

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 2



2009 Trial Examination

SOLUTIONS

SECTION A: Core

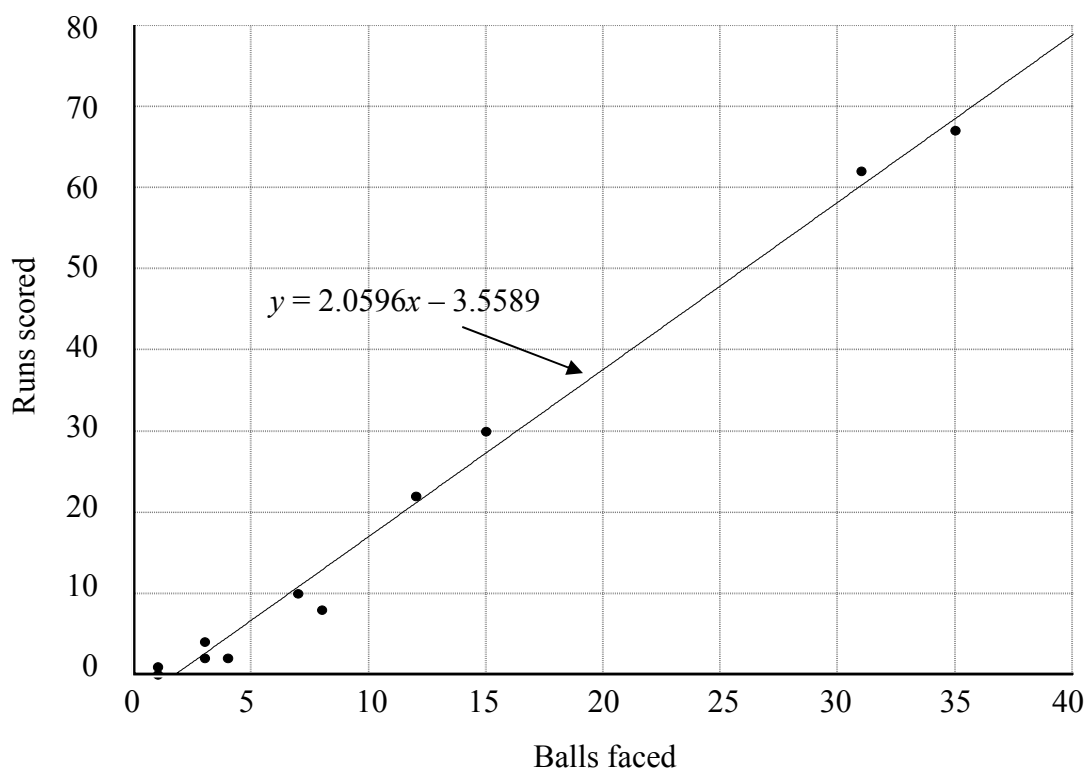
Question 1

- a. Negatively skewed. A1
- b.
- i. $11+12+11+4+1=39$ A1
- ii. $\frac{29}{60} \times 100\% = 48.3\%$ A1

Question 2

- a. Bar chart, segmented bar chart, pie chart. A1
These graphs allow the display of categorical data with less than 6 options. A1
- b. Runs scored A1

- c. Appropriately labelled axes A1
 Correct choice of x and y axes A1
 Appropriate placement of data points M1



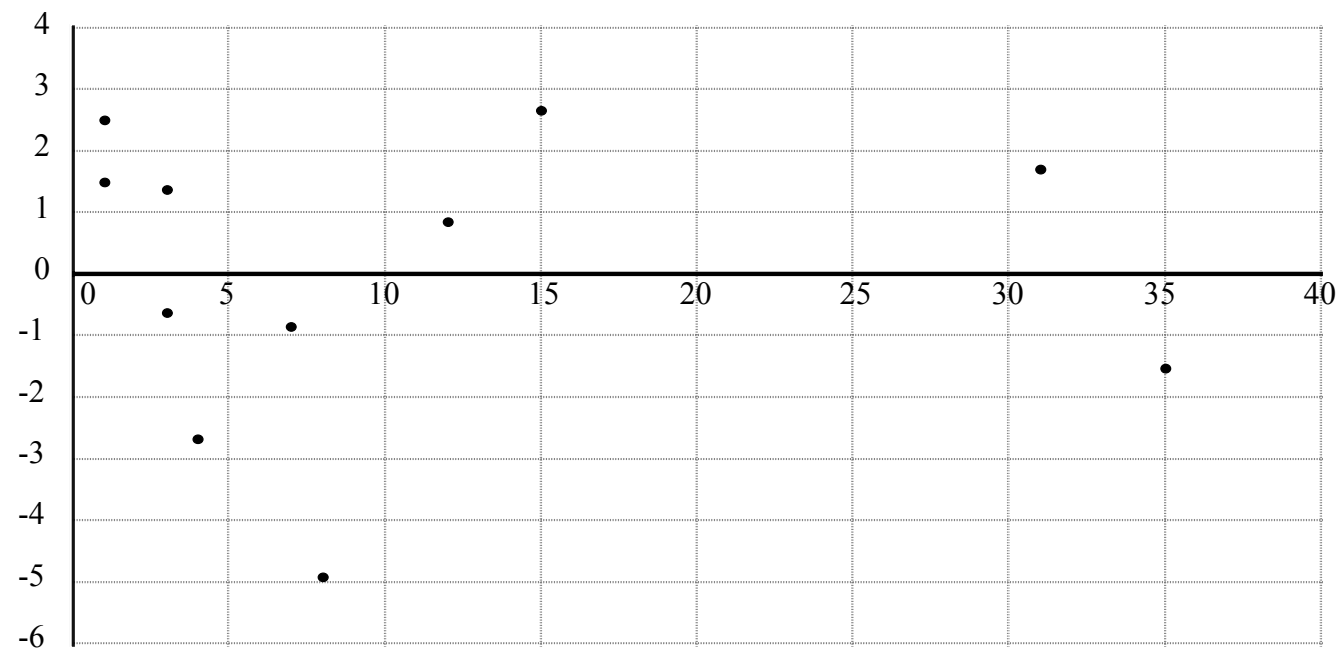
- d. Least squares regression line: number of runs = $2.06 \times$ number of balls faced – 3.56 A1
 Appropriate placement of regression line A1

- e.

Appropriate scale on graph

A1
- Appropriate placement of residuals

A1



- f.

Yes, it does suggest linearity.

A1
- There is a relatively random distribution of the residuals.

A1

SECTION B: Modules**Module 1: Number Patterns****Question 1**

a. $3 \times 140 = 420km$ A1

b. $a = 140$ A1

c.

$$3430 = 140n$$

$$\therefore n = \frac{3430}{140} \quad \text{A1}$$

$$n = 24.5days$$

d. $140n = \frac{n}{2}[2 \times 100 + (n-1)10]$ M1

$$\frac{140n}{n} = \frac{\frac{n}{2}[2 \times 100 + (n-1)10]}{n}$$

$$140 = \frac{1}{2}[2 \times 100 + (n-1)10]$$

$$140 = \frac{1}{2}[200 + 10n - 10]$$

$$140 = \frac{1}{2}[190 + 10n] \quad \text{M1}$$

$$140 = 95 + 5n$$

$$45 = 5n$$

$$\therefore n = 9days \quad \text{A1}$$

Question 2

a.

$$D_4 = 0.8 \times 350 + 100 = 380$$

$$D_5 = 0.8 \times 380 + 100 = 404 \text{ km}$$

A1

b.

$$\frac{350}{380} = 0.921$$

$$\frac{380}{404} = 0.941$$

$$380 - 350 = 30$$

$$404 - 380 = 24$$

M1

$\therefore$ not geometric

$\therefore$ not arithmetic

c.

$$D_n = \frac{D_{n+1} - 100}{0.8}$$

$$D_2 = \frac{350 - 100}{0.8} = 312.5$$

$$D_1 = \frac{312.5 - 100}{0.8}$$

$$D_1 = 265.625 \text{ km}$$

M1

A1

Question 3

a. $d_n = 0.9d_{n-1}$

A1

b. $d_5 = 400(0.9)^4$
 $= 262.44 \text{ km}$

A1

c. $3430 = \frac{a(1 - 0.9^{10})}{1 - 0.9}$

M1

$$3430 = \frac{a(1 - 0.9^{10})}{0.1}$$

$$3430 \times 0.1 = \frac{a(1 - 0.9^{10})}{0.1} \times 0.1$$

$$343 = a(1 - 0.9^{10})$$

M1

$$343 = 0.65132a$$

$$\frac{343}{0.65132} = \frac{0.65132a}{0.65132}$$

$$a = \frac{343}{0.65132}$$

$$\therefore a = 526.62 \text{ km}$$

A1

Module 2: Geometry and Trigonometry

Question 1

$$\text{a. } \tan \theta = \frac{O}{A}$$

$$\therefore \tan 18^\circ = \frac{x}{8.5}$$

$$\therefore x = 8.5 \tan 18^\circ$$

$$= 2.76 \text{ m}$$

A1

$$\text{b. Yes, total height} = 2.4 + 2.76 + 0.15 = 5.31 \text{ m}$$

A1

$$\text{c. The maximum allowed value of } x \text{ is } 5.5 - 2.4 - 0.15 = 2.95. \text{ Therefore}$$

$$\theta = \tan^{-1}\left(\frac{2.95}{8.5}\right) = 19.14^\circ$$

A1

Question 2

$$\begin{aligned} \text{a. } C &= \sqrt{a^2 + b^2 - 2ab \cos C} \\ &= \sqrt{28^2 + 42^2 - 2 \times 28 \times 42 \times \cos 70} \\ &= 41.76 \text{ m} \end{aligned}$$

A1

$$\text{b. } A = \frac{1}{2} ac \sin B$$

$$A = \frac{1}{2} \times 28 \times 42 \times \sin 70$$

$$= 552.54 \text{ m}^2 \approx 553 \text{ m}^2$$

A1

c. Scale factor 1:500, Area scale factor $1^2 : 500^2 = 1 : 25000$

M1

$$552.54 \text{ m}^2 = 5525400 \text{ cm}^2$$

$$\therefore \frac{5525400}{25000} = 221 \text{ cm}^2$$

A1

d. Volume scale factor $1^3 : 500^3 = 1 : 25000000$

$$16 \times 20 \times 2.4 = 768 \text{ m}^3 \times 100^3 = 768000000$$

$$\frac{768000000}{125000000} = 6 \text{ cm}^3$$

M1

A1

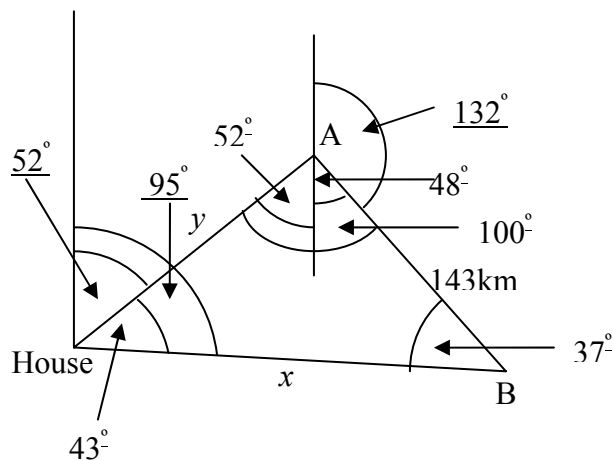
Question 3

a. Appropriately drawn diagram of points

A1

Underlined angles included

A1



b. i. $\angle BAH = 232^\circ - 132^\circ = 100^\circ$

$$\angle ABH = 180^\circ - 43^\circ - 100^\circ = 37^\circ$$

M1

ii. $\frac{135}{\sin 43} = \frac{x}{\sin 100} = \frac{y}{\sin 37}$

$$x = \frac{135 \sin 100}{\sin 43}$$

$$x = 194.94 \text{ km}$$

$$y = \frac{135 \sin 37}{\sin 43}$$

$$y = 119.13 \text{ km}$$

A2

c. $052^\circ T$

A1

Module 3: Graphs and Relations

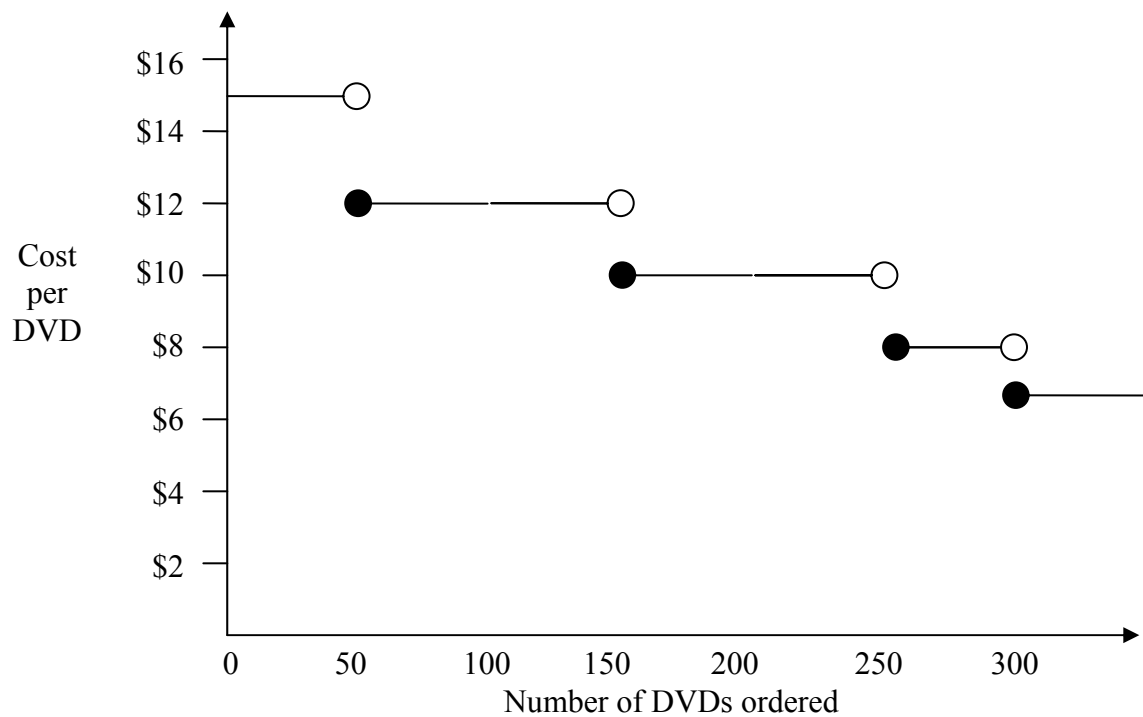
Question 1

a. \$12 A1

b. $150 \times \$10 = \1500 A1

c. 50 copies A1

d. Correct graph A1



Question 2

a. $C = 20000 + 3x$ A1

b. $R = 20x$ A1

c. $20x = 20000 + 3x$ M1

$$20x - 3x = 20000$$

$$17x = 20000$$

$$x = \frac{20000}{17}$$

$$x = 1176.47$$

$\therefore 1177$ DVDs A1

Question 3

a. $x \geq 10$ A1

$y \geq 8$ A1

$2x + 2.5y \leq 50$ A1

b. Correct positioning of lines A1

Correct calculation of corner points A1

Intersect of $x \geq 10$ and $y \geq 8$ is (10, 8)

Intersect of $x \geq 10$ and $2x + 2.5y \leq 50$ is $2 \times 10 + 2.5y = 50$

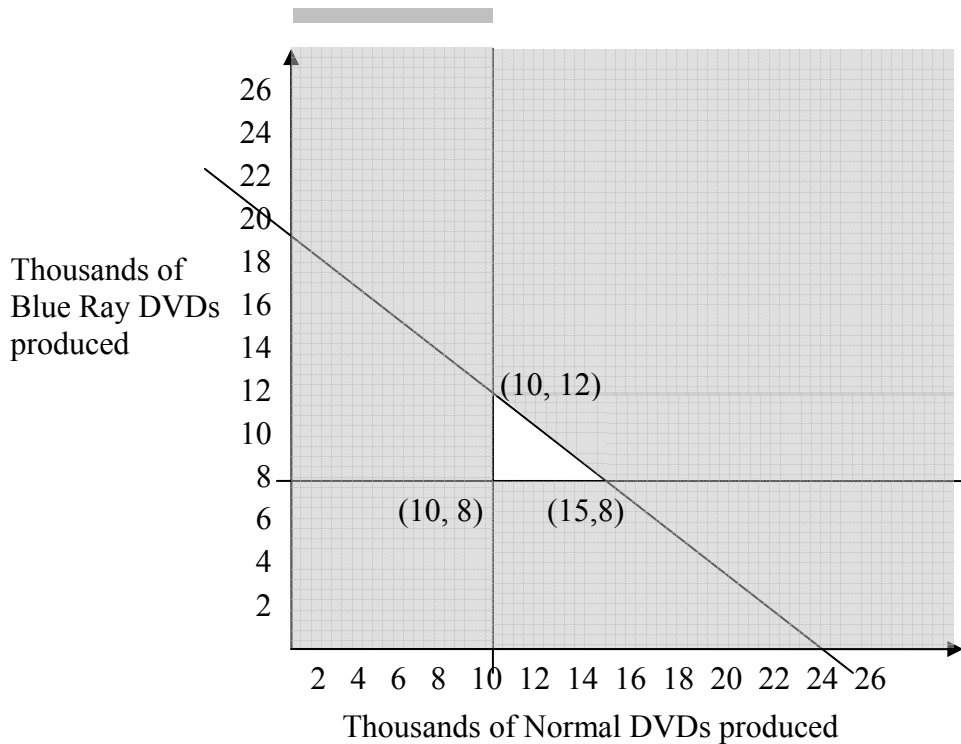
$$2.5y = 30$$

$$y = 12 \quad (10, 12)$$

Intersect of $y \geq 8$ and $2x + 2.5y \leq 50$ is $2x + 2.5 \times 8 = 50$

$$2x = 30 \quad (15, 8)$$

$$x = 15$$



- c. $(10, 8) = 10 \times 15 + 8 \times 25 = \350
 $(10, 12) = 10 \times 15 + 12 \times 25 = \450
 $(15, 8) = 15 \times 15 + 8 \times 25 = \425 M1
 Maximum profit of \$450 by producing 10000 Normal and 12000 Blue Ray DVDs. A1

Module 4: Business-Related Mathematics

Question 1

a.

i. $\$2800 \times 20\% = \560 A1

ii. $2800 - 560 = 2240$

$\frac{2240}{12} = \$186.67$ A1

b. $500 + 52 \times 50 = \$3100$

Better to go to the first store. A1

Will save $3100 - 2800 = \$300$ A1

c. $2950 \times 0.825 = \$2433.75$ A1

d. $2433.75 \times \frac{100}{110} = 2212.5$

$2433.75 - 2212.5 = \$221.25$

A1

Question 2

a. \$341.80 using TVM solver

A1

b. TVM solver gives weekly payments of \$78.73

$78.73 \times 364 = \$28657.72$ total payment by weekly instalments

M1

$341.80 \times 84 = \$28711.20$ total payment by monthly instalments

$28711.20 - 28657.72 = \$53.48$ saving

A1

c. Using TVM solver

Balance after 2 years (104 weeks) = \$16899.62

After \$5000 is paid he then owes \$11899.62

M1

Using TVM solver this will be paid off in 171.84 weeks

Total time is $171.84 + 104 = 275.84$ weeks $\rightarrow 276$ weeks = 5 years 16 weeks

A1

d. $\$28657.72 - (275.84 \times 78.73 + 5000) = \1935.84

A1

e. Option 1: $5000 = 22000 - (0.065 \times 22000)x$

$5000 - 22000 = 22000 - 1430x - 22000$

$-17000 = -1430x$

$x = 11.89$

$x = 12$ years

A1

Option 2: $y = 22000 \times (0.89)^x$

Using Table function on graphics calculator

Value first gets below 5000 after 13 years

A1

Answer: Option 1 is best

A1

Module 5: Networks and Decision Mathematics

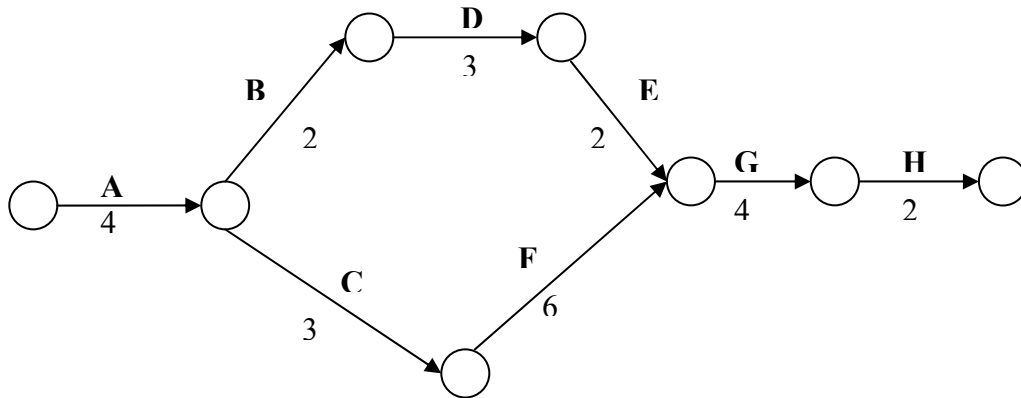
Question 1

a. Correct placement of nodes and activities

A1

Appropriate placement of times

A1



b. A, C, F, G, H. Project takes 19 weeks.

A1

c. A, B, D, E, G, H = $4 + 2 + 3 + 2 + 4 + 2 = 17$ weeks

20-17 weeks = 3 weeks

A1

d.

i. A, C, F, G, H = 19 weeks

Cheapest to crash on critical path is C = \$2000

A1

ii. There are two paths to complete this task A, C, F, G, H = 19 weeks

and A, B, D, E, G, H = 17 weeks

The only task available to be crashed in both paths is task G = 1 week = \$3000

$\therefore$ need to reduce A, C, F, G, H by 3 more weeks $\rightarrow$ crash C 1 week (\$2000) and F 2 weeks (\$10000)

Also, need to reduce A, B, D, E, G, H by 1 week $\rightarrow$ crash E by 1 week (\$3000)

Total = $\$3000 + \$2000 + \$10000 + \$3000 = \$18000$

A1

iii. Complete in 18 weeks crashing C. This gets a profit of $5000 - 2000 = \$3000$

versus crashing to 15 weeks gets a profit of $20000 - 18000 = \$2000$

A1

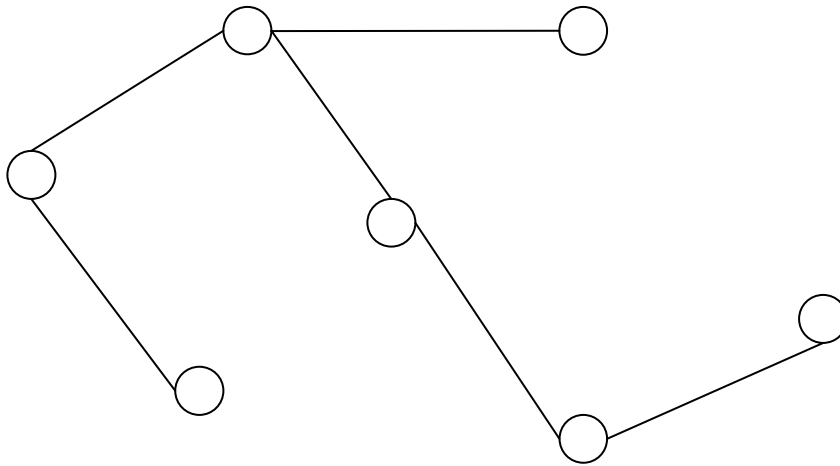
Question 2

a. $A - C - F - G = 65$ min

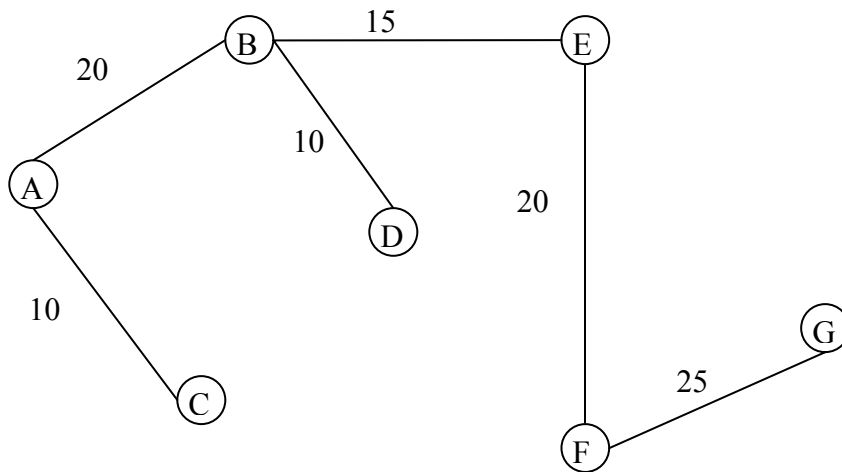
A1

i. Appropriate choice and drawing of minimum spanning tree.

A1



OR



ii. $10+20+10+15+20+25 = 100$

A1

Question 3

a. $8+20=28$ people

A1

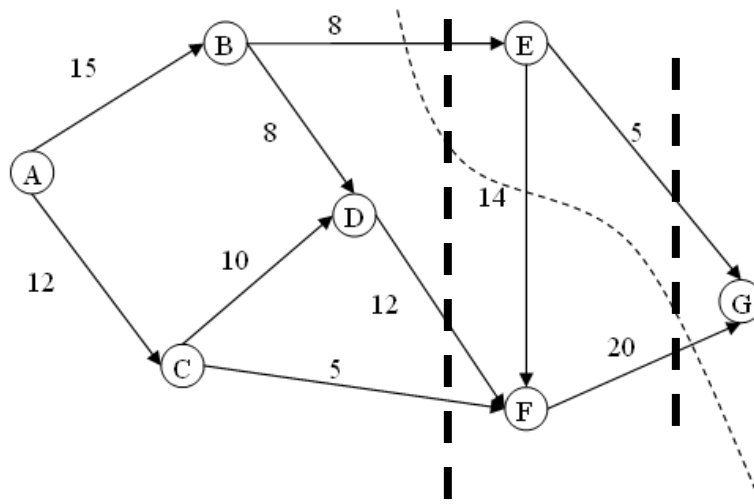
b. $20+5=25$ people

or $8+12+5=25$ people

A1

Appropriate placement of either cut

A1



c. No, it doesn't have an effect.

A1

$8+12+5=25$ people is still the minimum cut

A1

Module 6: Matrices**Question 1**

a. $\begin{bmatrix} 12 \\ 20 \\ 8 \end{bmatrix}$ A1

b. $\begin{bmatrix} 2 & 3 & 2 \\ 1 & 3 & 3 \\ 4 & 6 & 6 \\ 0 & 2 & 2 \end{bmatrix}$ A1

c.
$$OC = \begin{matrix} 2 \times 12 + 3 \times 20 + 2 \times 8 \\ 1 \times 12 + 3 \times 20 + 3 \times 8 \\ 4 \times 12 + 6 \times 20 + 6 \times 8 \\ 0 \times 12 + 2 \times 20 + 2 \times 8 \end{matrix} = \begin{bmatrix} 100 \\ 96 \\ 216 \\ 56 \end{bmatrix}$$
 A1

Bill 1 = \$100, bill 2 = \$96, bill 3 = \$216 and bill 4 = \$56 A1

d. The number of columns of C does not match the number of rows of O . A1

Question 2

a. $\begin{bmatrix} 3 & 2 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 90 \\ 204 \end{bmatrix}$ A1

b.
$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{3 \times 6 - 2 \times 6} \begin{bmatrix} 6 & -2 \\ -6 & 3 \end{bmatrix}$$
 M1

$$= \frac{1}{6} \begin{bmatrix} 6 & -2 \\ -6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -0.33 \\ -1 & 0.5 \end{bmatrix}$$
 A1

$$\text{c. } \begin{bmatrix} 3 & 2 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 90 \\ 204 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -0.33 \\ -1 & 0.5 \end{bmatrix} \begin{bmatrix} 90 \\ 204 \end{bmatrix}$$

M1

$$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 \\ 12 \end{bmatrix}$$

Main meal costs \$22 and dessert costs \$12.

A1

Question 3

$$\text{a. } \begin{bmatrix} 0.72 & 0.14 & 0.12 \\ 0.16 & 0.8 & 0.04 \\ 0.12 & 0.06 & 0.84 \end{bmatrix}$$

A1

$$\text{b. } \begin{bmatrix} 2000 \\ 1500 \\ 800 \end{bmatrix}$$

A1

$$\text{c. } \begin{bmatrix} 0.72 & 0.14 & 0.12 \\ 0.16 & 0.8 & 0.04 \\ 0.12 & 0.06 & 0.84 \end{bmatrix}^3 \begin{bmatrix} 2000 \\ 1500 \\ 800 \end{bmatrix}$$

M1

$$= \begin{bmatrix} 1504.0 \\ 1549.7 \\ 1246.3 \end{bmatrix}$$

A1

Therefore, after three weeks, 1504 diners are expected to dine at Applebox, 1550 at Baskerville and 1246 at The Club.

A1