

2009 Further Mathematics Trial Exam 1 Solutions

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SECTION A Core: Data analysis

1	2	3	4	5	6	7	8	9	10	11	12	13
C	B	D	B	A	D	D	C	A	C	C	E	A

SECTION B

Module 1: Number patterns and applications

1	2	3	4	5	6	7	8	9
E	A	D	A	C	C	E	E	C

Module 5: Networks and decision mathematics

1	2	3	4	5	6	7	8	9
D	E	D	A	C	C	B	E	D

Module 6: Matrices

1	2	3	4	5	6	7	8	9
C	A	C	D	E	D	A	D	C

SECTION A Core: Data analysis

Q1 The interval 160 -164 has the highest frequency. C

Q2 By inspection 164 cm cannot be the mean of the right side of the back-to-back stemplot. $\therefore$ the right side must be the data of male students. The mean is 157.8 cm. B

Q3 Mean height = $\frac{164 \times 42 + 158 \times 32}{74} = 161.4$ cm. D

Q4 A back-to-back stemplot is used to display the relationship between a numerical variable (height) and a two-valued categorical variable (gender). B

Q5 25% in the interval 15-23 mm, less than 25% in the interval 35-52 mm. A

Q6 Number of fish greater than 35 mm is 80. There are 4 outliers greater than 35 mm.

Percentage = $\frac{4}{80} \times 100\% = 5\%$. D

Q7 The mean is 250 and the standard deviation is about 50. The interval 200-400 is from $\mu - \sigma$ to $\mu + 3\sigma$ approximately.

Percentage $\approx \frac{68\%}{2} + 50\% = 84\%$. D

Q8 105 is about 3σ below μ . $\therefore z \approx -3$. C

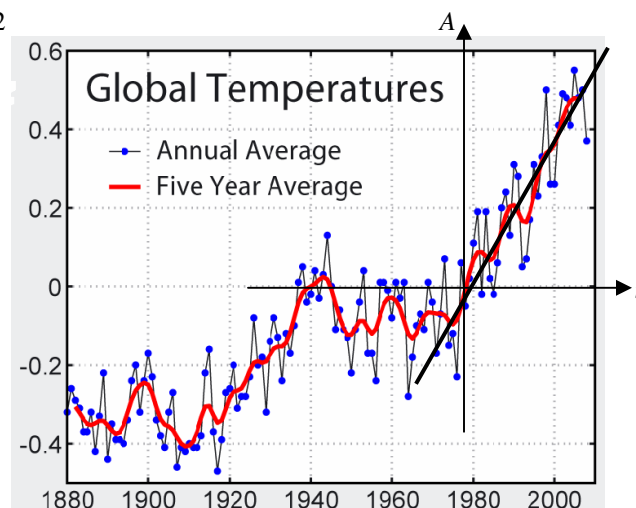
Q9 $\frac{s_y}{s_x} = \frac{b}{r}$, $r = 0.80$ (given), $b = 1.2$ (gradient).

$\frac{s_{(\log y)}}{s_x} = \frac{1.2}{0.80} = 1.5$. A

Q10 C

Q11 Sum $\approx -0.05 + 0.01 + 0.11 + 0.19 - 0.02 + 0.19 = 0.43$. C

Q12



Gradient of trend line ≈ 0.019 , vertical axis intercept ≈ -0.04 . E

Q13 A

SECTION B

Module 1: Number patterns and applications

Q1 There is no common ratio in each of the four series. E

Q2 Consider 71.5 and 67 as the first and second terms of the sequence. $\therefore a = 71.5$, $d = 67 - 71.5 = -4.5$.

$S_{108} = \frac{108}{2} (2 \times 71.5 + (108 - 1) \times -4.5) = -18279$.

The sum of the middle 54 terms = $\frac{1}{2} \times -18279 = -9139.5$. A

Q3 Common difference: $\frac{1}{a} - \frac{1}{2} = \frac{1}{4} - \frac{1}{a}$, $\therefore \frac{2}{a} = \frac{3}{4}$, $\therefore a = \frac{8}{3}$. D

Q4 By inspection, $t_{n+1} = t_n + 4n$. A

Q5 Given $t_4 = 25$, $t_5 = 25 + 4 \times 4 = 41$, ..., $t_9 = 145$, $t_{10} = 145 + 4 \times 9 = 181$. The tenth pattern has 181 ten-cent coins valued at \$18.10. C

Note: The choices in the question have the decimal point incorrectly placed. A. 14.50, B. 15.30, C. 18.10, D. 22.10, E. 22.50.

Q6 $11001.10011001..... = 10000 + 1001.10011001.....$
 $= 10000 + 1001 + 0.1001 + 0.00001001 + 0.000000001001 +$
 $= 10000 + \frac{1001}{1 - 0.0001}$. $\therefore a = 1001$. C

$$Q7 \text{ Common ratio} = \frac{t_2}{t_1} = \frac{-30}{45} = -\frac{2}{3}, \therefore \frac{t_{n+1}}{t_n} = -\frac{2}{3},$$

$$\therefore 3t_{n+1} + 2t_n = 0. \quad E$$

Q8 Use $u_{n+1} = a + bu_n$ and 20 and 10 to set up equation (1)

$10 = a + 20b$. Use $u_{n+1} = a + bu_n$ and 10 and 25 to set up

equation (2) $25 = a + 10b$.

Solve the 2 equations simultaneously to obtain $a = 40$ and

$$b = -1.5. \quad E$$

Q9 Third bounce distance = 98 cm.

Third fall distance = $98 + 5 = 103$ cm.

Total distance for the third fall and bounce = $103 + 98 = 201$ cm.

Total distance for the second fall and bounce

$$= 108 + 103 = 211 \text{ cm.}$$

Total distance for the first fall and bounce

$$= 113 + 108 = 221 \text{ cm.}$$

$$a = 221, d = -10, S_{18} = \frac{18}{2}(2 \times 221 + (18-1) \times -10) = 2448. \quad C$$

Module 5: Networks and decision mathematics

Q1 D

Q2 E

Q3 Euler's formula: $v - e + f = 2$. f is an even number.

$\therefore$ either both v and e are even numbers or both v and e are odd numbers. D

Q4 A

Q5 An Euler path is a path that includes every edge just once.

A Hamiltonian path is a path that passes through each vertex just once, and the starting vertex is different from the finishing vertex. C

Q6 A spanning tree is a subgraph and a tree containing all the vertices of the graph. There are 8 vertices in the graph. $\therefore$ exactly 7 edges. C

Q7 Maximum flow from P to Q = minimum cut = 60.

Maximum flow from R to Q = $20 + 20 + 10 = 50$.

Minimum flow from R to Q = $20 + 20 + 0 = 40$.

$$\text{Difference} = 50 - 40 = 10. \quad B$$

Q8 E

Q9 One-step dominance matrix:

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 1 & 0 & 0 & 0 \\ B & 0 & 0 & 1 & 0 & 0 \\ C & 1 & 0 & 0 & 1 & 1 \\ D & 1 & 1 & 0 & 0 & 0 \\ E & 1 & 1 & 0 & 1 & 0 \end{array}$$

Two-step dominance matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 \end{bmatrix}$$

$$\text{Sum of missing entries} = 3 + 0 = 3. \quad D$$

Module 6: Matrices

Q1 The order of product $X \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ must be the same as that of

$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 0 & 4 \end{bmatrix}$, i.e. 2×3 . $\begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ has an order of 4×3 , $\therefore X$ must have

an order of 2×4 . C

$$Q2 \begin{bmatrix} a & 2 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} -2a+2 & a+2b \\ 0 & \frac{1}{2}+b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$\therefore -2a + 2 = 1 \text{ and } \frac{1}{2} + b = 1.$$

$$\therefore a = \frac{1}{2} \text{ and } b = \frac{1}{2}. \quad A$$

$$Q3 \begin{bmatrix} -1 & -2 \\ 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{3}{5} & -\frac{1}{5} \end{bmatrix}. \therefore X = \begin{bmatrix} -6 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{3}{5} & -\frac{1}{5} \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -1 & -1 \end{bmatrix}. \quad C$$

Q4 D

Q5 A transition matrix (i) is a square matrix, (ii) has no negative elements, and (iii) the sum of elements in each column must be 1. E

Q6 D

$$Q7 [a \ b \ c] = [2 \ -3 \ 1] \begin{bmatrix} -1 & 2 & 1 \\ 1 & -5 & -3 \\ 1 & 0 & 2 \end{bmatrix}^{-1} = [0 \ 0.6 \ 1.4]. \quad A$$

Q8 Apply the transition matrix repeatedly:

$$\begin{bmatrix} 3500 \\ 2800 \\ 3800 \\ 2500 \end{bmatrix} \rightarrow \begin{bmatrix} 2966 \end{bmatrix} \rightarrow \begin{bmatrix} 3095 \end{bmatrix} \rightarrow \dots$$

$$\% \text{ change in the first transition} = \frac{2966 - 2800}{2800} \times 100\% = 5.9\%.$$

$$\% \text{ change in the next transition} = \frac{3095 - 2966}{2966} \times 100\% = 4.3\%.$$

D

$$Q9 \text{ For large enough } n, \begin{bmatrix} 0.88 & 0.02 & 0.04 & 0.01 \\ 0.05 & 0.92 & 0.05 & 0.01 \\ 0.03 & 0.03 & 0.85 & 0.01 \\ 0.04 & 0.03 & 0.06 & 0.97 \end{bmatrix}^n \begin{bmatrix} 3500 \\ 2800 \\ 3800 \\ 2500 \end{bmatrix} \rightarrow \begin{bmatrix} 7193 \end{bmatrix}.$$

$$\% \text{ of 12600 customers} = \frac{7193}{12600} \times 100\% \approx 57\%. \quad C$$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors