

2009

VCE Further Mathematics Trial Examination 2

Suggested Solutions

© Kilbaha Multimedia Publishing 2009



Kilbaha Multimedia Publishing ABN 47 065 111 373
PO Box 2227
Kew Vic 3101
Australia
Tel: (03) 9817 5374

Fax: (03) 9817 4334

kilbaha@gmail.com

<http://kilbaha.googlepages.com>

IMPORTANT COPYRIGHT NOTICE

- This material is copyright. Subject to statutory exception and to the provisions of the relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Kilbaha Multimedia Publishing.
- The contents of this work are copyrighted. Unauthorised copying of any part of this work is illegal and detrimental to the interests of the author.
- For authorised copying within Australia please check that your institution has a licence from **Copyright Agency Limited**. This permits the copying of small parts of the material, in limited quantities, within the conditions set out in the licence.

Reproduction and communication for educational purposes The Australian Copyright Act 1968 (the Act) allows a maximum of one chapter or 10% of the pages of this work, to be reproduced and/or communicated by any educational institution for its educational purposes provided that educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.

For details of the CAL licence for educational institutions contact

CAL, Level 15, 233 Castlereagh Street, Sydney, NSW, 2000

Tel: (02) 9394 7600

Fax: (02) 9394 7601

Email: info@copyright.com.au

- Teachers and students are reminded that for the purposes of school requirements and external assessments, students must submit work that is clearly their own.
- Schools which purchase a licence to use this material may distribute this electronic file to the students at the school for their exclusive use. This distribution can be done either on an Intranet Server or on media for the use on stand-alone computers.
- Schools which purchase a licence to use this material may distribute this printed file to the students at the school for their exclusive use.

- **The Word file is for use ONLY within the school**
- **It may be modified to suit the school syllabus and for teaching purposes.**
- **All modified versions of the file must carry this copyright notice**
- **Commercial use of this material is expressly prohibited**

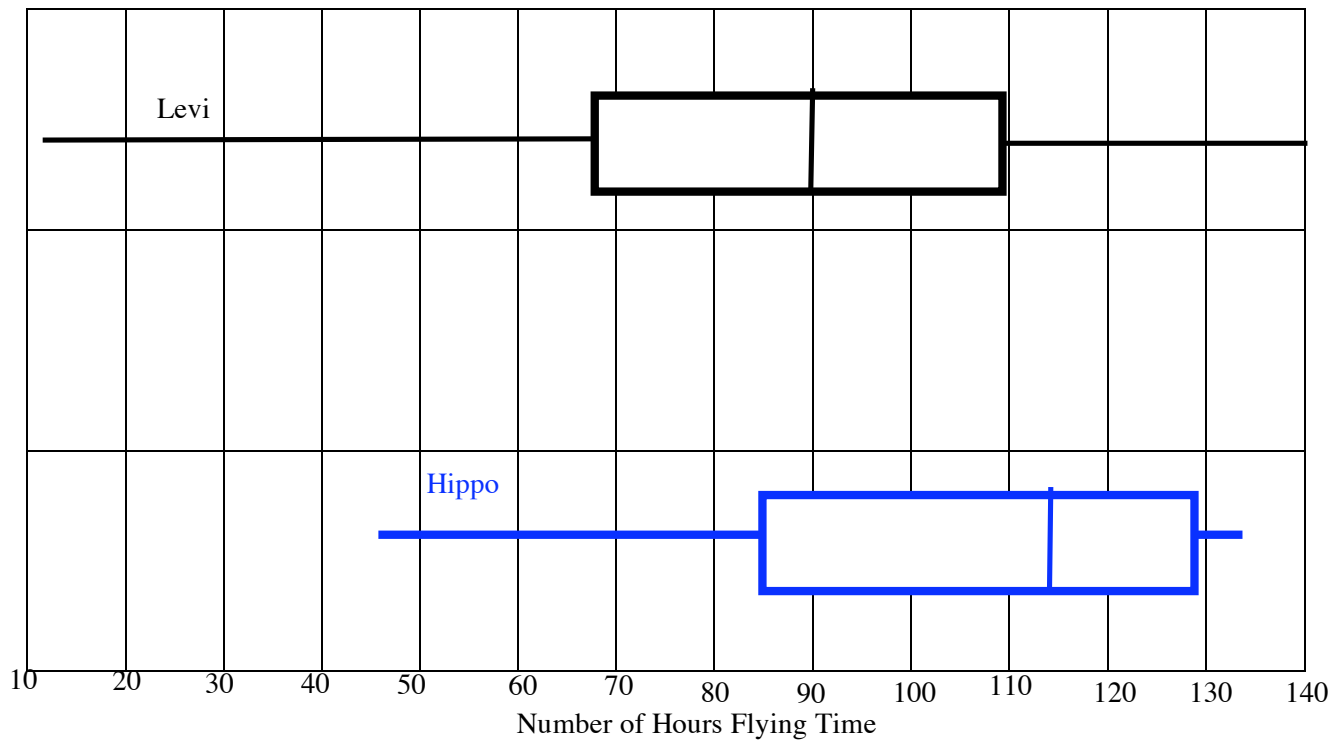
Core**Question 1**

a. $\frac{9}{13} \times 100 = 69.2\%$		(1 mark)
b. For T183 calculator enter data in stat edit in column L_1 . Go to stat calc, 1-var stats and enter L_1 . This gives standard deviation = 26.9 (1 mark)	c. The Levi stewards have a much larger spread in their number of flying hours than the Hippo stewards. (1 mark)	
d. Using calculator window from 1 b. gives $\bar{x} = 106.1, Q_1 = 85, Q_3 = 129$ $\bar{x} \pm 1.5IQR = 106.1 \pm 1.5 \times (129 - 85)$ So values outside 40.1 and 172.1 are outliers. Hence, there are no outliers. (1 mark)		

Core**Question 1 (continued)****e.**

Using the calculator window from 1(b) gives $Q_1 = 85$, $\text{Med} = 114$, $Q_3 = 129$
 $\text{Min} = 45$, $\text{Max} = 135$

(1 mark)

**f.**

Hours spent flying by Hippo stewards is more negatively skewed than hours spent flying by Levi stewards.

(1 mark)

Core**Question 2**

a.	
Advertising is the independent variable (1 mark)	
b.	c.
The variables are basically linearly related, and there are no outliers. (Only one of these points required for a mark)	For T183 calculator, enter data in stat edit in column L_1 and L_2 Go to stat calc, LinReg($ax+b$) and enter L_1, L_2 to get
(1 mark)	$Sales = \boxed{8609.0} + Advertising \times \boxed{0.5}$ (1 mark)
d.	e.
Predicted Value = $0.5 \times 6000 + 8609 = 11609$ Residual = $12500 - 11609 = \$891$	Using same calculator screen as for 2c. gives $r^2 = 0.98$
(1 mark)	(1 mark)
f.	g.
98% of the variation in sales income can be explained by the variation in advertising expenditure.	$20000 = 0.5 \times \text{advertising} + 8609$ $11391 = 0.5 \times \text{advertising}$ Advertising = \$22782
(1 mark)	(1 mark)

Core**Question 3****a.**

Year	2005	2006	2007	2008
Average	162.5	163.75	163	164.25
Autumn	$\frac{172}{162.5}$ =1.06	$\frac{170}{163.75}$ =1.04	$\frac{171}{163}$ =1.05	$\frac{174}{164.25}$ =1.06

$$SI = \frac{1.06 + 1.04 + 1.05 + 1.06}{4}$$

$$= 1.1$$

(1 mark)

b.

In autumn, sales are 10% higher than sales in an average season.

(1 mark)

Module 1 Number patterns and applications**Question 1**

a. 300, 350, 400... Arithmetic sequence $a = 300, d = 50, n = 10$ $t_{10} = 300 + 9 \times 50 = 750$ tonnes (1 mark)	b. $t_n = 300 + (n-1)50$ $t_n = 300 + 50n - 50 = 250 + 50n$ tonnes (1 mark)
c. This is an arithmetic sequence $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{20} = \frac{20}{2}[2 \times 300 + 19 \times 50]$ $S_{20} = 10 \times 1550 = 15500$ tonnes (1 mark)	d. Use T1-83 graphics calculator in sequence mode. $y =$ enter $nMin = 1$ $\mu_n = 300 + (n-1) \times 50$ $\mu(nMin) = 300$ Press 2nd table Scroll down to $n = 20$ This gives the value $\mu_n = 1250$ This is amount extracted in 20th year. Amount extracted in 21st year = $1250 - 75 = 1175$ tonnes (1 mark)

Module 1 Number patterns and applications**Question 1 (continued)**

e. Use graphics calculator in sequence mode. $y = \text{enter}$ $nMin = 1$ $\mu_n = 1175 - (n - 1)75$ $\mu(nMin) = 1175$ Press 2nd table Scroll down to find the first μ_n value which is less than 300 This is 275 when $n = 13$ Since $n = 1$ corresponds to year 21 then $n = 13$ corresponds to year 33 (1 mark)	f. Use same calculator screen as for 1e. If first year is 2000 then 21st. year is 2020. This corresponds to $n = 1$ on calculator. Cannot mine a negative amount of copper, So mining ceases when $n = 16$ which is year 2035 (1 mark)
---	---

Module 1 Number patterns and applications**Question 2**

a. Geometric sequence $a = 800, r = 1 - 0.085 = 0.915$ $t_2 = ar = 800 \times 0.915 = 732$ tonnes (1 mark)	b. $S_n = \frac{a(1-r^n)}{(1-r)}$ $S_{10} = \frac{800(1-0.915^{10})}{(1-0.915)} = 5540.24 \text{ tonnes}$ (1 mark)
c. $S_{\infty} = \frac{a}{(1-r)}$ $\Rightarrow S_{\infty} = \frac{800}{(1-0.915)}$ $\Rightarrow S_{\infty} = 9411.76 \text{ tonnes}$ (1 mark)	d. Use calculator in sequence mode. Press $y = nMin = 1$ $\mu_n = 800 \times 0.915^{(n-1)}$ $\mu(nMin) = 800$ Press 2nd table Scroll down to find the first μ_n value which is less than 100 This is 94.885 when $n = 25$ So mine will close in 25th year of production (1 mark)

Module 1 Number patterns and applications**Question 2 (continued)**

e. Use calculator in sequence mode and keep same input as for d. Then add $v_n = 300 + (n - 1)50$ $\mu(nMin) = 300$ Press 2nd table Scroll down to find the first v_n value which is greater than μ_n This is $550 > 513.09$ when $n = 6$ So in the 6th year of production (1 mark)	f. Use same screen as e. When $n = 10$ <i>Acme</i> is 750 and <i>Beta</i> is 359.65 $750 - 359.65 = 390.35$ tonnes. (1 mark)
--	--

Module 1 Number patterns and applications**Question 3**

a. $P_n = 1.15P_{n-1}$ $P_0 = 250000$ (1 mark)	b. $P_4 = 1.15^4 \times 250000$ $= \$437251.56$ (1 mark)
c. $t_4 = 1100 + 700 = 1800$ $t_5 = 1800 + 1100 = 2900$ $t_6 = 2900 + 1800 = 4700 \text{ m.}$ (1 mark)	

Module 2 Geometry and trigonometry**Question 1****a.**

$$AC^2 = 10^2 + 12^2$$

$$AC = \sqrt{10^2 + 12^2} = 15.62 \text{ km.}$$

(1 mark)

b.

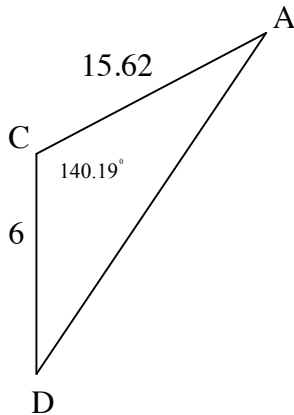
$$\tan \angle ACB = \frac{12}{10}$$

$$\angle ACB = \tan^{-1}\left(\frac{12}{10}\right) = 50.19^\circ$$

$$\angle ACD = 50.19^\circ + 90^\circ = 140.19^\circ$$

1 mark for $\angle ACB$, 1 mark for $\angle ACD$

(2 marks)

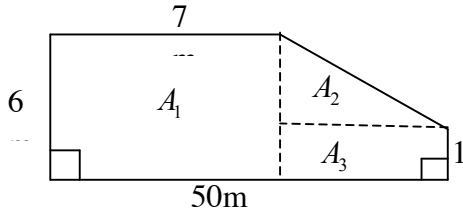
c.

$$AD^2 = 6^2 + 15.62^2 - 2 \times 6 \times 15.62 \times \cos 140.19$$

$$AD = \sqrt{6^2 + 15.62^2 - 2 \times 6 \times 15.62 \times \cos 140.19}$$

$$AD = 20.6 \text{ km.}$$

(1 mark)

Module 2 Geometry and trigonometry**Question 2****a.**

$$A_1 = 6 \times 7 = 42$$

$$A_2 = \frac{43 \times 5}{2} = 107.5$$

$$A_3 = 43 \times 1 = 43$$

$$\text{Total area} = 42 + 107.5 + 43 = 192.5 \text{ m}^2$$

(1 mark)

b.Volume = Area of base $\times$ height

$$\text{Volume} = 192.5 \times 12 = 2310 \text{ m}^3$$

(1 mark)

c.

$$\text{Area to be tiled} = 2(12 \times 8) + 2(50 \times 8) + (12 \times 50)$$

$$\text{Area to be tiled} = 1592 \text{ m}^2$$

(1 mark)

d.

$$\text{Number tiles} = \frac{1592 \times 100 \times 100}{300} = 53066.67$$

Need to buy 53100 tiles.

(1 mark)

e.

$$\text{Number of packs} = \frac{53100}{1000} = 53.1$$

Need 54 packs

$$\text{Cost} = 120 \times 54 = \$6480$$

(1 mark)

f.

Area under water

$$= 2(192.5) + (6 \times 12) + (1 \times 12) + (50 \times 12)$$

$$\text{Area under water} = 1069$$

$$\% \text{ under water} = \frac{1069}{1592} \times 100 = 67.1\%$$

(1 mark) for Area under water

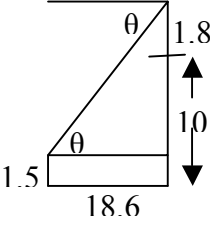
(1 mark) for percentage.

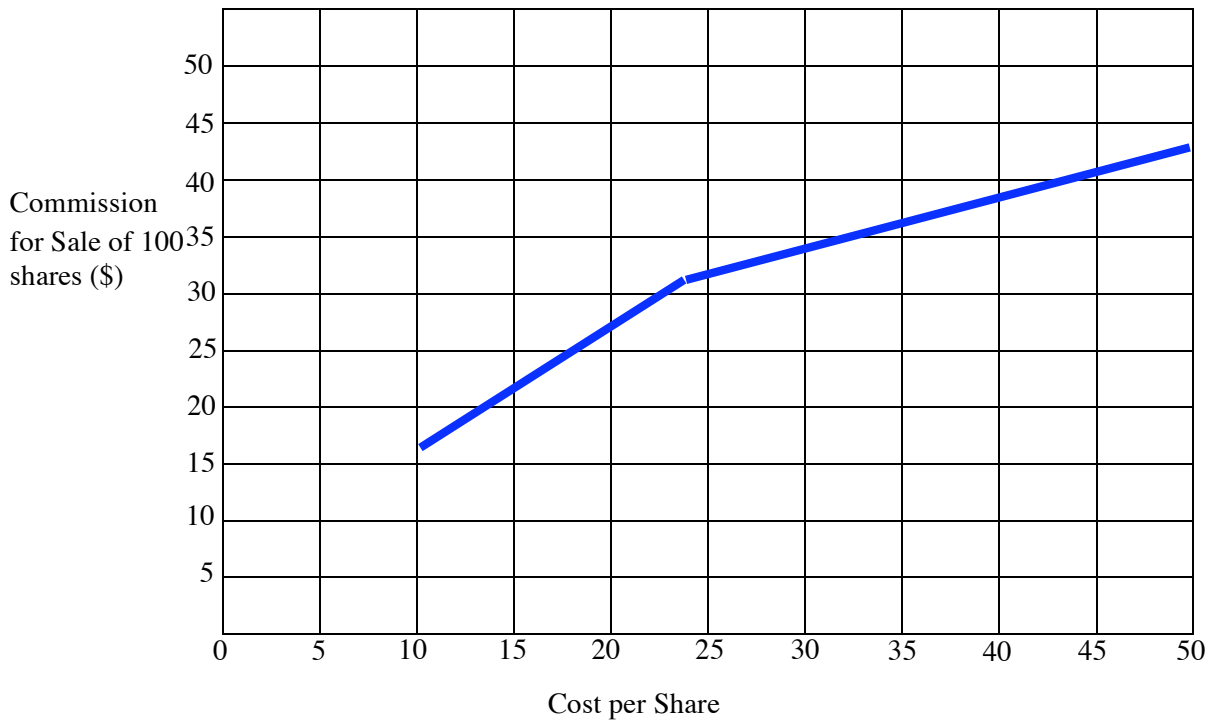
(2 marks)

Module 2 Geometry and trigonometry**Question 3**

a. From 2b. Vol. of water in pool = 2310 7.5 m^3 in 1 min 2310 m^3 in $\frac{2310}{7.5} = 308$ mins = $\frac{308}{60} = 5.1$ hrs. (1 mark)	b. $V = \pi r^2 h$ $50 = \pi r^2 4.9$ $r^2 = \frac{50}{4.9\pi}$ $r = \sqrt{\frac{50}{4.9\pi}} = 1.8$ $d = 2 \times 1.8 = 3.6 \text{ m}$ (1 mark)
---	--

Question 4

a. $XY = \frac{1}{2} AC$ $AC = \sqrt{30^2 + 22^2}$ $XY = \frac{1}{2} \sqrt{30^2 + 22^2} = 18.6 \text{ m}$ (1 mark)	b.  $\tan \theta = \frac{10.3}{18.6}$ $\theta = \tan^{-1} \left(\frac{10.3}{18.6} \right) = 29^\circ$ (1 mark)
--	--

Module 3 Graphs and relations**Question 1 (continued)****d.**When $p = 10$, $C = 7 + 10 = 17$ When $p = 23.9$, $C = 7 + 23.9 = 30.9$ When $p = 24$, $C = 19 + 0.5 \times 24 = 31$ When $p = 49.9$, $C = 19 + 0.5 \times 49.9 = 43.95$ When $p = 50$, $C = 39 + 0.1 \times 50 = 44$

Plot points on graph and join with straight lines since all equations are equations of straight lines.

For correct straight line from 10 to 24

(1 mark)

For correct straight line from 24 to 50

(1 mark)

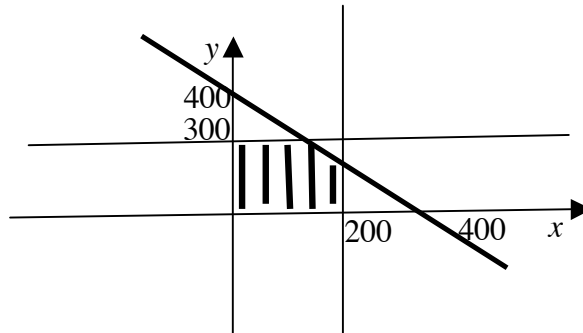
Module 3 Graphs and relations**Question 2**

a. $C = 4245 + 5.40x$ (1 mark)	b. $S = 62x$ (1 mark)
c. $4245 + 5.4x = 62x$ $56.6x = 4245$ $x = 75$ 75 shirts must be sold to break even. (1 mark)	d. $P = SP - CP$ $P = 62x - 4245 - 5.4x$ $P = 56.6x - 4245$ (1 mark)
e. $56.6x - 4245 = 25639.80$ $56.6x = 29884.80$ $x = 528$ (1 mark)	

Module 3 Graphs and relations**Question 3****a.**

$$x + y \leq 400$$

(1 mark)

b. and c.

For the straight line (1 mark)

For the shaded region (1 mark)

d.

$$P = 52x + 48.5y$$

(1 mark)

e.

Corner point (0,0) $P = 52 \times 0 + 48.5 \times 0 = 0$

Corner point (0,300) $P = 52 \times 0 + 48.5 \times 300 = 14550$

Corner point (100,300) $P = 52 \times 100 + 48.5 \times 300 = 19750$

Corner point (200,200) $P = 52 \times 200 + 48.5 \times 200 = 20100$

Corner point (200,0) $P = 52 \times 200 + 48.5 \times 0 = 10400$

Maximum profit is \$20100

(1 mark)

Module 4 Business-related mathematics**Question 1**

a. Tax paid = $18000 + 0.40(8000) = \$21,200$ (1 mark)	
b. i. Last year paid $0.15(33000 - 6000) = 4050$ This year pays $4200 + 0.30 \times 4000 = 5400$ Extra tax paid = $5400 - 4050 = \$1350$ (1 mark)	b. ii. Last year take home = $33000 - 4050 = 28950$ This year take home = $38000 - 5400 = 32600$ Extra take home this year = $32600 - 28950 = 3650$ Gets to actually take home \$3650 more. (1 mark)

Question 2

a. $\frac{20}{100} \times 7200 = \1440 (1 mark)	b. Amount for work = $7200 \times \frac{100}{110} = 6545.45$ GST = $7200 - 6545.45 = \$654.55$ (1 mark)
c. Amount repaid = $475.20 \times 20 = \$9504$ (1 mark)	d. Interest paid = $9504 - (7200 - 1440) = 3744$ (1 mark) $R = \frac{100I}{PT} = \frac{100 \times 3744}{(7200 - 1440) \times 5} = 13\%$ (1 mark)

Module 4 Business-related mathematics**Question 3**

a. Use graphics calculator Press Apps Finance Enter TVM Solver Enter $N = 120$ $I = 6.5$ $PV = 25000$ $PMT =$ $FV = 0$ $P / Y = 12$ $C / Y = 12$ End Put cursor on PMT and press alpha solve. $PMT = \\$283.87$ (1 mark)	b. Total amount repaid = 120×283.86994 $= 34064.39$ Interest = $34064.39 - 25000 = \\$9064.39$ $= \\$9064$ to nearest dollar. (1 mark)
c. Use graphics calculator Press Apps Finance Enter TVM Solver Enter $N = 60$ $I = 6.5$ $PV = 25000$ $PMT = -283.86994.....$ $FV =$ $P / Y = 12$ $C / Y = 12$ End Put cursor on PMT and press alpha solve. $FV = \\$14508.21818$ (1 mark)	Again use graphics calculator Press Apps Finance Enter TVM Solver Enter $N =$ $I = 6.5$ $PV = 14508.21818$ $PMT = -400$ $FV = 0$ $P / Y = 12$ $C / Y = 12$ End Put cursor on PMT and press alpha solve. $N = 40.4911$ months = 3.4 years (1 mark)

Module 4 Business-related mathematics

Question 4

a. $0.15 \times 100000 = \$15000$ (1 mark)	b. $\text{Depreciation} = 25000 - 5000 = 20000$ $20000 = x \times 0.15$ $x = \frac{20000}{0.15} = 133333 \text{ km.}$ (1 mark)
c. With calculator in function mode enter $y = 25000(0.92)^x$ Press 2nd table and scroll down to $y = 11804$ This corresponds to $x = 9$ Hence, car will be valued at less than \$12000 after 9 years. (1 mark)	

Module 5 Networks and decision mathematics**Question 1****a.**

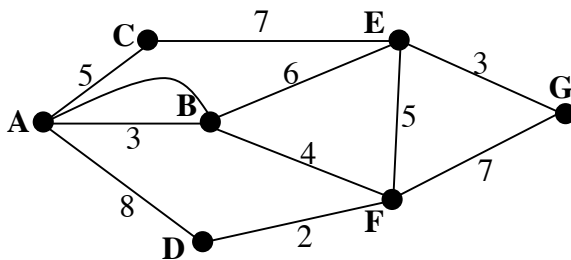
Since all the vertices are not of an even degree,
then an Euler circuit does not exist.

(1 mark)

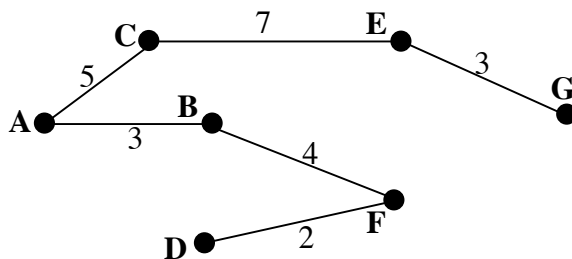
b.

B as this is the only other vertex of odd degree.

(1 mark)

c.

(1 mark)

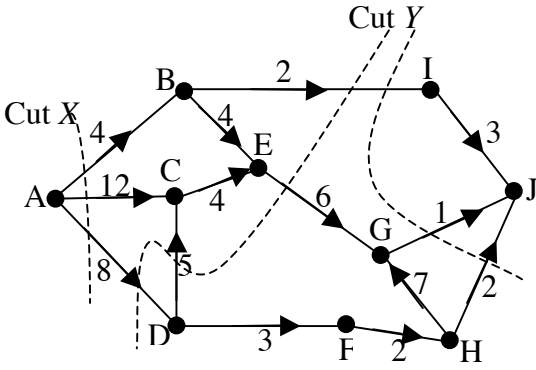
d.

$$2 + 4 + 3 + 5 + 7 + 3 = 24$$

There are other routes that will also give 24.

(1 mark)

Module 5 Networks and decision mathematics**Question 2**

a. i. $4 + 12 + 8 = 24$ (1 mark)	a. ii. $2 + 6 + 8 = 16$ (1 mark)
b. $2 + 1 + 2 = 5$ (1 mark)	c.  (1 mark)

Module 5 Networks and decision mathematics**Question 3****a.**

Activity	Immediate Predecessor	Earliest Starting Time (days)
A	None	0
B	A	5
C	A , E	24
D	B	11
E	A, D	19
F	C , E , D	37

(3 marks)

(1 mark) for each of the three values

b. Critical path is the longest path. A – B – D – E – C – F – G (1 mark)	c. There is no slack time for E because E lies on the critical path. (1 mark)
d. i. There is now a new critical path: A – E – C – F – G This takes 35 days instead of the original 38 days, so 3 days are saved. (1 mark)	d. ii. He saves \$3000 by saving 3 days but he has to pay 4 workers \$200 per day for 1 day, so he would save $3000 - 800 = \$2200$. It is worth doing. (1 mark)

Module 6 Matrices**Question 1**

a. $\begin{matrix} R \\ P \end{matrix} \begin{bmatrix} x \\ y \end{bmatrix}$ (1 mark)	b. 3×2 (1 mark)
c. i. $\begin{bmatrix} 7 & 3 \\ 2 & 5 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 200 \\ 120 \end{bmatrix}$ (1 mark)	c. ii. $\begin{bmatrix} 7 & 3 \\ 2 & 5 \\ 4 & 1 \end{bmatrix} \times \begin{bmatrix} 200 \\ 120 \end{bmatrix} = \begin{bmatrix} 1760 \\ 1000 \\ 920 \end{bmatrix}$ 1760 of a, 1000 of b and 920 of c. (1 mark)

Module 6 Matrices**Question 2****a.**

$$\begin{bmatrix} 4.80 \\ 5.00 \\ 5.20 \end{bmatrix}$$

(1 mark)

b.

Let \$x\$ be the price per kilogram of raisins

Let \$y\$ be the price per kilogram of sultanas

Let \$z\$ be the price per kilogram of currants

$$\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4.80 \\ 5.00 \\ 5.20 \end{bmatrix}$$

For correct matrices (1 mark)

For correct equation (1 mark)

c.

Use graphics calculator.

Enter

$$\begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \end{bmatrix} \text{ in } A$$

Enter

$$\begin{bmatrix} 4.80 \\ 5.00 \\ 5.20 \end{bmatrix} \text{ in } B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B = \begin{bmatrix} 7.20 \\ 1.20 \\ 5.20 \end{bmatrix}$$

1 kg. raisins cost \$7.20, 1kg. sultanas cost \$1.20,
1 kg currants cost \$5.20

(1 mark)

d.

$$X = Q - P$$

$$X = \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \end{bmatrix} - \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.2 \\ 0.4 & 0.2 & 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} -0.3 & -0.1 & 0.4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(1 mark)

e.

X says that the mixture in Home brand is changed by increasing the currants by 0.4 kg and decreasing the raisins and sultanas by 0.3 kg and 0.1 kg respectively.

(1 mark)

Module 6 Matrices**Question 3**

a. This Week Buy Not Buy Next Week Buy $\begin{bmatrix} 0.68 & 0.25 \\ 0.32 & 0.75 \end{bmatrix}$ (1 mark)	b. $S_0 = \begin{bmatrix} 500 \\ 200 \end{bmatrix}$ (1 mark)
c. $S_3 = T^3 S_0$ $S_{30} = \begin{bmatrix} 0.68 & 0.25 \\ 0.32 & 0.75 \end{bmatrix}^3 \begin{bmatrix} 500 \\ 200 \end{bmatrix} = \begin{bmatrix} 322.4 \\ 377.6 \end{bmatrix}$ Expect 322 people to buy goats cheese in three weeks time. (1 mark)	d. $S_{100} = T^{100} S_0$ $S_{100} = \begin{bmatrix} 0.68 & 0.25 \\ 0.32 & 0.75 \end{bmatrix}^{100} \begin{bmatrix} 500 \\ 200 \end{bmatrix} = \begin{bmatrix} 307 \\ 393 \end{bmatrix}$ In the long term expect 393 customers to not buy goats cheese. (1 mark)
e. Let number of people who bought blue, cheddar, brie and gouda last week be x, $3x$, $2x$ and x, respectively. $\begin{bmatrix} 0.1 & 0.2 & 0.2 & 0.9 \end{bmatrix} \begin{bmatrix} x \\ 3x \\ 2x \\ x \end{bmatrix} = 0.1x + 0.6x + 0.4x + 0.9x = 2x = 800$ $x = 400$ Total number to buy four cheeses = $x + 3x + 2x + x = 7x$ $7x = 7 \times 400 = 2800$ (1 mark)	

End of suggested solutions 2009 Further Mathematics VCE Trial Examination 2

Kilbaha Multimedia Publishing PO Box 2227 Kew Vic 3101 Australia	Tel: (03) 9817 5374 Fax: (03) 9817 4334 kilbaha@gmail.com http://kilbaha.googlepages.com
---	---