



Victorian Certificate of Education

2009

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

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FURTHER MATHEMATICS

Written examination 2

Wednesday 4 November 2009

Reading time: 11.45 am to 12.00 noon (15 minutes)

Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

QUESTION AND ANSWER BOOK

Structure of book

Core		
<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
4	4	15
Module		
<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
6	3	45
		Total 60

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 33 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

This examination consists of a core and six modules. Students should answer **all** questions in the core and then select **three** modules and answer **all** questions within the modules selected.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π , surds or fractions.

Diagrams are not to scale unless specified otherwise.

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Core

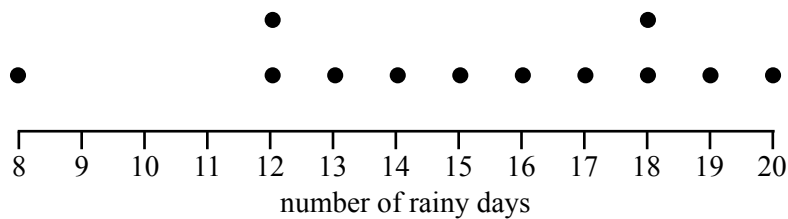
Question 1

Table 1 shows the number of rainy days recorded in a high rainfall area for each month during 2008.

Table 1

Month	Number of rainy days
January	12
February	8
March	12
April	14
May	18
June	18
July	20
August	19
September	17
October	16
November	15
December	13

The dot plot below displays the distribution of the number of rainy days for the 12 months of 2008.



- a. **Circle** the dot on the dot plot that represents the number of rainy days in April 2008.

1 mark

- b. For the year 2008, determine

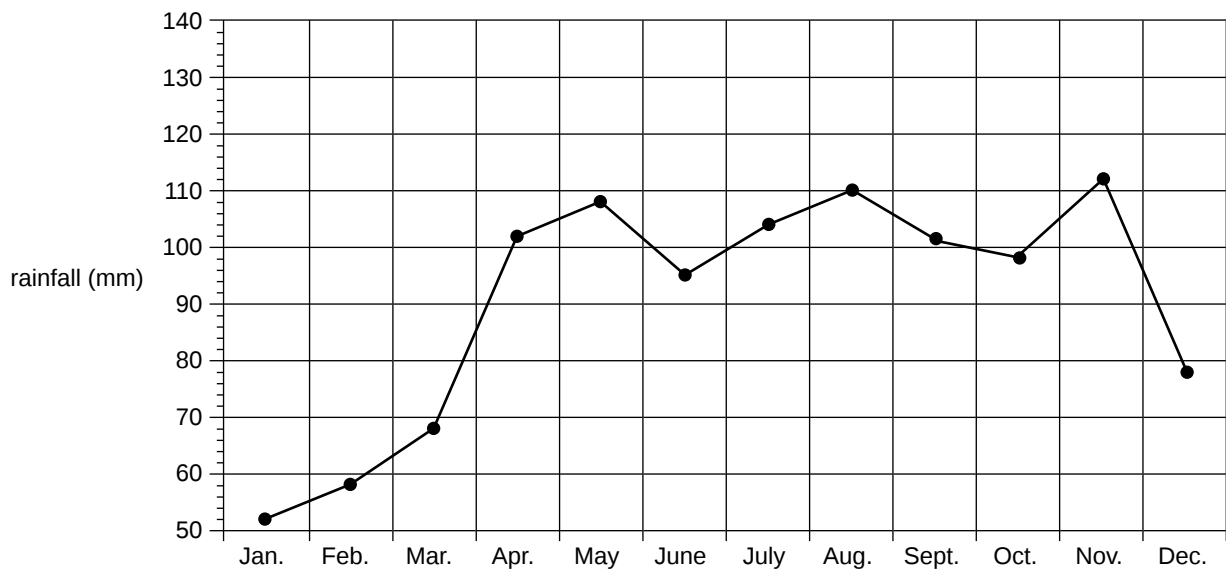
- i. the median number of rainy days per month

- ii. the percentage of months that have more than 10 rainy days. Write your answer correct to the nearest per cent.

1 + 1 = 2 marks

Question 2

The time series plot below shows the rainfall (in mm) for each month during 2008.



- a. Which month had the highest rainfall?

1 mark

- b. Use three-median smoothing to smooth the time series. Plot the smoothed time series on the plot above. Mark each smoothed data point with a cross (×).

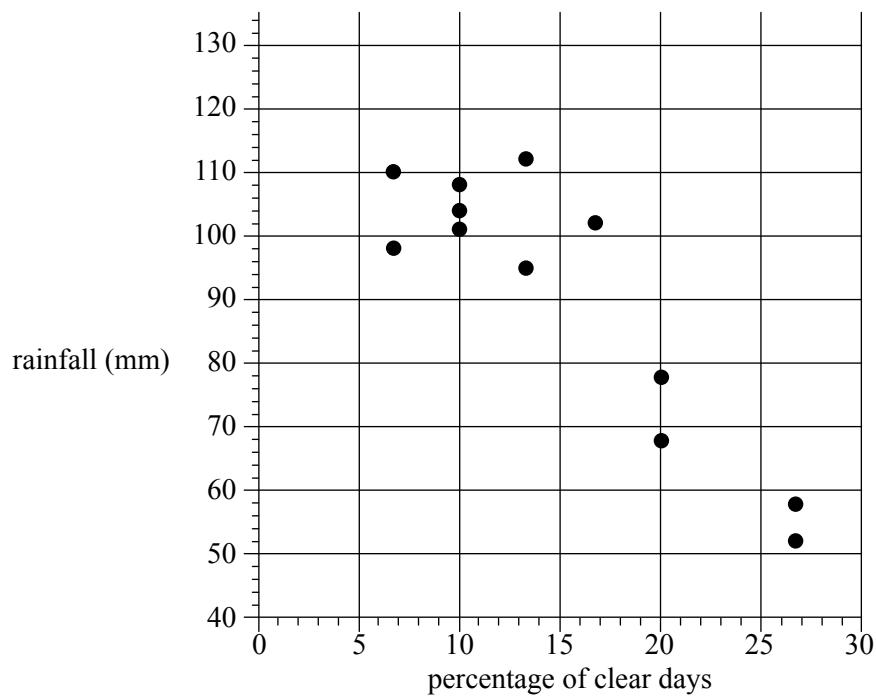
2 marks

- c. Describe the general pattern in rainfall that is revealed by the smoothed time series plot.

1 mark

Question 3

The scatterplot below shows the *rainfall* (in mm) and the *percentage of clear days* for each month of 2008.



An equation of the least squares regression line for this data set is

$$\text{rainfall} = 131 - 2.68 \times \text{percentage of clear days}$$

- a. Draw this line on the scatterplot.

1 mark

- b. Use the equation of the least squares regression line to predict the rainfall for a month with 35% of clear days. Write your answer in mm correct to one decimal place.

1 mark

- c. The coefficient of determination for this data set is 0.8081.

- i. Interpret the coefficient of determination in terms of the variables *rainfall* and *percentage of clear days*.

- ii. Determine the value of Pearson's product moment correlation coefficient. Write your answer correct to three decimal places.

1 + 2 = 3 marks

Question 4

- a. Table 2 shows the seasonal indices for rainfall in summer, autumn and winter. Complete the table by calculating the seasonal index for spring.

Table 2

Seasonal indices			
summer	autumn	winter	spring
0.78	1.05	1.07	

1 mark

- b. In 2008, a total of 188 mm of rain fell during summer.

Using the appropriate seasonal index in Table 2, determine the deseasonalised value for the summer rainfall in 2008. Write your answer correct to the nearest millimetre.

1 mark

- c. What does a seasonal index of 1.05 tell us about the rainfall in autumn?

1 mark

Total 15 marks

Module 1: Number patterns

In a concert hall there are 28 seats in row one, 29 seats in row two, 30 seats in row three, and so on. The number of seats in successive rows of the concert hall form the terms of an arithmetic sequence.

Question 1

- a. How many seats are in row 10?

1 mark

- b. There are 70 seats in the last row of the concert hall.
How many rows of seats does the concert hall contain?

1 mark

- c. Find the total number of seats in the first 20 rows.

1 mark

For a sell-out rock concert, the revenue obtained from successive rows of seats forms the terms of a geometric sequence with a common ratio of 1.01.

The revenue obtained from row 1 is \$2800.

- d. Find the revenue, in dollars, from
- i. row 2

- ii. row 9.

1 + 1 = 2 marks

- e. Calculate the total revenue obtained from the first 30 rows. Write your answer correct to the nearest dollar.

1 mark

- f. Each seat in the same row of the concert hall costs the same amount.
Determine the price of a single seat in row 26. Write your answer correct to the nearest cent.

2 marks

Question 2

For a sell-out gala concert, the revenue, in dollars, from each row, is generated by the difference equation

$$R_{n+1} = 1.02 R_n \quad R_6 = 2601$$

where R_n is the revenue, in dollars, from the n th row.

- a. By what percentage does the revenue from successive rows increase?

1 mark

- b. Determine the revenue obtained from row 4.

1 mark

Question 3

The longer a performance season runs, the fewer people attend.

The difference equation below provides a model for predicting the weekly attendance at a variety concert.

$$T_{n+1} = 0.8T_n + 1000 \quad T_1 = 12\,000 \quad \text{where } T_n \text{ is the attendance in week } n.$$

- a. Use the difference equation to predict the attendance in week 3.

1 mark

- b. Show that the sequence generated by this difference equation is not arithmetic.

1 mark

- c. In which week will the attendance first fall below 6000 people?

1 mark

- d. The performance season will continue as long as the weekly attendance is at least 5000 people.
What does the difference equation indicate about the long-term future of this variety concert?
Justify your answer by showing appropriate working.

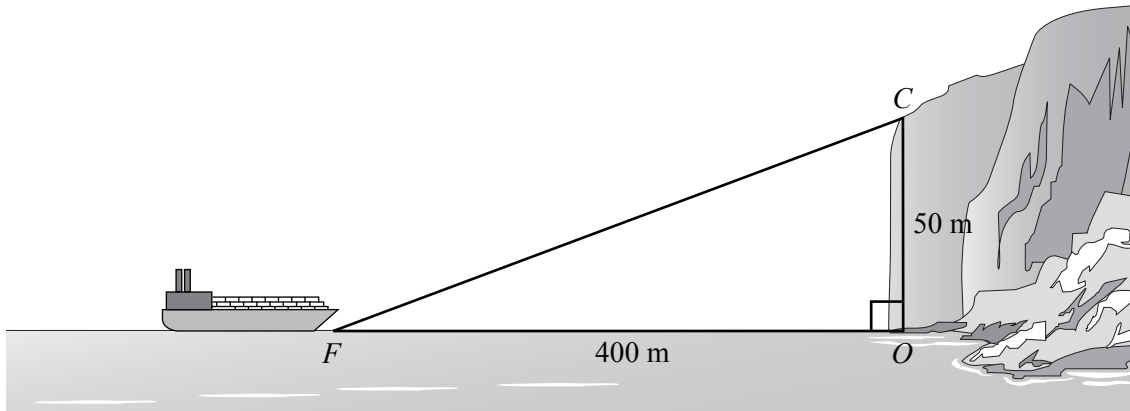
2 marks

Total 15 marks

Module 2: Geometry and trigonometry

Question 1

A ferry, F , is 400 metres from point O at the base of a 50 metre high cliff, OC .



- a. Show that the gradient of the line FC in the diagram is 0.125.

1 mark

- b. Calculate the angle of elevation of point C from F .
Write your answer in degrees, correct to one decimal place.

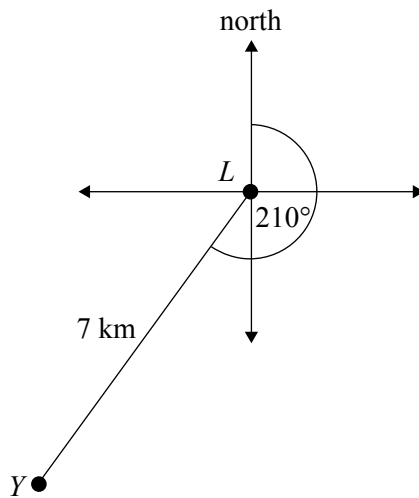
1 mark

- c. Calculate the distance FC , in metres, correct to one decimal place.

1 mark

Question 2

A yacht, Y , is 7 km from a lighthouse, L , on a bearing of 210° as shown in the diagram below.



- a. A ferry can also be seen from the lighthouse. The ferry is 3 km from L on a bearing of 135° . On the diagram above, label the position of the ferry, F , and show an angle to indicate its bearing.

1 mark

- b. Determine the angle between LY and LF .

1 mark

- c. Calculate the distance, in km, between the ferry and the yacht correct to two decimal places.

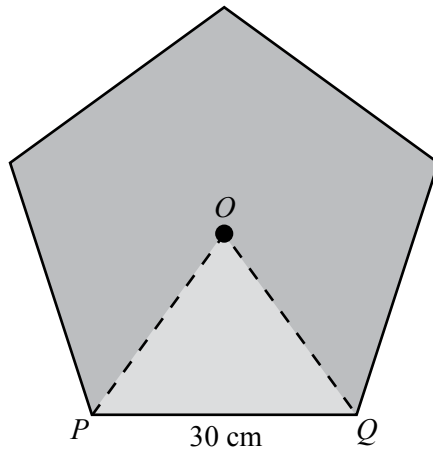
1 mark

- d. Determine the bearing of the lighthouse from the ferry.

1 mark

Question 3

The ferry has a logo painted on its side. The logo is a regular pentagon with centre O and side length 30 cm. It is shown in the diagram below.



- a. Show that angle POQ is equal to 72° .

1 mark

- b. Show that, correct to two decimal places, the length OP is 25.52 cm.

1 mark

- c. Find the area of the pentagon. Write your answer correct to the nearest cm^2 .

2 marks

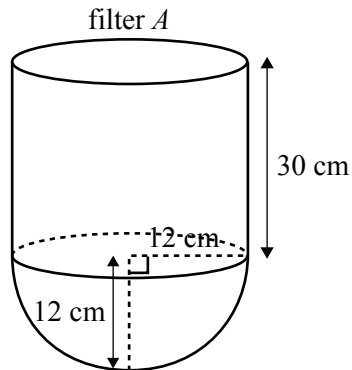
Working space

Question 4

The ferry has two fuel filters, A and B .

Filter A has a hemispherical base with radius 12 cm.

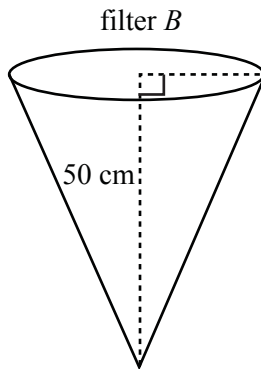
A cylinder of height 30 cm sits on top of this base.



- a. Calculate the volume of filter A . Write your answer correct to the nearest cm^3 .

2 marks

Filter B is a right cone with height 50 cm.



- b. Originally filter B was full of oil, but some was removed.

If the height of the oil in the cone is now 20 cm, what percentage of the original volume of oil was removed?

2 marks

Total 15 marks

END OF MODULE 2
TURN OVER

Module 3: Graphs and relations

Question 1

Fair Go Airlines offers air travel between destinations in regional Victoria.

Table 1 shows the fares for some distances travelled.

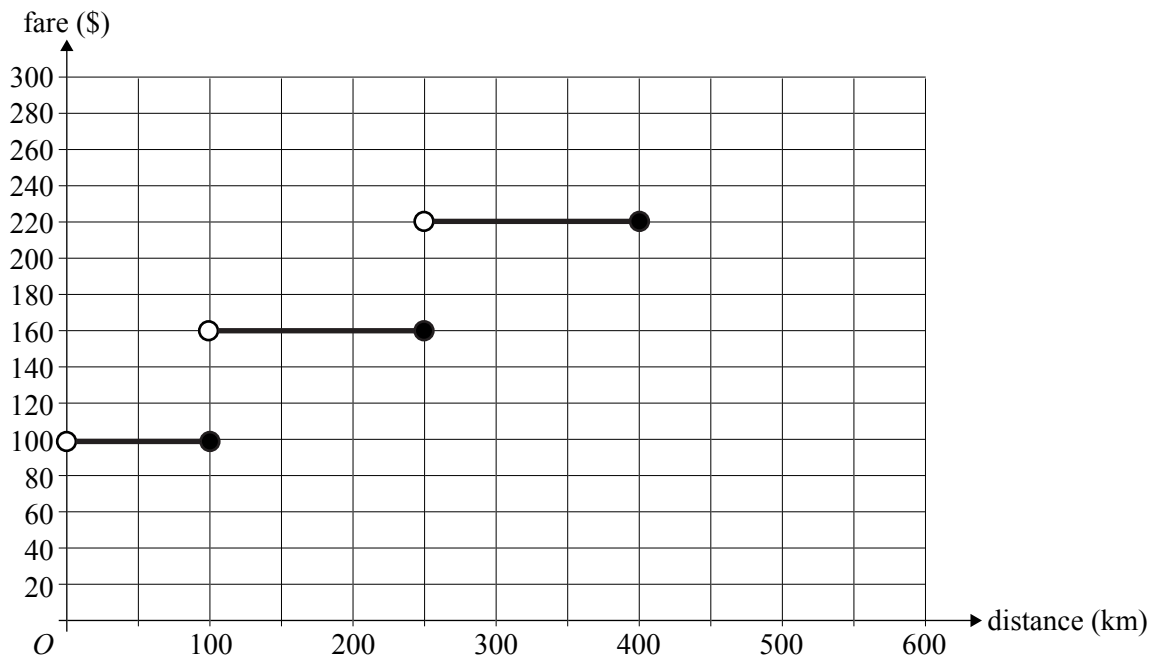
Table 1

Distance (km)	Fare
$0 < \text{distance} \leq 100$	\$100
$100 < \text{distance} \leq 250$	\$160
$250 < \text{distance} \leq 400$	\$220

- a. What is the maximum distance a passenger could travel for \$160?

1 mark

The fares for the distances travelled in Table 1 are graphed below.



- b. The fare for a distance longer than 400 km, but not longer than 550 km, is \$280. Draw this information on the graph above.

1 mark

Fair Go Airlines is planning to change its fares.

A new fare will include a service fee of \$40, plus 50 cents per kilometre travelled.

An equation used to determine this new fare is given by

$$\text{fare} = 40 + 0.5 \times \text{distance}.$$

- c. A passenger travels 300 km.

How much will this passenger save on the fare calculated using the equation above compared to the fare shown in Table 1?

1 mark

- d. At a certain distance between 250 km and 400 km, the fare, when calculated using either the new equation or Table 1, is the same.

What is this distance?

2 marks

- e. An equation connecting the maximum distance that may be travelled for each fare in **Table 1** on page 16 can be written as

$$\text{fare} = a + b \times \text{maximum distance}.$$

Determine a and b .

2 marks

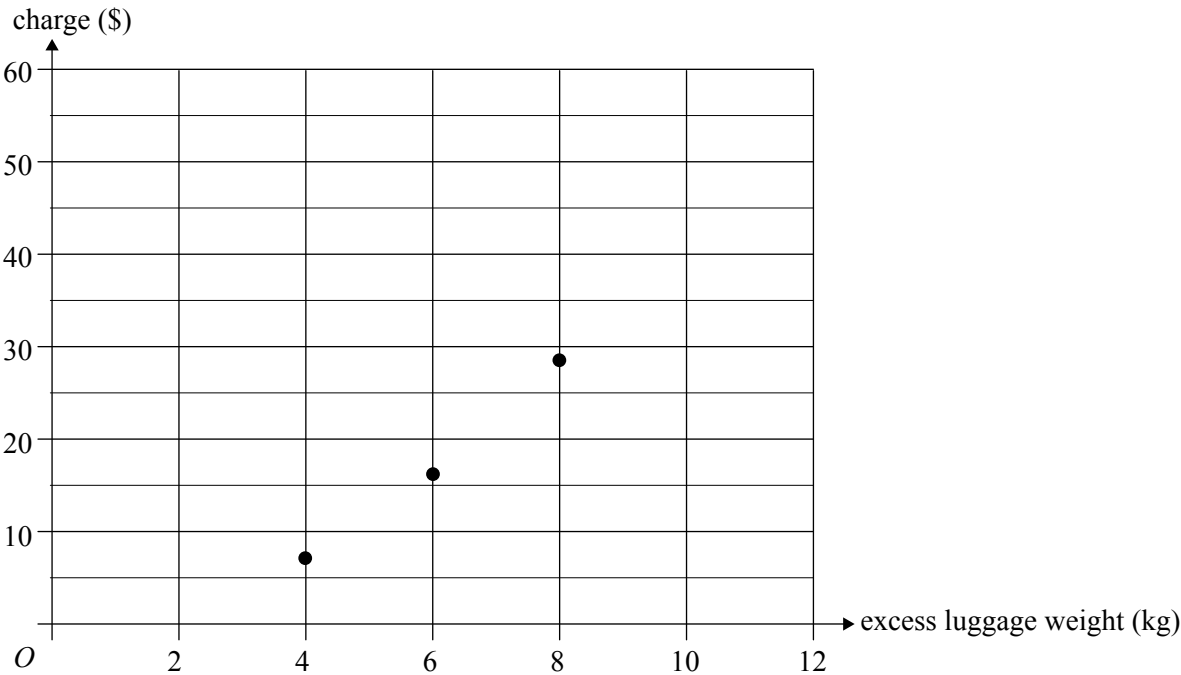
Question 2

Luggage over 20 kg in weight is called excess luggage.
Fair Go Airlines charges for transporting excess luggage.
The charges for some excess luggage weights are shown in Table 2.

Table 2

<i>excess luggage weight (kg)</i>	4	6	8	10
<i>charge (\$)</i>	\$7.20	\$16.20	\$28.80	\$45.00

- a. Complete this graph by plotting the charge for excess luggage weight of 10 kg. Mark this point with a cross (×).



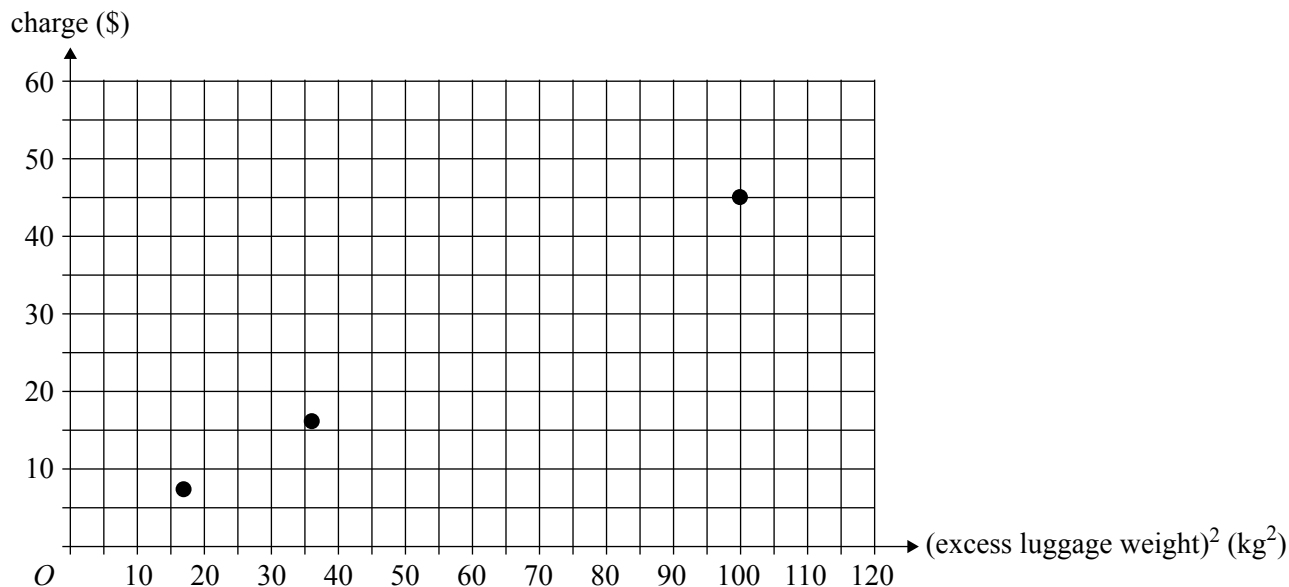
1 mark

- b. A graph of the *charge* against $(\text{excess luggage weight})^2$ is to be constructed.

Fill in the missing $(\text{excess luggage weight})^2$ value in Table 3 and **plot** this point with a cross (×) on the graph below.

Table 3

<i>excess luggage weight</i> (kg)	4	6	8	10
$(\text{excess luggage weight})^2$ (kg ²)	16	36		100
<i>charge</i> (\$)	\$7.20	\$16.20	\$28.80	\$45.00



1 mark

- c. The graph above can be used to find the value of k in the equation below.

$$\text{charge} = k \times (\text{excess luggage weight})^2$$

Find k .

1 mark

- d. Calculate the charge for transporting 12 kg of excess luggage.

Write your answer in dollars correct to the nearest cent.

1 mark

Question 3

Another company, Cheapstar Airlines, uses the two equations below to calculate the total cost of a flight.

The passenger fare, in dollars, for a given distance, in km, is calculated using the equation

$$\text{fare} = 20 + 0.47 \times \text{distance}.$$

The charge, in dollars, for a particular excess luggage weight, in kg, is calculated using the equation

$$\text{charge} = m \times (\text{excess luggage weight})^2.$$

Suzie will fly 450 km with 15 kg of excess luggage on Cheapstar Airlines.

She will pay \$299 for this flight.

Determine the value of m .

2 marks

Question 4

Cheapstar Airlines wishes to find the optimum number of flights per day on two of its most popular routes: Alberton to Bisley and Alberton to Crofton.

Let x be the number of flights per day from Alberton to Bisley

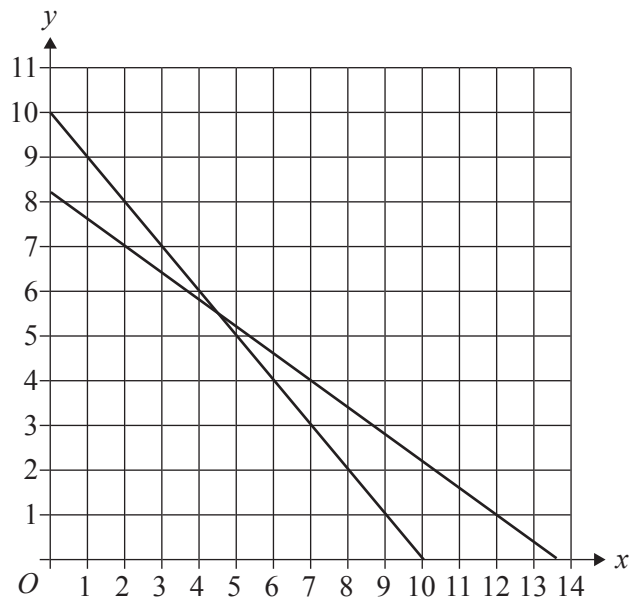
y be the number of flights per day from Alberton to Crofton

Table 4 shows the constraints on the number of flights per day and the number of crew per flight.

Table 4

	Alberton to Bisley	Alberton to Crofton	Maximum per day	Constraint
Number of flights per day	x	y	10	$x + y \leq 10$
Number of crew per flight	3	5	41	$3x + 5y \leq 41$

The lines $x + y = 10$ and $3x + 5y = 41$ are graphed below.



A profit of \$1300 is made on each flight from Alberton to Bisley and a profit of \$2100 is made on each flight from Alberton to Crofton.

Determine the maximum total profit that Cheapstar Airlines can make per day from these flights.

2 marks

Total 15 marks

Module 4: Business-related mathematics**Question 1**

The recommended retail price of a golf bag is \$500. Rebecca sees the bag discounted by \$120 at a sale.

- a. What is the price of the golf bag after the \$120 discount has been applied?

_____ 1 mark

- b. Find the discount as a percentage of the recommended retail price.

_____ 1 mark

Question 2

Rebecca will need to borrow \$250 to buy the golf bag.

- a. If she borrows the \$250 on her credit card, she will pay interest at the rate of 1.5% per month.

Calculate the interest Rebecca will pay in the first month.

Write your answer correct to the nearest cent.

_____ 1 mark

- b. If Rebecca borrows the \$250 from the store's finance company she will pay \$6 interest per month.

Calculate the **annual** flat interest rate charged. Write your answer as a percentage correct to one decimal place.

_____ 1 mark

Question 3

The golf club's social committee has \$3400 invested in an account which pays interest at the rate of 4.4% per annum compounding quarterly.

- a. Show that the interest rate per quarter is 1.1%.

_____ 1 mark

- b. Determine the value of the \$3400 investment after three years.

Write your answer in dollars correct to the nearest cent.

_____ 1 mark

- c. Calculate the interest the \$3400 investment will earn over **six** years.

Write your answer in dollars correct to the nearest cent.

2 marks

Question 4

The golf club management purchased new lawn mowers for \$22 000.

- a. Use the flat rate depreciation method with a depreciation rate of 12% per annum to find the depreciated value of the lawn mowers after four years.

2 marks

- b. Use the reducing balance depreciation method with a depreciation rate of 16% per annum to calculate the depreciated value of the lawn mowers after four years. Write your answer in dollars correct to the nearest cent.

1 mark

- c. After 4 years, which method, flat rate depreciation or reducing balance depreciation, will give the greater depreciation? Write down the greater depreciation amount in dollars correct to the nearest cent.

1 mark

Question 5

In order to drought-proof the course, the golf club will borrow \$200 000 to develop a water treatment facility. The club will establish a reducing balance loan and pay interest monthly at the rate of 4.65% per annum.

- a. \$1500 per month will be paid on this loan.

How much of the principal will be left to pay after five years?

Write your answer in dollars correct to the nearest cent.

1 mark

- b. Determine the total interest paid on the loan over the five-year period.

Write your answer in dollars correct to the nearest cent.

1 mark

- c. When the amount outstanding on the loan has reduced to \$95 200, the interest rate increases to 5.65% per annum.

Calculate the new monthly repayment that will fully repay this amount in 60 equal instalments.

Write your answer in dollars correct to the nearest cent.

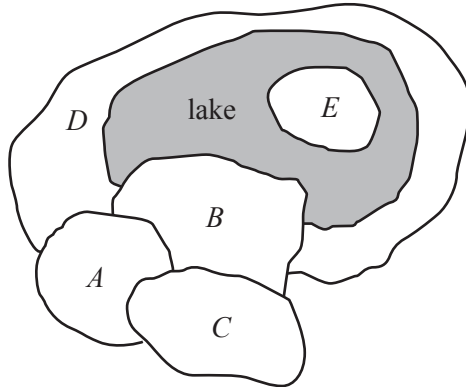
1 mark

Total 15 marks

Module 5: Networks and decision mathematics

Question 1

The city of Robville is divided into five suburbs labelled as A to E on the map below. A lake which is situated in the city is shaded on the map.



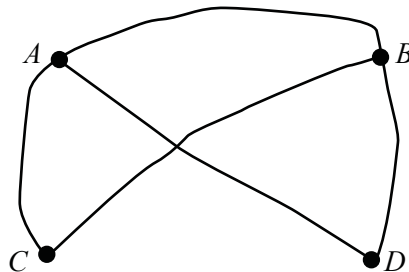
An adjacency matrix is constructed to represent the number of land borders between suburbs.

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	1	2	0
C	1	1	0	0	0
D	1	2	0	0	0
E	0	0	0	0	0

- a. Explain why all values in the final row and final column are zero.

1 mark

In the network diagram below, vertices represent suburbs and edges represent land borders between suburbs. The diagram has been started but is not finished.

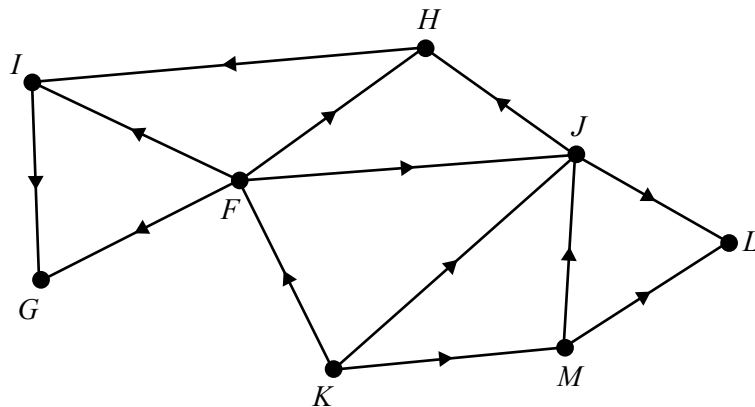


- b. The network diagram is missing one edge and one vertex.
On the diagram
- draw the missing edge
 - draw and label the missing vertex.

1 + 1 = 2 marks

Question 2

One of the landmarks in the city is a hedge maze. The maze contains eight statues. The statues are labelled F to M on the following directed graph. Walkers within the maze are only allowed to move in the directions of the arrows.



- a. Write down the two statues that a walker could not reach from statue M .

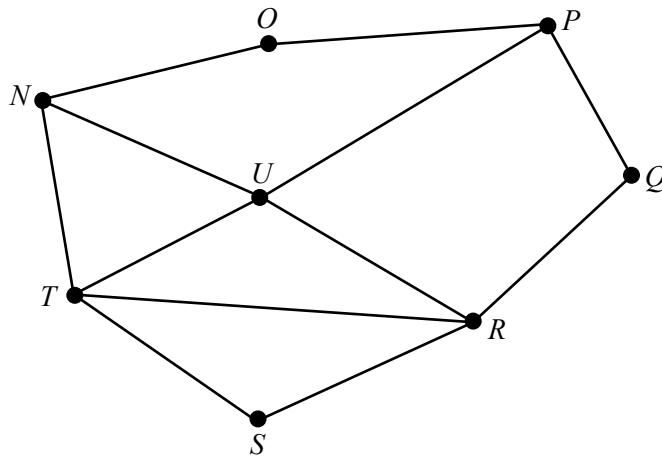
1 mark

- b. One way that statue H can be reached from statue K is along path KFH .
List the three other ways that statue H can be reached from statue K .

1 mark

Question 3

The city of Robville contains eight landmarks denoted as vertices N to U on the network diagram below. The edges on this network represent the roads that link the eight landmarks.



- a. Write down the degree of vertex U .

1 mark

- b. Steven wants to visit each landmark, but drive along each road only once. He will begin his journey at landmark N .

- i. At which landmark must he finish his journey?

- ii. Regardless of which route Steven decides to take, **how many** of the landmarks (including those at the start and finish) will he see on exactly two occasions?

1 + 1 = 2 marks

- c. Cathy decides to visit each landmark only once.

- i. Suppose she starts at S , then visits R and finishes at T .

Write down the order Cathy will visit the landmarks.

- ii. Suppose Cathy starts at S , then visits R but does not finish at T .

List three different ways that she can visit the landmarks.

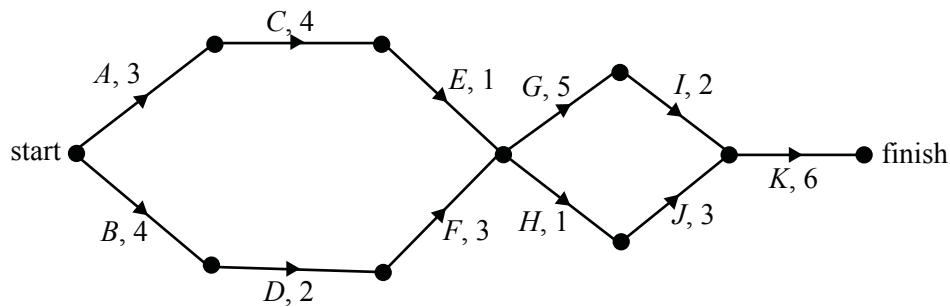
1 + 1 = 2 marks

Question 4

A walkway is to be built across the lake.

Eleven activities must be completed for this building project.

The directed network below shows the activities and their completion times in weeks.



- a. What is the earliest start time for activity *E*?

1 mark

- b. Write down the critical path for this project.

1 mark

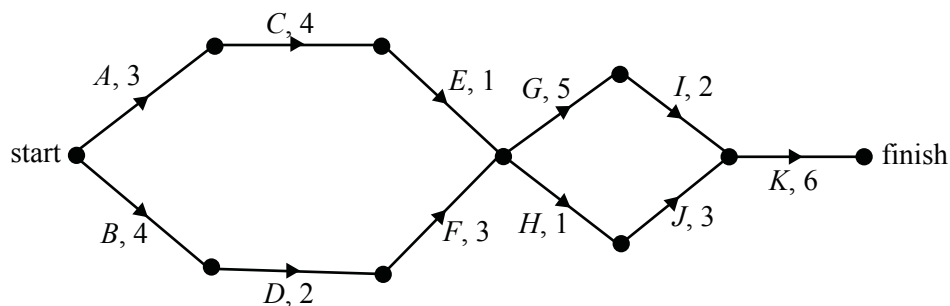
- c. The project supervisor correctly writes down the float time for each activity that can be delayed and makes a list of these times.

Determine the longest float time, in weeks, on the supervisor's list.

1 mark

A twelfth activity, *L*, with duration three weeks, is to be added without altering the critical path.

Activity *L* has an **earliest** start time of four weeks and a **latest** start time of five weeks.



- d. Draw in activity *L* on the network diagram above.

1 mark

- e. Activity *L* starts, but then takes four weeks longer than originally planned.

Determine the total overall time, in weeks, for the completion of this building project.

1 mark

Total 15 marks

END OF MODULE 5

Module 6: Matrices

Question 1

Three types of cheese, Cheddar (C), Gouda (G) and Blue (B), will be bought for a school function.

The cost matrix P lists the prices of these cheeses, in dollars, at two stores, Foodway and Safeworth.

$$P = \begin{bmatrix} 6.80 & 5.30 & 6.20 \\ 7.30 & 4.90 & 6.15 \end{bmatrix} \begin{matrix} \text{Foodway} \\ \text{Safeworth} \end{matrix}$$

- a. What is the order of matrix P ?

1 mark

The number of packets of each type of cheese needed is listed in the quantity matrix Q .

$$Q = \begin{bmatrix} 8 \\ 11 \\ 3 \end{bmatrix} \begin{matrix} C \\ G \\ B \end{matrix}$$

- b. i. Evaluate the matrix $W = PQ$.

- ii. At which store will the total cost of the cheese be lower?

1 + 1 = 2 marks

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Question 2

Tickets for the function are sold at the school office, the function hall and online.

Different prices are charged for students, teachers and parents.

Table 1 shows the number of tickets sold at each place and the total value of sales.

Table 1

	School office	Function hall	Online
Student tickets	283	35	84
Teacher tickets	28	4	3
Parent tickets	5	2	7
Total sales	\$8712	\$1143	\$2609

For this function

- student tickets cost \$ x
- teacher tickets cost \$ y
- parent tickets cost \$ z .

- a. Use the information in Table 1 to complete the following matrix equation by inserting the missing values in the shaded boxes.

$$\begin{bmatrix} 283 & 28 & 5 \\ \text{ } & 4 & \text{ } \\ 84 & 3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8712 \\ 1143 \\ 2609 \end{bmatrix}$$

1 mark

- b. Use the matrix equation to find the cost of a teacher ticket to the school function.

2 marks

Question 3

In 2009, the school entered a Rock Eisteddfod competition.

When rehearsals commenced in February, all students were asked whether they thought the school would make the state finals. The students' responses, 'yes', 'no' or 'undecided' are shown in the initial state matrix S_0 .

$$S_0 = \begin{bmatrix} 160 \\ 120 \\ 220 \end{bmatrix} \begin{matrix} \text{yes} \\ \text{no} \\ \text{undecided} \end{matrix}$$

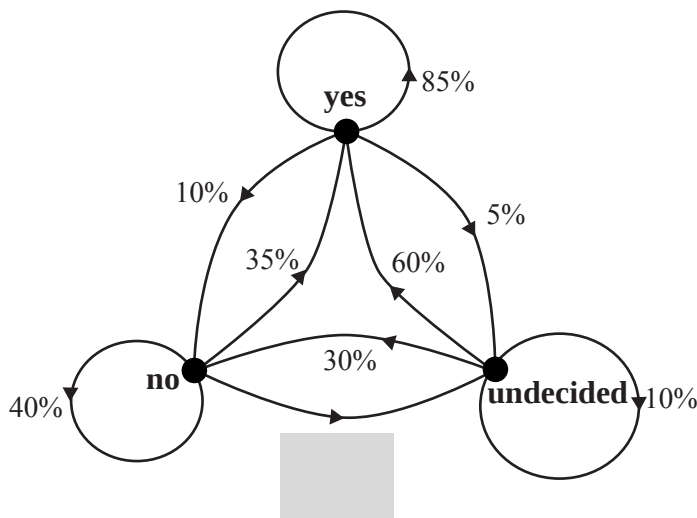
- a. How many students attend this school?

1 mark

Each week some students are expected to change their responses. The changes in their responses from one week to the next are modelled by the transition matrix T shown below.

$$T = \begin{matrix} \begin{matrix} \text{response this week} \\ \text{yes} & \text{no} & \text{undecided} \end{matrix} \\ \begin{bmatrix} 0.85 & 0.35 & 0.60 \\ 0.10 & 0.40 & 0.30 \\ 0.05 & 0.25 & 0.10 \end{bmatrix} \begin{matrix} \text{yes} \\ \text{no} \\ \text{undecided} \end{matrix} \end{matrix} \quad \begin{matrix} \text{response} \\ \text{next week} \end{matrix}$$

The following diagram can also be used to display the information represented in the transition matrix T .



- b. i. Complete the diagram above by writing the missing percentage in the shaded box.
- ii. Of the students who respond 'yes' one week, what percentage are expected to respond 'undecided' the next week when asked whether they think the school will make the state finals?
- iii. In total, how many students are **not** expected to have changed their response at the end of the first week?

1 + 1 + 2 = 4 marks

- c. Evaluate the product $S_1 = T S_0$, where S_1 is the state matrix at the end of the first week.

1 mark

- d. How many students are expected to respond 'yes' at the end of the third week when asked whether they think the school will make the state finals?

1 mark

Question 4

A series of extra rehearsals commenced in April. Each week participants could choose extra dancing rehearsals or extra singing rehearsals.

A matrix equation used to determine the number of students expected to attend these extra rehearsals is given by

$$L_{n+1} = \begin{bmatrix} 0.85 & 0.25 \\ 0.15 & 0.75 \end{bmatrix} \times L_n - \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

where L_n is the column matrix that lists the number of students attending in week n .

The attendance matrix for the first week of extra rehearsals is given by

$$L_1 = \begin{bmatrix} 95 \\ 97 \end{bmatrix} \begin{matrix} \text{dancing} \\ \text{singing} \end{matrix}$$

- a. Calculate the number of students who are expected to attend the extra singing rehearsals in week 3.

1 mark

- b. Of the students who attended extra rehearsals in week 3, how many are not expected to return for any extra rehearsals in week 4?

1 mark

Total 15 marks

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \frac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \dots + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \dots = \frac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc \sin A$

Heron's formula: $A = \sqrt{s(s - a)(s - b)(s - c)}$ where $s = \frac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base $\times$ height

volume of a pyramid: $\frac{1}{3}$ area of base $\times$ height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times \text{flat rate}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; \quad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix: $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$