for January 2010 was closest to

A. 0.96

B. 0.99

C. 1.00

D. 1.01

E. 1.04

Question 12

The 4-point moving average of the seasonally adjusted unemployment rate for January 2010 was closest to

A. 5.50 %

B. 5.45 %

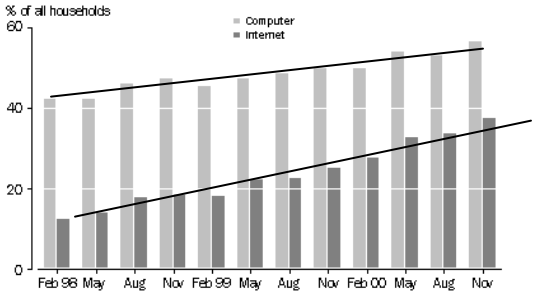
C. 5.40 %

D. 5.38 %

E. 5.35 %

Question 13

Household computer and Internet access 1998 – 2000 Australia (Copied from www.abs.gov.au)



Two trend lines for 1998 – 2000 data are shown above. Following the trends the percentage of households with computers would be approximately the same as the percentage of households with internet access by year

A. 2006

B. 2011

C. 2016

D. 2021

E. 2026

SECTION B Instructions

Answer **all** questions
A correct answer scores 1, an incorrect answer scores 0.
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Module 1: Number patterns

Question 1

Spheres are stacked to form a *triangular* pyramid, i.e. a tetrahedron. The pyramid has five layers of spheres. The number of spheres used to form the pyramid is

A. 35

B. 37

C. 39

D. 40

E. 41

Question 2

The 3^{rd}, 4^{th}, 5^{th} and 6^{th} terms of a sequence are 132, 159, 240 and 483 respectively. The first term of the sequence is

A. 120

B. 123

C. 126

D. 127

E. 128

Question 3

The value of the series $1-2+3-4+5-6+.....-378+379$ is

A. 188

B. 189

C. 190

D. 191

E. 192

The following information relates to Questions 4 and 5

A "superball" is dropped from a height of 5 metres above the ground. At each successive bounce it regains 90% of the previous height.

Question 4

The total distance (in metres) travelled by the "superball" *just before* the sixth bounce is closest to

A. 34

B. 36

C. 38

D. 40

E. 42

Question 5

The total distance (in metres) travelled by the "superball" for it to stop bouncing is closest to

A. 55

B. 75

C. 85

D. 95

E. 105

Question 6

Given $t_{n+1} = \frac{1}{2}(10 - t_n)$ and $t_1 = -2$, the value of t_5 is

A. 3

B. 3.25

C. 3.5

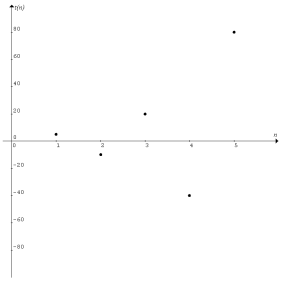
D. 4

E. 6

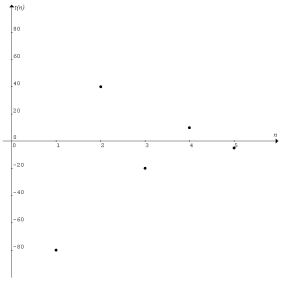
Question 7

Which one of the following diagram *CANNOT* be the graph of a sequence defined by $t_{n+1} = at_n$, where a is a real number?

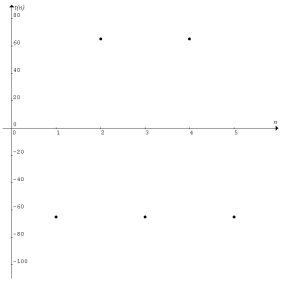
A.



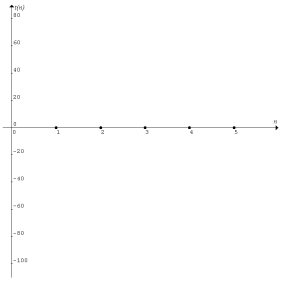
B.



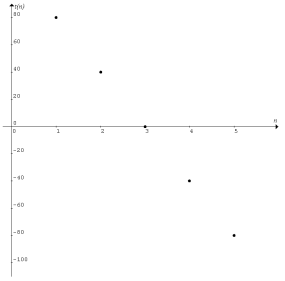
C.



D.



E.



The following information relates to Questions 8 and 9

A school had 200 students to start the first year of its operation. Each year 5% of the student population left the school but 50 new students were enrolled to start a new year.

Question 8

The total number of students to start the third year of operation of the school was

A. 240

B. 273

C. 278

D. 285

E. 314

Question 9

Let t_n be the student population of the school at the start of its nth year in operation. A difference equation to model the student population is

A. $t_{n+1} = 0.05t_n + 50,\ t_1 = 200$

B. $t_{n+1} = t_n - 5\% + 50,\ t_1 = 200$

C. $t_{n+1} = 0.95t_n + 50,\ t_1 = 200$

D. $t_{n+1} = 0.95(t_n + 50),\ t_1 = 200$

E. $t_{n+1} = 1.05(t_n + 50),\ t_1 = 200$

Module 5: Networks and decision mathematics

Question 1 A connected planar graph has 11 edges. The total number of vertices and regions is

A. 9

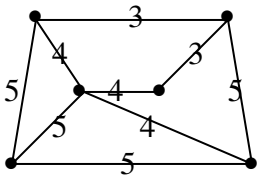
B. 10

C. 11

D. 12

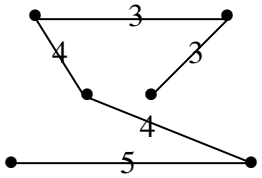
E. 13

Question 2

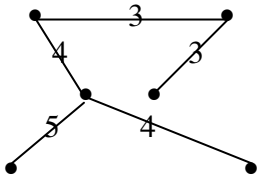


Which one of the following networks is NOT a minimum spanning tree of the graph above?

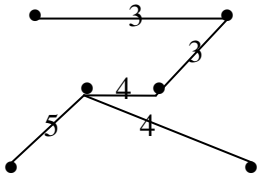
A.



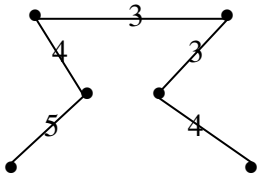
B.



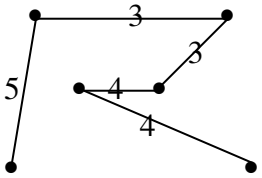
C.



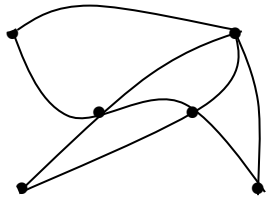
D.



E.



Question 3



Which one of the following statements is NOT true about the planar graph shown above?

A. The graph has an Eulerian path.

B. The graph has an Eulerian circuit.

C. The graph has an Eulerian path but not an Eulerian circuit.

D. The graph has a Hamiltonian path and a Hamiltonian circuit.

E. The graph has an Eulerian circuit and a Hamiltonian circuit.

Question 4 Which one of the following matrices CANNOT be an adjacency matrix of an **undirected** graph?

A. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

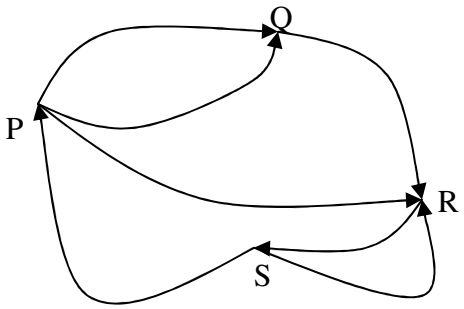
B. $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

Question 5



The digraph above represents a one-way-road network connecting locations P, Q, R and S. A matrix that shows the number of two-step paths between each pair of the locations is

A.

	P	Q	R	S
P	0	0	2	1
Q	0	0	0	1
R	1	0	1	0
S	0	2	1	1

B.

	P	Q	R	S
P	0	0	2	1
Q	0	0	0	1
R	1	0	0	0
S	0	2	1	0

C.

	P	Q	R	S
P	0	0	1	1
Q	0	0	0	1
R	1	0	1	0
S	0	1	1	1

D.

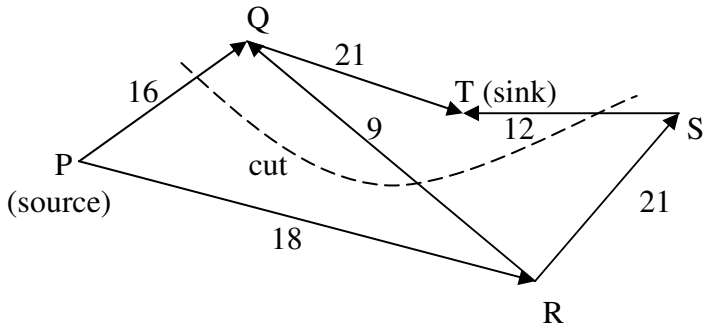
	P	Q	R	S
P	0	0	2	1
Q	0	0	0	1
R	1	0	1	0
S	0	2	1	0

E.

	P	Q	R	S
P	0	2	1	0
Q	0	0	1	0
R	0	0	0	1
S	1	0	1	0

The following information relates to Questions 6 and 7

The weighted digraph below shows the flow of water through a pipe network connecting five locations P, Q, R, S and T. Each number next to a pipe indicates the maximum flow rate of the pipe. A cut (dotted curve) is drawn across three pipes.



Question 6 From location P to location T the capacity of the cut shown above is

A. 37

B. 19

C. 13

D. 5

E. 0

Question 7 From location P to location T the maximum flow rate is

A. 48

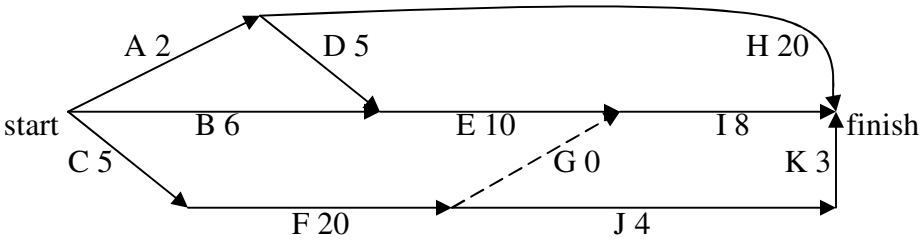
B. 46

C. 39

D. 34

E. 33

The following information relates to Questions 8 and 9



The directed graph above indicates the activities (A to K) which are required to complete a project. The number next to an activity indicates the time in hours required for the activity.

Question 8 The critical path for the project is

A. A-H

B. B-E-I

C. A-D-E-I

D. C-F-G-I

E. C-F-J-K

Question 9 If it is possible to shorten the time for activity H by an hour and activity F by 9 hours, the time (hours) required to complete the project is

A. 21

B. 22

C. 23

D. 24

E. 25

Module 6: Matrices

Question 1

If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} - 3\begin{bmatrix} -2 & a \\ b & 1 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ -2 & -3 \end{bmatrix}$ then a is equal to

A. -2

B. -1

C. 0

D. 1

E. 2

Question 2

If $\begin{bmatrix} 1 & b \\ a & -1 \end{bmatrix}\begin{bmatrix} -4 & b \\ b & 2 \end{bmatrix} = \begin{bmatrix} 0 & 3b \\ -2 & -4 \end{bmatrix}$ then the value of b is

A. ± 2

B. 2

C. -2

D. ± 16

E. undefined

Question 3

$\begin{bmatrix} 3 & m \\ -2 & n \end{bmatrix}$ does not have an inverse when $\frac{m}{n}$ is equal to

A. 1

B. -1

C. 0

D. -1.5

E. 1.5

Question 4

The system of simultaneous equations

$$z + y + 2 = 0$$
$$x - 2y - 3 = 0$$
$$3z - y = 0$$

can be solved by using the matrix equation

A. $\begin{bmatrix} 1 & 1 & 2 \\ 1 & -2 & -3 \\ 3 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 1 \\ 1 & -2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 & 0 \\ 1 & -2 & 0 \\ 3 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 & 3 \\ 1 & -2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 0 & 1 \\ 3 & 0 & -1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} z \\ x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$

Question 5

The cost prices of three different notebook computers are \$649, \$749 and \$849 respectively. Each computer is marked up to 1.5 times its cost price and then discounted by \$45. The selling price of each of these three notebook computers can be determined by evaluating

A. $[1.5] \begin{bmatrix} 649 \\ 749 \\ 849 \end{bmatrix} - [45]$

B. $1.5 \times \begin{bmatrix} 649 \\ 749 \\ 849 \end{bmatrix} - 45$

C. $[1.5] \begin{bmatrix} 649 \\ 749 \\ 849 \end{bmatrix} - \begin{bmatrix} 45 \\ 45 \\ 45 \end{bmatrix}$

D. $1.5 \times \begin{bmatrix} 649 \\ 749 \\ 849 \end{bmatrix} - \begin{bmatrix} 45 \\ 45 \\ 45 \end{bmatrix}$

E. $\begin{bmatrix} 1.5 \\ 1.5 \\ 1.5 \end{bmatrix} \begin{bmatrix} 649 \\ 749 \\ 849 \end{bmatrix} - \begin{bmatrix} 45 \\ 45 \\ 45 \end{bmatrix}$

The following information relates to Questions 6 and 7

Everyday a person must have exactly one cup of hot drink (either tea or coffee). If tea is consumed in one day, the probability that tea is consumed again the next day is 0.80. If coffee is consumed in one day, the probability that tea instead is consumed the next day is 0.30.

Question 6

A transition matrix that can be used to represent this situation is

A. $\begin{matrix} & \text{one day} \\ \begin{bmatrix} 0.80 & 0.30 \\ 0.20 & 0.70 \end{bmatrix} & \text{next day} \end{matrix}$

B. $\begin{matrix} & \text{one day} \\ \begin{bmatrix} 0.80 & 0.20 \\ 0.30 & 0.70 \end{bmatrix} & \text{next day} \end{matrix}$

C. $\begin{matrix} & \text{one day} \\ \begin{bmatrix} 0.80 & 0.70 \\ 0.20 & 0.30 \end{bmatrix} & \text{next day} \end{matrix}$

D. $\begin{matrix} & \text{one day} \\ \begin{bmatrix} 0.80 & 0.20 \\ 0.70 & 0.30 \end{bmatrix} & \text{next day} \end{matrix}$

E. $\begin{matrix} & \text{one day} \\ \begin{bmatrix} 0.70 & 0.30 \\ 0.20 & 0.80 \end{bmatrix} & \text{next day} \end{matrix}$

Question 7

In a year (365 days) the total number of cups of tea consumed by the person is closest to

A. 290

B. 250

C. 220

D. 190

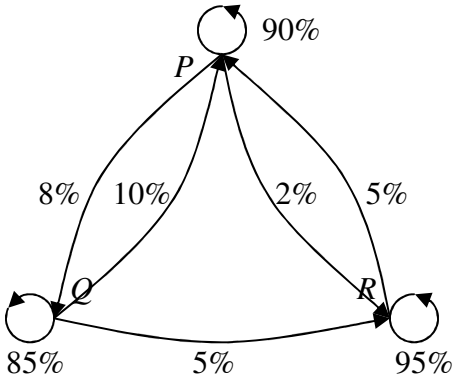
E. 183

The following information relates to Questions 8 and 9

A car rental company operates in three cities P, Q and R to full capacity. Customers can return the hired car in any one of the three cities. A transition matrix and an equivalent transition diagram are constructed according to the company's monthly statistics. It can be used to predict the number of rental cars of the company available for hiring from each of the three cities in subsequent months.

	this month P	Q	R	
	0.90	0.10	0.05	P
	0.08	0.85	0	Q
	0.02	0.05	0.95	R

next month



In October 2010 the company has 50 cars available for hiring in city P, 60 cars in city Q and 40 cars in city R.

Question 8

In December 2010 the number of cars (nearest whole number) available for hiring in city Q is predicted to be

A. 45

B. 46

C. 51

D. 56

E. 57

Question 9

In August 2010 the number of cars (nearest whole number) available for hiring in city Q was

A. 70

B. 74

C. 78

D. 82

E. 86

End of Exam 1