

Student Name: _____

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 1



2008 Trial Examination

Reading Time: 15 minutes

Writing Time: 1 hour and 30 minutes

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of modules</i>	<i>Number of modules to be answered</i>	<i>Number of marks</i>
A	13	13			13
B	54	27	6	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers, one bound reference (may be a textbook) that may be annotated (typed or handwritten), one approved graphics calculator or CAS and if desired a scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 35 pages.
- Working space provided throughout the book.

Instructions

- Print your **name** in the space provided on the top of this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

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Working space

TURN OVER

SECTION A**Instructions for Section A**

Answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Core – Data analysis**Question 1**

Maurie compared his marks in five subjects with the class average and standard deviation. In which subject did Maurie have the best standard score?

Subject	Maurie's grade	Class average	Class standard deviation
Art	60	52	6
Biology	72	65	7
Chemistry	78	72	3
Drawing	65	48	15
English	82	80	10

- A. Art
- B. Biology
- C. Chemistry
- D. Drawing
- E. English

Question 2

Autonomous Secondary College senior students are voting on a logo for the 2009 jumper. The results are shown in the table below.

	Male	Female	Total
For	39	37	76
Against	29	45	74
Total	68	82	150

Of the females who voted, the percentage who are in favour of the design is closest to

- A. 45%
- B. 49%
- C. 51%
- D. 55%
- E. 61%

SECTION A- continued

The following data relates to questions 3 and 4.

<i>x</i>	1	2	3	4	5	6	7	8	9	10
<i>y</i>	8	7	5	5	6	5	3	4	1	2

Question 3

A scatterplot of this data would show

- A. A strong positive correlation
- B. A moderate positive correlation
- C. No correlation
- D. A moderate negative correlation
- E. A strong negative correlation

Question 4

The coefficient of determination for this data would be closest to

- A. -0.91
- B. -0.83
- C. -0.65
- D. 0.83
- E. 0.91

Question 5

The mean height of basketballers in our local team is 190 cm with a standard deviation of 2.5 cm. Last season the mean number of points scored per game was 52 with a standard deviation of 8. There is a correlation factor of $r = 0.86$ between height and points scored.

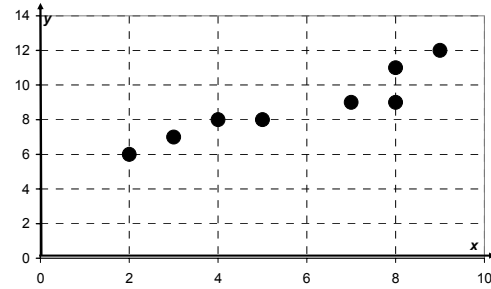
The equation of the least squares regression line that would enable you to predict basketballers score given his height is closest to

- A. $S = 0.7 + 0.27H$
- B. $S = 47 + 2.75H$
- C. $S = -470 + 2.75H$
- D. $S = 138 + 0.86H$
- E. $S = -47 + 0.27H$

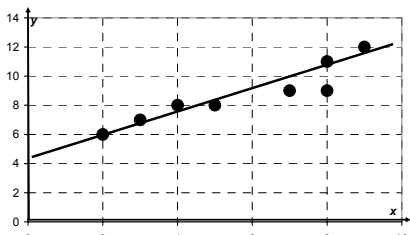
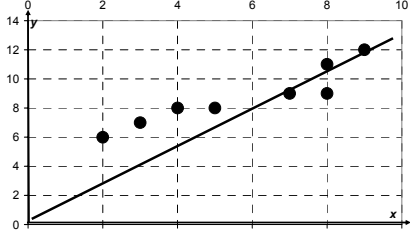
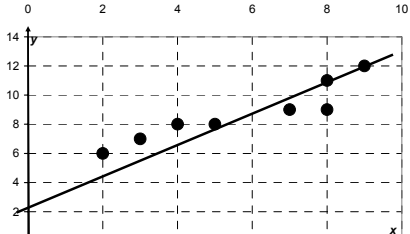
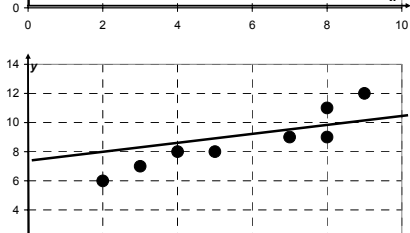
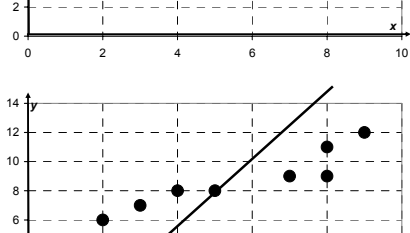
SECTION A – continued
TURN OVER

Question 6

A three median regression line is to be applied to the data graphed at right.



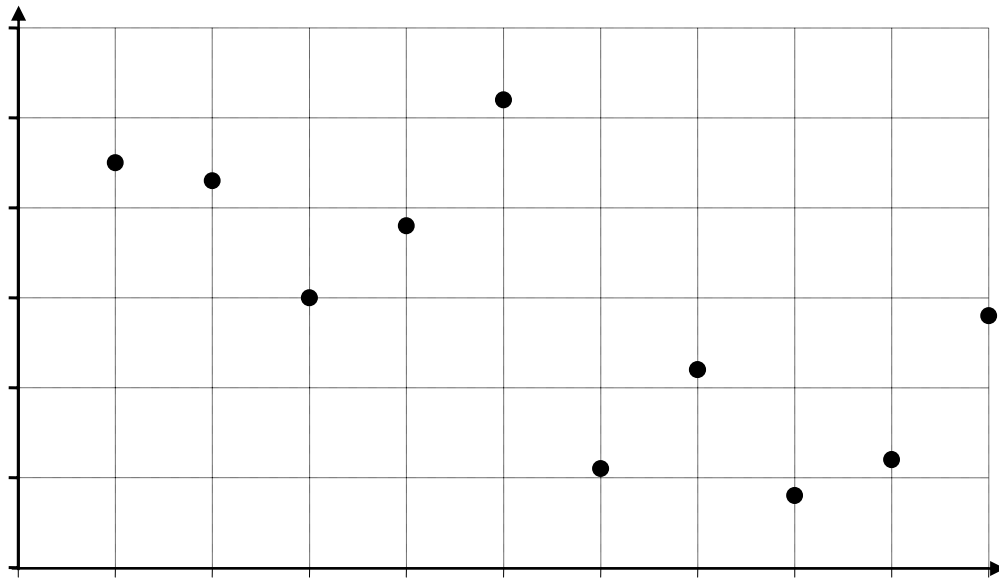
The graph which best illustrates this line is

- A. 
- B. 
- C. 
- D. 
- E. 

SECTION A – continued

The following information relates to questions 7 and 8.

The graph shows the total number of absences each week for a ten week school term.



<i>Week</i>	1	2	3	4	5	6	7	8	9	10
<i>Absences</i>	85	83	70	78	92	51	62	48	52	68

Question 7

From this scatterplot it can be said that over the ten week period absences showed

- A. No overall trend.
- B. A decreasing secular trend
- C. An increasing secular trend
- D. A decreasing cyclic trend
- E. An increasing cyclic trend.

Question 8

Using a three point moving mean method to smooth this data, the smoothed values for weeks 4 and 8 would be closest to

- A. 74, 56
- B. 77, 54
- C. 78, 52
- D. 80, 54
- E. 179, 127

**SECTION A – continued
TURN OVER**

The following information relates to questions 9 and 10.

The Cuppacafe Coffee Shop is open from Wednesday to Sunday. The following table shows their annual profit for each week day from 2006 and the calculated seasonal indices for each day.

	Wednesday	Thursday	Friday	Saturday	Sunday
Profit (2006)	\$12000	\$14000	\$13500	\$20000	\$25000
Seasonal Indices	0.78	0.85	0.82		1.35

Question 9

The seasonal index for Saturday is closest to

- A. 0.20
- B. 0.24
- C. 1.10
- D. 1.20
- E. 4.23.

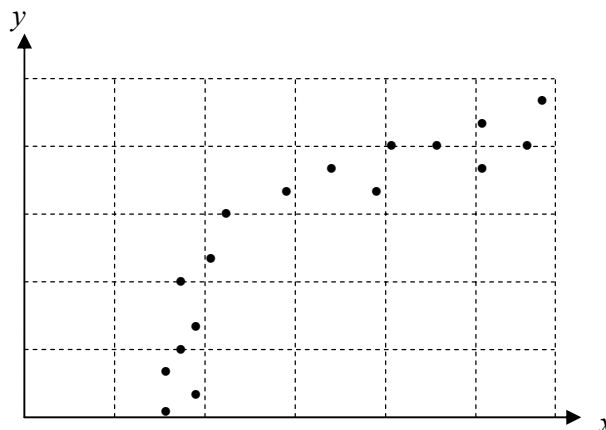
Question 10

The deseasonalised profit for Fridays in 2006 is closest to

- A. 11100
- B. 15800
- C. 16500
- D. 16900
- E. 84500

Question 11

The graph below shows a non-linear scatterplot.



The best transformation which could be applied to this data to make it more linear would be

- A. x^2
- B. x^3
- C. $\log x$
- D. $\log y$
- E. $\frac{1}{y}$

SECTION A – continued

The following information relates to questions 12 and 13.

A company was investigating their monthly advertising budget and total sales for the first six months of 2007.

Advertising budget (\$'000)	0.9	0.7	0.8	1.0	0.5	0.3
Total sales (\$'000)	100.2	74.6	99.8	89.1	57.2	42.0

Question 12

The value of Pearson's correlation coefficient which indicates how total sales are affected by the advertising budget is closest to

- A. 0.24
- B. 0.26
- C. 0.84
- D. 0.92
- E. 23.8

Question 13

The equation of the least squares regression line is

$$Sales = 18.45 + 83.85 \times Advertising.$$

In the next month the company spent 0.7 (\$'000) on advertising. Their total sales for this month were 94.5 (\$'000).

Comparing this result to the one predicted using the regression line produces a residual in (\$'000) closest to

- A. 2.55
- B. 17.36
- C. 19.90
- D. 66.15
- E. 77.15

**END OF SECTION A
TURN OVER**

SECTION B**Instructions for Section B**

Select **three** modules and answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Module		Page
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SECTION B- continued

Module 1: Number patterns

Answer all questions in this module.

Question 1

The sequence -8, 4, -2, 1, -0.5 could be described as

- A. A finite arithmetic sequence
- B. An infinite arithmetic sequence
- C. A finite geometric sequence
- D. An infinite geometric sequence
- E. A second order sequence

Question 2

The 51st term in the sequence -12, -9.5, -7, -4.5, is

- A. 50
- B. 51
- C. 110.5
- D. 113
- E. 115.5

Question 3

If the third term of a geometric sequence is 18, the fifth term is 162 and the common ratio is positive, the sum of the first six terms is

- A. 57
- B. 153
- C. 486
- D. 728
- E. 6556

Question 4

A sequence is defined by the difference equation $t_{n+1} = 3t_n + 2$ $t_0 = -2$.

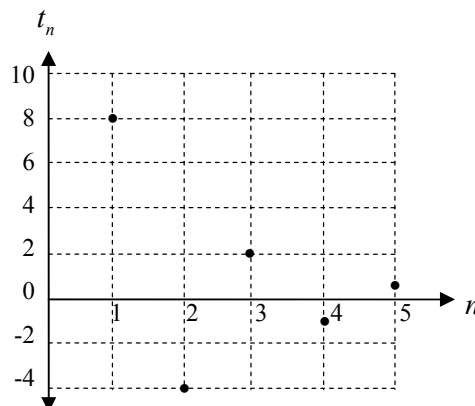
The fourth term in the sequence is

- A. -82
- B. -28
- C. 4
- D. 28
- E. 82

SECTION B- Module 1: Number patterns- continued
TURN OVER

Question 5

The first five terms of a sequence are plotted on the graph.



The sequence can be described as

- A. An arithmetic sequence with $a = 8$ and $d = 0.5$
- B. An arithmetic sequence with $a = 8$ and $d = -0.5$
- C. A geometric sequence with $a = 8$ and $r = 0.5$
- D. A geometric sequence with $a = 8$ and $r = -0.5$
- E. A second order sequence with $t_{n+2} = t_{n+1} \div t_n$

Question 6

The sequence 3, 6, 12, 24, 48, ... can be defined by the difference equation

- A. $t_{n+1} = t_n + 2$, $t_1 = 3$
- B. $t_{n+1} = t_n + 3$, $t_1 = 3$
- C. $t_{n+1} = 2t_n$, $t_1 = 3$
- D. $t_{n+1} = 3t_n$, $t_1 = 3$
- E. $t_{n+1} = t_n^2$, $t_1 = 3$

Question 7

The office manager has estimated that the number of files (A) increases by 5% each year. Initially they had 2500 files. The difference equation that would describe this is

- A. $A_n = 0.05A_{n-1} + 2500$ where $A_0 = 0$
- B. $A_n = 0.05A_{n-1}$ where $A_0 = 2500$
- C. $A_n = 0.95A_{n-1}$ where $A_0 = 2500$
- D. $A_n = 1.05A_{n-1} + 2500$ where $A_0 = 0$
- E. $A_n = 1.05A_{n-1}$ where $A_0 = 2500$

Question 8

For a Fibonacci sequence given by the equation

$$f_{n+2} = f_n + f_{n+1} \quad \text{where} \quad f_1 = 1, \quad f_2 = 4,$$

The value of f_6 is

- A. 21
- B. 23
- C. 37
- D. 60
- E. 1024

Question 9

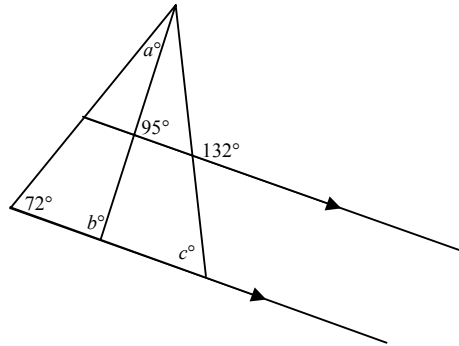
Mark is saving to buy a new car. He invests \$6000 into an account paying 4% per annum with interest compounded quarterly. The amount of money he will have in his account after 3 years can be calculated by

- A. $6(12000 + 11 \times 1.04)$
- B. 6000×1.01^{12}
- C. 6000×1.04^3
- D. $\frac{6000(1.01)^{12}}{0.01}$
- E. $\frac{6000(1.04)^3}{0.04}$

SECTION B- continued
TURN OVER

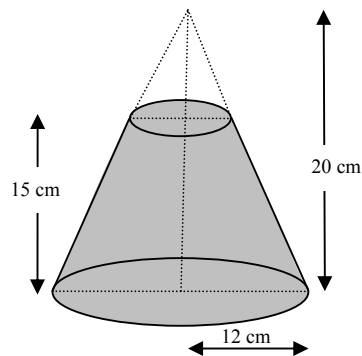
Module 2: Geometry and trigonometry

Answer all questions in this module.

Question 1

For the diagram above, the size of angles a° , b° and c° are respectively

- A. 23° , 85° , 48°
- B. 37° , 85° , 48°
- C. 72° , 85° , 48°
- D. 72° , 95° , 132°
- E. 95° , 95° , 132°

Question 2

The volume in cm^3 of the truncated cone of radius 12 cm shown in the diagram above is closest to

- A. 9
- B. 47
- C. 2592
- D. 2969
- E. 3016

SECTION B- Module 2: Geometry and trigonometry- continued

Question 3

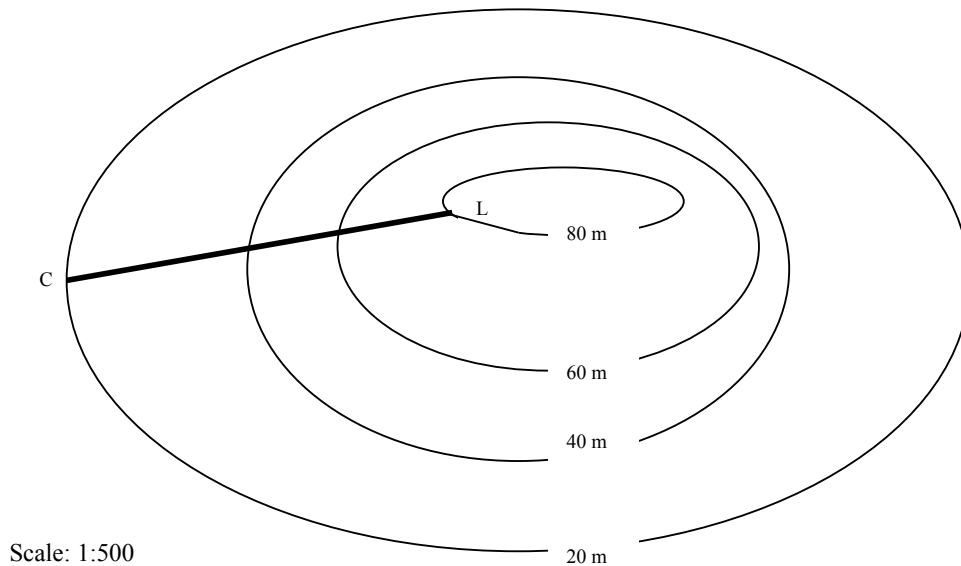
A model house is built so that the volume of the model is $\frac{1}{2000}$ of the volume of the actual house. If the length of the model is 220 cm the actual length of the house in metres is closest to

- A. 22 m
- B. 28 m
- C. 44 m
- D. 2800 m
- E. 4400 m



Question 4

The flat topped hill shown on the contour map below has a lookout (L) at the top and a car park (C) at the bottom.



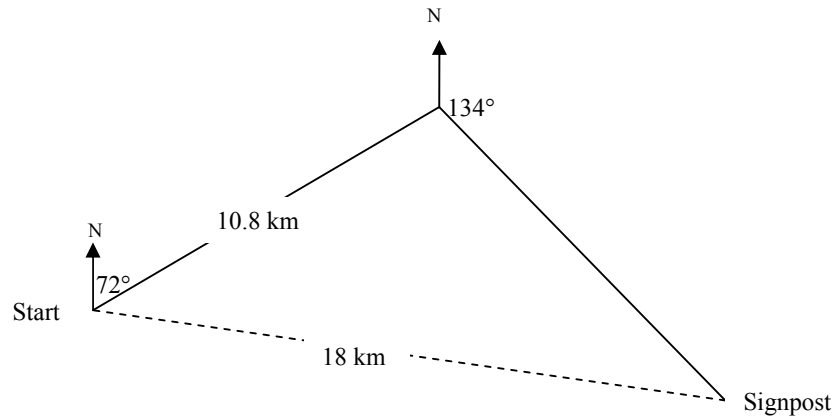
CL represents a straight path to the lookout. If the distance of CL on the map is 6 cm, the walk from the car park to the lookout is closest to

- A. 30 m
- B. 45 m
- C. 60 m
- D. 67 m
- E. 85 m

SECTION B- Module 2: Geometry and trigonometry- continued
TURN OVER

The following information relates to questions 5 and 6.

A scout troop set off on a bearing of 072° for a 10.8 km hike. They then travelled on a bearing of 134° until they came to a signpost which indicated they were 18km in a straight line from their starting point.



Question 5

The bearing from the signpost to their starting point is closest to

- A. 032°
- B. 102°
- C. 118°
- D. 282°
- E. 328°

Question 6

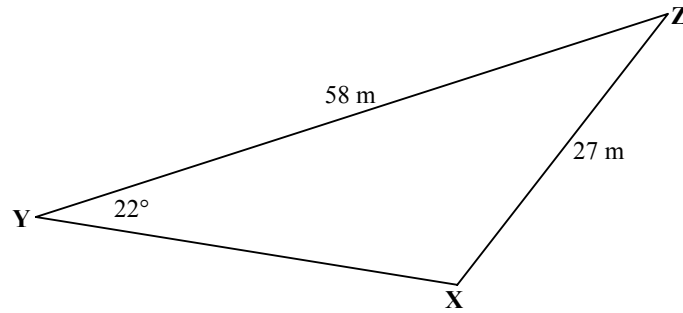
The distance they travelled on the second leg of their journey is closest to

- A. 7.2 km
- B. 10.2 km
- C. 10.8 km
- D. 11.4 km
- E. 14.3 km

SECTION B- Module 2: Geometry and trigonometry- continued

Question 7

A block of land is in the shape of an obtuse angled triangle (XYZ) as shown.

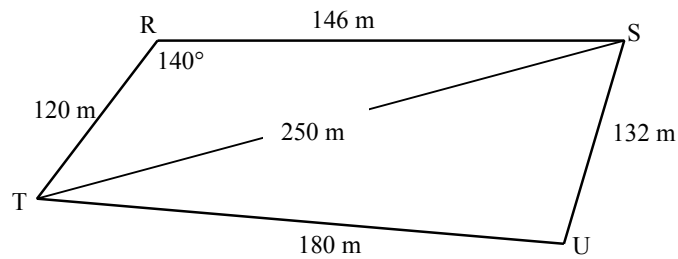


The value of angle X is closest to

- A. 32°
- B. 47°
- C. 54°
- D. 104°
- E. 126°

The following information relates to questions 8 and 9.

An irregularly shaped block of land has the measurements shown in the diagram.

**Question 8**

If the distance from S to T is 250 m, the value of angle U is closest to:

- A. 75°
- B. 105°
- C. 106°
- D. 124°
- E. 136°

SECTION B- Module 2: Geometry and trigonometry- continued
TURN OVER

Question 9

By finding the areas of the triangles RST and STU, the area of the block of land is closest to:

- A. 5000 m^2
- B. 6000 m^2
- C. 11000 m^2
- D. 17000 m^2
- E. 25000 m^2

SECTION B- continued

Module 3: Graphs and relations

Answer all questions in this module.

Question 1

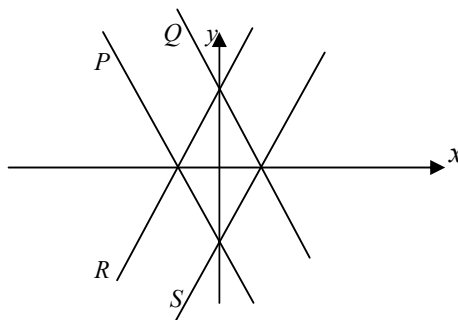
The equations below have been sketched on the graph as shown.

$$k: y = 2x + 1$$

$$l: y = -2x - 1$$

$$m: y = 2x - 1$$

$$n: y = -2x + 1$$



The best match between graphs P , Q , R and S and equations k , l , m and n is given by

- A. Pl , Qm , Rk , Sn
- B. Pm , Qn , Rk , Sl
- C. Pl , Qk , Rn , Sm
- D. Pm , Qk , Rn , Sl
- E. Pl , Qn , Rk , Sm

Question 2

A mathematics tutor charges \$50 for the first hour or part thereof and then \$8.50 for every subsequent 15 minutes (or part thereof). He started work at 10:30 am. If he charged \$75.50 for his time, at which of the following times did he stop work?

- A. 11:32 am
- B. 11:58 am
- C. 12:07 pm
- D. 12:16 pm
- E. 12:26 pm

Question 3

Jodi has just had an interview to negotiate her wages for the next year. She has been offered two packages and asked to choose one of them.

Package 1 Base salary of \$250 plus \$15 per hour.

Package 2 Base salary of \$150 plus \$17.50 per hour.

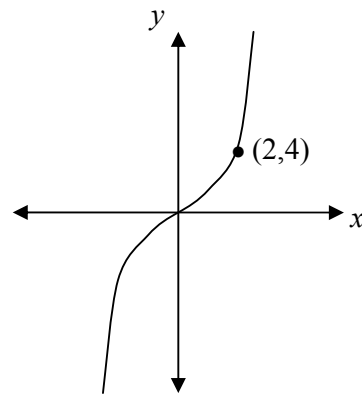
How many hours a week does Jodi have to work before package 2 becomes the better option?

- A. 30
- B. 38
- C. 40
- D. 41
- E. Package 2 will never be a better option.

SECTION B- Module 3: Graphs and relations- continued
TURN OVER

Question 4

The equation which best describes the graph shown is



- A. $y = 2x$
- B. $y = x^2$
- C. $y = \frac{x^3}{2}$
- D. $y = x^3$
- E. $y = 2x^3$

Question 5

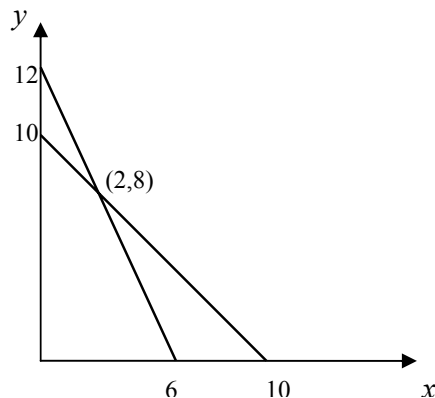
$$x \geq 0 \quad y \leq 0 \quad 2x + y \geq 4 \quad x - y < 5$$

A point that is found in the solution region of these four linear equations is

- A. (3,-1)
- B. (3,1)
- C. (1,-3)
- D. (1,3)
- E. (0,0)

Question 6

This graph below shows the lines
 $x + y = 10$ and $2x + y = 12$.



For the inequations:

$$x \geq 0, \quad y \geq 0, \quad x + y \geq 10, \quad 2x + y \leq 12,$$

the maximum value of the objective function $3x + y$ is

- A. 10
- B. 12
- C. 14
- D. 18
- E. 30

The following information relates to questions 7, 8 and 9.

Manny is a wine maker. He is making two blends of wine using cabernet, shiraz and merlot grapes.

BLEND 1: 80% cabernet and 20% shiraz

BLEND 2: 40% cabernet, 40% shiraz and 20% merlot.

He has 260 litres of cabernet, 200 litres of shiraz and 95 litres of merlot wine available for blending. It costs Manny \$2.75 to make each litre of blend 1 and \$2.70 to make each litre of blend 2. Manny makes x litres of blend 1 and y litres of blend 2.

Question 7

The mathematical constraints on this system can be written as

- A. $x + y + z \leq 550$
- B. $2.75x + 2.5y \leq 550$
- C. $0.8x + 0.4y \leq 260$ $0.2x + 0.4y \leq 200$ $0.2y \leq 95$
- D. $0.8x + 0.2y \leq 260$ $0.4x + 0.4y + 0.2z \leq 200$
- E. $0.8 \times 2.75x + 0.4 \times 2.50y \leq 260$ $0.2 \times 2.75x + 0.4 \times 2.50y \leq 200$ $0.2 \times 2.50y \leq 95$

SECTION B- Module 3: Graphs and relations- continued
TURN OVER

Question 8

The equation for the total cost (C) of making the wine is

- A. $C = 2.75x + 2.7y$
- B. $C = 0.80x + 0.20y$
- C. $C = 0.40x + 0.40y + 0.20z$
- D. $C = 0.20z$
- E. $C = 260x + 200y + 95z$

Question 9

The total volume of wine that can be made if costs are kept under \$1500 is

- A. 325 litres
- B. 450 litres
- C. 475 litres
- D. 525 litres
- E. 550 litres

SECTION B- continued

Module 4: Business-related mathematics

Answer all questions in this module.

Question 1

Maxie purchased some textbooks for his accountancy course. He wants to calculate the total amount of GST (10%) he paid. If the total cost of his books was \$227.40, the amount of GST he paid is closest to

- A. \$20.47
- B. \$20.67
- C. \$22.74
- D. \$25.01
- E. \$206.73

Question 2

Maxie has just finished working out his taxable income for the last financial year. He must pay tax on \$42500. The tax scales are as follows

\$1 - \$6000	Nil
\$6001 - \$30000	15c for each \$1 over 6000
\$30001 - \$75000	\$3600 + 30c for each \$1 over 30000
\$75001 - \$150000	\$17100 + 40c for each \$1 over 75000
\$150001 and above	\$47100 + 45c for each \$1 over 150000
Medicare	1.5% of taxable income

The total amount Maxie pays in tax including the Medicare levy is closest to

- A. \$638
- B. \$3750
- C. \$4388
- D. \$7350
- E. \$7988

Question 3

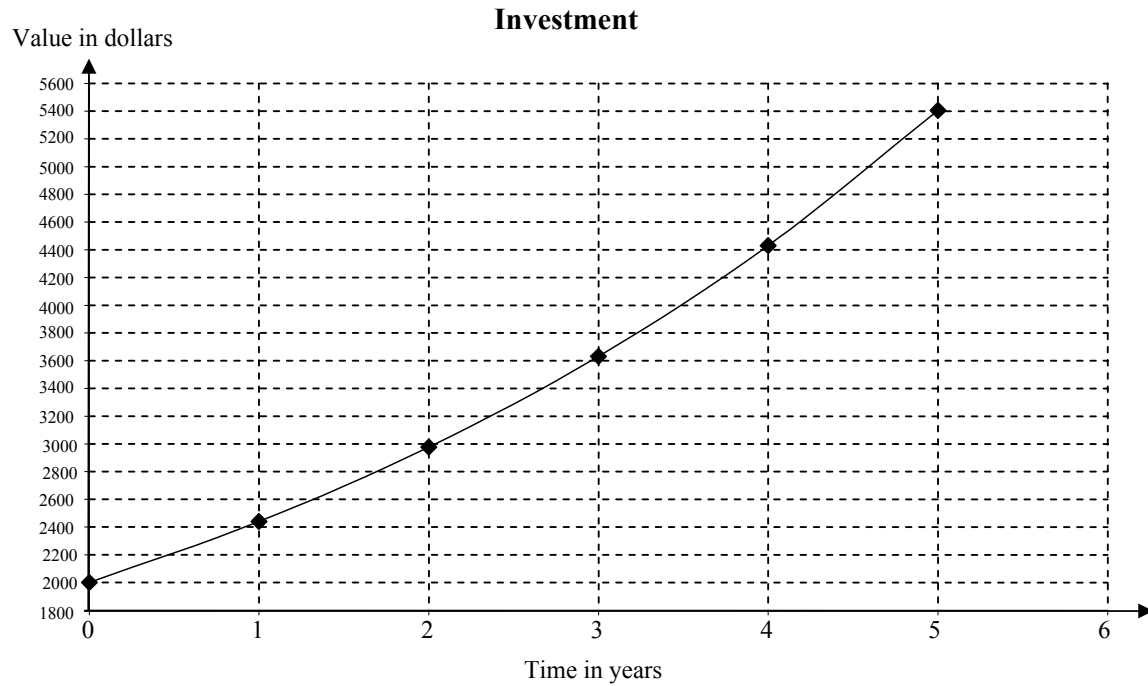
The current inflation rate is expected to remain at 3% pa for the next 5 years. Maxie's salary of \$45000 is indexed to increase with inflation. After 5 years Maxie's salary would be closest to

- A. \$50600
- B. \$51800
- C. \$52200
- D. \$109400
- E. \$16700

SECTION B- Module 4: Business-related mathematics- continued
TURN OVER

Question 4

The graph below shows the income from an investment as a function of time.



This investment shows

- A. Simple Interest at a rate of 6.8 %
- B. Compound Interest at a rate of 6.8 %
- C. Compound Interest at a rate of 20 %
- D. Compound Interest at a rate of 22 %
- E. Compound Interest at a rate of 25 %

Question 5

Maxie wants to invest \$1000 in a 4 year term deposit paying 4% interest compounded annually. To receive the same return in a simple interest account he would need an annual simple interest rate closest to

- A. 1.70 %
- B. 4.00 %
- C. 4.25 %
- D. 4.55 %
- E. 29.25 %

Question 6

Maxie wants a perpetuity of \$3000 per month. If the interest rate is 5.75%, the amount he needs to invest is closest to

- A. \$17250
- B. \$52000
- C. \$207000
- D. \$280000
- E. \$626000

Question 7

Maxie buys a new entertainment system at a cost of \$6400. He pays a deposit of \$900 and then pays the remainder back at a rate of \$250 per month over 4 years. The flat rate of interest per month that Maxie is paying is closest to

- A. 2.12 %
- B. 2.46 %
- C. 5.50 %
- D. 6.50 %
- E. 11.36 %

Question 8

Maxie also takes out a loan of \$26000 to buy a car. He pays this off over 5 years with monthly payments of \$550. The equivalent annual reducing balance rate of interest is closest to

- A. 6.9 %
- B. 7.0 %
- C. 9.8 %
- D. 9.9 %
- E. 14.0 %

Question 9

Maxie depreciates his \$26000 car at a rate of 18%. Its book value after 4 years is closest to

- A. \$2700
- B. \$5040
- C. \$5330
- D. \$11800
- E. \$18700

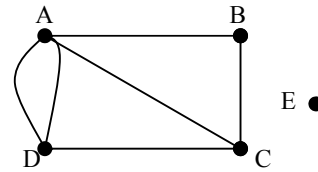
SECTION B- continued
TURN OVER

Module 5: Network and decision mathematics

Answer all questions in this module.

Question 1

The adjacency matrix for the graph at right is given by



A.
$$\begin{bmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

B.
$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

C.
$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

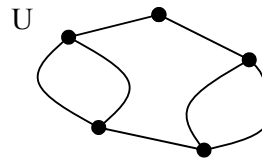
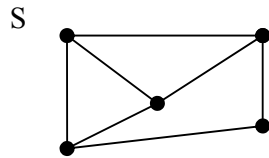
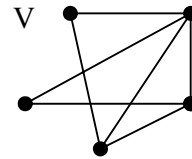
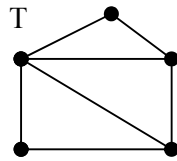
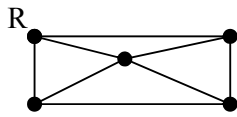
D.
$$\begin{bmatrix} 1 & 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

E.
$$\begin{bmatrix} 4 \\ 2 \\ 3 \\ 3 \\ 0 \end{bmatrix}$$

SECTION B- Module 5: Networks and decision mathematics- continued

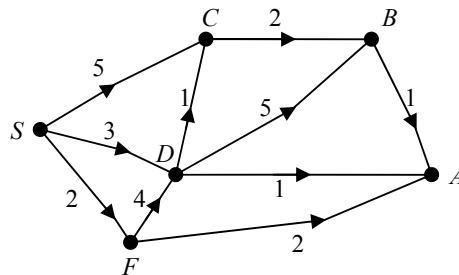
Question 2

Which of the following graphs are isomorphic?



- A. R and T
- B. R and S
- C. R and U
- D. T and U
- E. T and V

Question 3

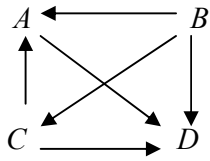


The shortest distance from S to A is

- A. 5
- B. 6
- C. 7
- D. 8
- E. 9

Question 4

In a tournament the outdegree of a vertex represents a win for the team.



The dominance vector that represents the outcome of this tournament is

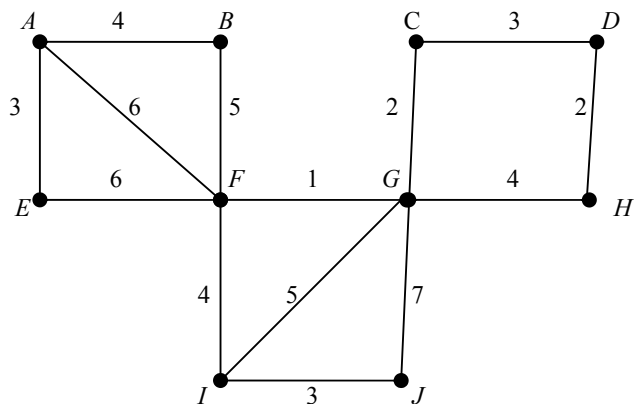
A. $\begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \end{bmatrix}$

B. $\begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 3 \\ 2 \\ 0 \end{bmatrix}$

D. $\begin{bmatrix} 2 \\ 4 \\ 3 \\ 1 \end{bmatrix}$

E. $\begin{bmatrix} 6 \\ 3 \\ 5 \\ 9 \end{bmatrix}$

Question 5

The last two points to be connected into a minimum spanning tree using Prim's algorithm on the network above would be

- A. G and J
- B. A and E
- C. E and F
- D. G and I
- E. G and H

Question 6

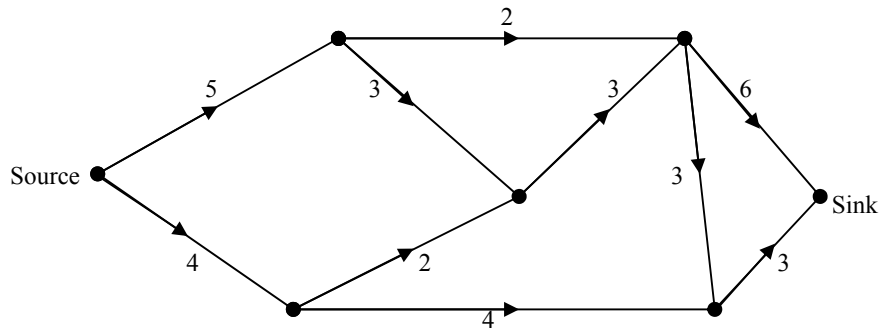
The Victorian Sports Newspaper needs to send reporters to four cities to cover the All Australian finals. From experience the editor realises that each journalist has different expenses in different cities. The editor summarises the expenses in a matrix.

<i>Journalist:</i>	<i>Colin</i>	<i>Dave</i>	<i>Ed</i>	<i>Frank</i>
<i>Sydney</i>	700	800	500	500
<i>Melbourne</i>	500	500	300	500
<i>Adelaide</i>	200	200	400	500
<i>Brisbane</i>	500	600	700	500

If the editor authorises the minimum possible amount on expenses it will cost the newspaper

- A. \$2300
- B. \$1700
- C. \$1600
- D. \$1500
- E. \$1200

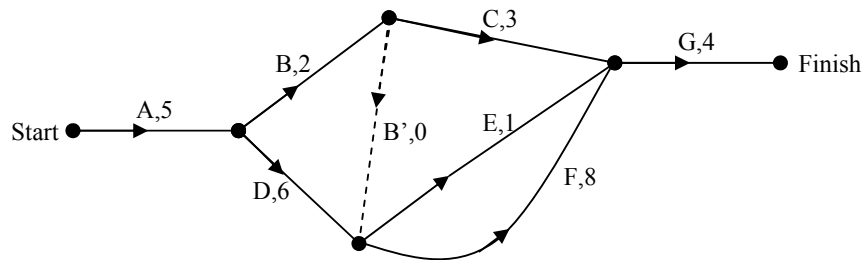
Question 7



The maximum flow in the network above is

- A. 2
- B. 8
- C. 9
- D. 11
- E. 13

The following information relates to questions 8 and 9



Question 8

The critical path for this project is given by

- A. A, B, C, G
- B. A, B, (B'), E, G
- C. A, B, (B'), F, G
- D. A, D, F, G
- E. A, B, C, D, E, F, G

Question 9

It is possible to crash the project for activities A, C & G using the information in the table.

Activity	Number of Days saved	Cost per day (\$/day)
A	1	500
C	2	100
G	2	300

To complete the project two days earlier for the least amount of money you should crash:

- A. Activity C by 2 days
- B. Activity C by 1 day and Activity G by 1 day
- C. Activity G by 2 days
- D. Activity G by 1 day and Activity A by 1 day
- E. Activity A by 2 days

SECTION B- continued
TURN OVER

Module 6: Matrices

Answer all questions in this module.

Question 1

Given that $2 \begin{bmatrix} a & 3b \\ -c & \frac{d}{2} \end{bmatrix} = \begin{bmatrix} -2 & 24 \\ 4 & -9 \end{bmatrix}$ then the values of a , b , c and d are given by

- A. -2, 8, -4, -18
- B. -1, 4, -2, -9
- C. -4, 16, -8, -18
- D. -1, 12, 2, -4.5
- E. -2, 24, 4, -9

The following information relates to questions 2 and 3.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 5 & 2 \\ -2 & -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 4 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 3 \\ -1 & 4 \\ 2 & -2 \end{bmatrix}.$$

Question 2

Which of the following matrix products is non-conformable?

- A. $A.C.B$
- B. $C.B.A$
- C. $B.C.A$
- D. $B.A.C$
- E. All of the above

Question 3

In the matrix product $C \times B$, element a_{22} is found by calculating

- A. 2×2
- B. $1 \times -1 + 3 \times 4$
- C. $1 \times 2 + -1 \times 4 + 2 \times -1$
- D. $-1 \times -1 + 4 \times 4$
- E. $2 \times 3 + 4 \times 4 + -1 \times -2$

Question 4

$$A = \begin{bmatrix} 2 & 3 \\ -2 & -4 \end{bmatrix} \quad B = \begin{bmatrix} -4 & -3 \\ 2 & 2 \end{bmatrix}$$

Which of the following statements is true for the two matrices given above?

- A. $B = A^{-1}$
- B. $A = 2B^{-1}$
- C. B has no determinant
- D. $|A| = -\frac{1}{2}$
- E. $A \times B = -2I$

Question 5

The matrix solution to the equations

$$3x - 4y = 15$$

$$x + 5y = -14$$

can be found by

- A. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 15 \\ -14 \end{bmatrix}$
- B. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ -14 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & 5 \end{bmatrix}$
- C. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 15 \\ -14 \end{bmatrix}$
- D. $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{19} \begin{bmatrix} 15 \\ -14 \end{bmatrix} \begin{bmatrix} 5 & 4 \\ -1 & 3 \end{bmatrix}$
- E. $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{19} \begin{bmatrix} 5 & 4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 15 \\ -14 \end{bmatrix}$

SECTION B- Module 6: Matrices- continued
TURN OVER

The following information relates to questions 6 and 7.

Mark is in training for a 25km fun run. Last weekend Mark covered 20 km on Saturday, where he ran for $1\frac{1}{2}$ hours and walked for the last $1\frac{1}{4}$ hours. On Sunday he covered 15 km, running for 1 hour and again walking for $1\frac{1}{4}$ hours.

Question 6

If D represents a 2×1 distance matrix and T represent the times in hours for running and walking each route in a 2×2 matrix, then D and T can be represented as

- A. $D = \begin{bmatrix} 20 & 15 \end{bmatrix}$ $T = \begin{bmatrix} 1.5 & 1.25 \\ 1 & 1.25 \end{bmatrix}$
- B. $D = \begin{bmatrix} 20 \\ 15 \end{bmatrix}$ $T = \begin{bmatrix} 1.5 & 1.25 \\ 1 & 1.25 \end{bmatrix}$
- C. $D = \begin{bmatrix} 20 & 15 \end{bmatrix}$ $T = \begin{bmatrix} 1 & 1.25 \\ 1.25 & 1.5 \end{bmatrix}$
- D. $D = \begin{bmatrix} 20 \\ 15 \end{bmatrix}$ $T = \begin{bmatrix} 1 & 1.25 \\ 1.25 & 1.5 \end{bmatrix}$
- E. $D = \begin{bmatrix} 2.75 \\ 2.25 \end{bmatrix}$ $T = \begin{bmatrix} 25 & 20 \\ 15 & 0 \end{bmatrix}$

Question 7

Given that speed = distance $\div$ time, if matrix V represents the speed at which Mark walks and runs, then V can be found by

- A. $V = 1.6 \begin{bmatrix} 1.5 & 1.25 \\ 1 & 1.25 \end{bmatrix} \begin{bmatrix} 20 & 15 \end{bmatrix}$
- B. $V = 1.6 \begin{bmatrix} 1.5 & 1.25 \\ 1 & 1.25 \end{bmatrix} \begin{bmatrix} 20 \\ 15 \end{bmatrix}$
- C. $V = 1.6 \begin{bmatrix} 1.25 & -1.25 \\ -1 & 1.5 \end{bmatrix} \begin{bmatrix} 20 \\ 15 \end{bmatrix}$
- D. $V = 16 \begin{bmatrix} 1 & 1.25 \\ 1.25 & 1.5 \end{bmatrix} \begin{bmatrix} 20 \\ 15 \end{bmatrix}$
- E. $V = 16 \begin{bmatrix} -1.5 & 1.25 \\ 1.25 & -1 \end{bmatrix} \begin{bmatrix} 20 \\ 15 \end{bmatrix}$

The following information relates to questions 8 and 9.

The Flexible Fitness Swimming Club has just installed both a chlorinated pool and a salt water pool. Recently the manager noticed that 65% of people who use the chlorinated pool last week used the chlorinated pool this week. The others changed to the salt water pool. 15% of the people who used the salt water pool last week changed to the chlorinated pool and the rest continued to use the salt water pool. Last week 80% of the membership used the chlorinated pool and the remainder used the salt water pool.

Question 8

The transition matrix which demonstrates this information is given by

A. $T = \begin{bmatrix} 0.65 & 0.15 \\ 0.35 & 0.85 \end{bmatrix}$

B. $T = \begin{bmatrix} 0.65 & 0.35 \\ 0.15 & 0.85 \end{bmatrix}$

C. $T = \begin{bmatrix} 0.65 \\ 0.15 \\ 0.80 \end{bmatrix}$

D. $T = \begin{bmatrix} 0.55 \\ 0.45 \end{bmatrix}$

E. $T = \begin{bmatrix} 0.65 & 0.15 \\ 0.80 & 0.20 \end{bmatrix}$

Question 9

In the long term the proportion of members who use the salt water pool is closest to

- A. 30%
- B. 35%
- C. 45%
- D. 70%
- E. 85%

END OF QUESTION BOOK

FURTHER MATHEMATICS

Units 3 & 4 – Written examination 1



2008 Trial Examination

SOLUTIONS

SECTION A – Multiple-choice questions (1 mark each)

Core – Data analysis

Question 1

Answer: C

Explanation:

See last column of the table.

Subject	Maurie's grade	Class average	Class standard deviation	$z = \frac{x - \bar{x}}{s_x}$
Art	60	52	6	1.33
Biology	72	65	7	1
Chemistry	78	72	3	2
Drawing	65	48	15	1.13
English	82	80	10	0.2

Question 2

Answer: A

Explanation:

$\left(\frac{37 \times 100}{82}\right)$ Number of females in favour as a percentage of the total female vote.

Question 3

Answer: E

Explanation:

The graph slopes down and $r = -0.91$

Question 4

Answer: D

Explanation:

This is r^2 not r so it is positive.

Question 5

Answer: C

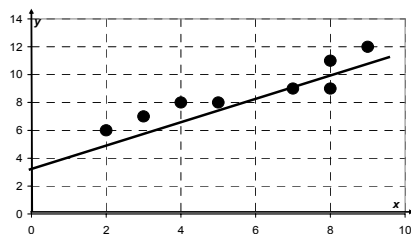
Explanation:

$$\begin{aligned}\bar{x} &= 190 & s_x &= 2.5 \\ \bar{y} &= 52 & s_y &= 8 & r &= 0.86 \\ m &= \frac{0.86 \times 8}{2.5} = 2.752 \\ c &= 52 - 2.752 \times 190 = -470\end{aligned}$$

Question 6

Answer: A

Explanation:



The equation of the line should be $y = 4.133 + 0.8x$
or The line is parallel to the line joining the upper and lower summary points and one third of the way towards the middle summary point.

Question 7

Answer: B

Explanation:

The graph shows a decreasing trend without a true cyclic nature.

Question 8

Answer: D

Explanation:

$$\text{Week 4} \quad \frac{70 + 78 + 92}{3} = 80 \quad \text{Week 8} \quad \frac{62 + 48 + 52}{3} = 54$$

Question 9

Answer: D

Explanation:

$$5 - 0.78 - 0.85 - 0.82 - 1.35 = 1.2$$

Question 10

Answer: C

Explanation:

$$\frac{13500}{0.82} = 16463$$

Question 11

Answer: C

Explanation:

A $\log x$ transformation will compress the upper end of the x scale thus linearising the points.
Or by recognising that $\log x$ is the only possible *Answer*.

Question 12

Answer: D

Explanation:

Calculated using a TI graphics calculator. Advertising in L1, sales in L2 and use STAT CALC 8.

Question 13

Answer: B

Explanation:

$$S = 18.45 + 83.85 \times 0.7 = 77.145 \quad 94.5 - 77.145 = 17.355 = 17.36$$

SECTION B: Multiple-choice questions (1 mark each)**Module 1: Number patterns****Question 1***Answer:* C*Explanation:*A finite geometric sequence – the sequence does not continue, $a = -8$ & $r = 0.5$ **Question 2***Answer:* D*Explanation:*

$$a = -12 \quad d = 2.5 \quad t_{51} = -12 + 50 \times 2.5 = 113$$

Question 3*Answer:* D*Explanation:*

$$ar^4 = 162 \quad ar^2 = 18 \quad r^2 = 9 \quad r = 3 \quad a = 2$$

$$S_6 = \frac{2(3^6 - 1)}{3 - 1} = 728$$

Question 4*Answer:* B*Explanation:*The fourth term is t_3

t_0	t_1	t_2	t_3
-2	$3 \times -2 + 2 = -4$	$3 \times -4 + 2 = -10$	$3 \times -10 + 2 = -28$

Question 5*Answer:* D*Explanation:*No common difference so cannot be arithmetic. There is a common ratio of -0.5 so the sequence is geometric with $a = 8$ and $r = -0.5$

Question 6

Answer: C

Explanation:

Check by trial and error. Also, the sequence is geometric and C gives the correct value of r

Question 7

Answer: E

Explanation: Increase of 5% each year gives $A_n = 1.05A_{n-1}$ and initial value is 2500.

Question 8

Answer: B

Explanation:

The sequence generated is 1, 4, 5, 9, 14, 23, 37, 60..... Thus 23 is the sixth term.

Question 9

Answer: B

Explanation:

Use $A = P(1+r)^n$ with $a = 6000$ $n = 3 \times 4 = 12$ and $r = \frac{0.04}{4} = 0.01$

Module 2: Geometry and trigonometry

Question 1

Answer: A

Explanation:

$$b = 180 - 95 \quad c = 180 - 132 \quad a = 180 - 72 - 85$$

Question 2

Answer: D

Explanation:

$$r_{small} = \frac{5 \times 12}{20} = 3cm$$

$$V_{truncated} = V_{large} - V_{small} = \frac{\pi \times 12^2 \times 20}{3} - \frac{\pi \times 3^2 \times 5}{3} = 3016 - 47 = 2969cm^3$$

Question 3

Answer: B

Explanation:

$$\frac{V_2}{V_1} = \frac{2000}{1} = \frac{x^3}{2.2^3} \quad x = \sqrt[3]{(2000 \times 2.2^3)} = 28$$

Question 4

Answer: D

Explanation:

$$d = \sqrt{(60^2 + (6 \times 5)^2)} = 67m$$

Question 5

Answer: D

Explanation:

$$\frac{\sin a}{10.8} = \frac{\sin 118}{18} \quad a = 32^\circ$$

$$bearing = 180 + 134 - 32 = 282^\circ$$

Question 6

Answer: B

Explanation:

$$b^\circ = 180 - 32 - 118 = 30^\circ \quad y = \frac{18 \sin 30}{\sin 118} = 10.2 \text{ km}$$

Question 7

Answer: E

Explanation:

$$\frac{\sin x}{58} = \frac{\sin 22}{27} \quad x = \sin^{-1}\left(\frac{58 \sin 22}{27}\right) = 54^\circ$$

$$\text{because } x > 90^\circ \quad x = 180 - 54 = 126^\circ$$

Question 8

Answer: B

Explanation:

$$U = \cos^{-1}\left(\frac{180^2 + 132^2 - 250^2}{2 \times 180 \times 132}\right) = 105^\circ$$

Question 9

Answer: D

Explanation:

$$\Delta RST = \frac{120 \times 146 \times \sin 140}{2} = 5630$$

$$s = \frac{250 + 132 + 180}{2} = 281$$

$$\Delta STU = \sqrt{(281 \times 31 \times 101 \times 149)} = 11450$$

$$\text{Area} = 11450 + 5630 \approx 17000$$

Module 3: Graphs and relations

Question 1

Answer: E

Explanation:

R & S have positive gradients with +1 & -1 intercepts, Q & P have negative gradients with +1 & -1 intercepts.

Question 2

Answer: C

Explanation:

$75.50 - 50 = 25.50$ $\frac{25.50}{8.5} = 3$. Therefore the tutor worked for more than one and a half hours i.e. after 12 pm, but less than 12:16 pm.

Question 3

Answer: D

Explanation:

$$17.5x + 150 = 15x + 250 \quad 2.5x = 100 \quad x = 40$$

Package 2 becomes the better option at more than 40 hours.

Question 4

Answer: C

Explanation:

From the shape this is a cubic graph. From the point given the coefficient of x^3 must be $\frac{1}{2}$.

Question 5

Answer: A

Explanation:

$$3 \geq 0 \quad -1 \leq 0 \quad 6 + -1 \geq 4 \quad 3 - -1 < 5$$

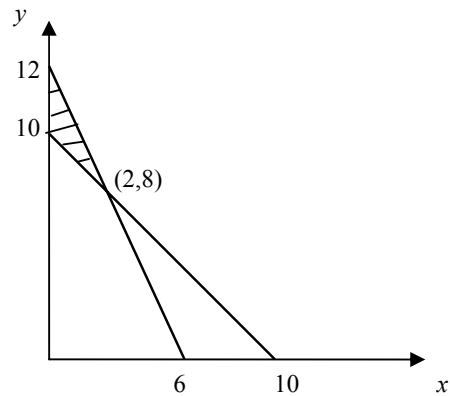
Question 6*Answer:* C*Explanation:*

Region required is shaded.

Valid points are (0,10), (0,12) and (2,8)

Maximum value is obtained from (2,8):

$$3 \times 2 + 8 = 14$$

**Question 7***Answer:* C*Explanation:*

The first equation shows the constraints for cabernet. The second equation shows the constraints for shiraz and the third equation shows the constraints for merlot.

Question 8*Answer:* A*Explanation:*

The cost of x litres of blend 1 is $2.75x$ and the cost of y litres of blend 2 is $2.7y$.

Question 9*Answer:* E*Explanation:*

Optimal points (x,y)	Volume litres	Cost (\$) = $2.75x + 2.7y$
(0,475)	475	1282.50
(50,475)	$50 + 475 = 525$	1420
(100,450)	$100 + 450 = 550$	1490
(375, 0)	$375 + 0 = 375$	1031.25

Module 4: Business-related mathematics

Question 1

Answer: B

Explanation:

$$\frac{227.4}{11} = 20.67$$

Question 2

Answer: E

Explanation:

$$3600 + 0.3 \times (42500 - 30000) + 0.015 \times 42500$$

Question 3

Answer: C

Explanation:

$$45000 \times 1.03^5$$

Question 4

Answer: D

Explanation:

Use table feature of graphics calculator trialling each interest rate given in turn.

Or

Recognise from the shape of the graph that it is compound interest and use $P = \$2000$ and note that the investment value is \$5400 after 5 years.

$$2000R^5 = 5400$$

$$R = \sqrt[5]{\frac{5400}{2000}} = 1.22$$

And this gives an interest rate of 22%.

Or

Using the TVM Solver:

$$N = 5$$

$$I = 0 \text{ * Alpha enter } 21.9755$$

$$PV = -2000$$

$$PMT = 0$$

$$FV = 5400$$

$$P/Y = C/Y = 1$$

Question 5

Answer: C

Explanation:

$$A = PR^n = 1000 \times 1.04^4 = 1169.86$$

$$r = \frac{100I}{PT} = \frac{100 \times 169.86}{1000 \times 4} = 4.25$$

Question 6

Answer: E

Explanation:

$$P = \frac{100Q}{r} = \frac{100 \times 3000}{\frac{1}{12} \times 5.75} = 626000$$

Question 7

Answer: B

Explanation:

$$6400 - 900 = 5500 \quad r = \frac{100I}{PT} = \frac{100 \times 6500}{5500 \times 4 \times 12}$$

$$250 \times 48 = 12000$$

$$12000 - 5500 = 6500$$

Question 8

Answer: D

Explanation:

Use the TVM solver with $N = 60$, $PV = 26000$, $PMT = -550$,
 $FV = 0$, $\frac{P}{Y} = 12$, $\frac{C}{Y} = 12$

Question 9

Answer: D

Explanation:

$$\text{book value} = PR^n = 26000 \times 0.82^4 \approx 11800$$

Module 5: Network and decision mathematics

Question 1

Answer: A

Explanation:

The adjacency matrix shows the number of connections from each point A, B, C, D & E to each point.

Question 2

Answer: E

Explanation:

These are the only two of the listed graphs which can be rearranged to look the same.

Question 3

Answer: B

Explanation:

$$SDCA \ 3 + 1 + 2 = 6$$

Question 4

Answer: C

Explanation:

This matrix shows that A has one arrow pointing away, B has 3 etc.

Question 5

Answer: B

Explanation:

Order would be $FG, CG, CD, DH, FI, IJ, BF, AB, AE$

Question 6

Answer: D

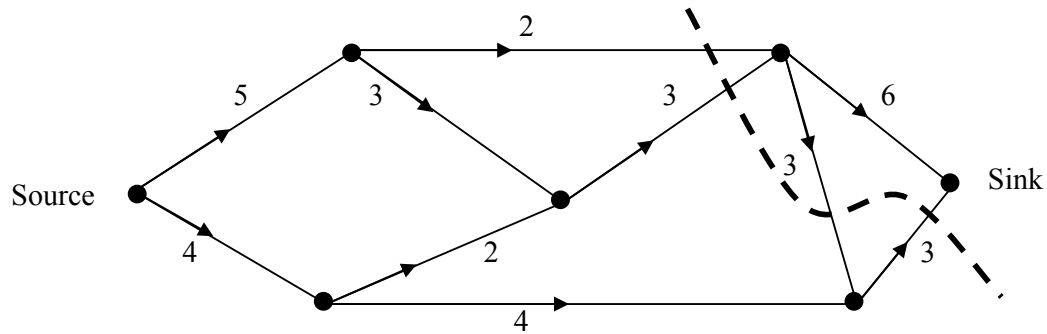
Explanation:

$$CB, DA, EM, FS = 500 + 200 + 300 + 500 = 1500$$

Question 7

Answer: B

Explanation:



Minimum cut of 8 shown above is $2 + 3 + 3 = 8$

Question 8

Answer: D

Explanation:

This path takes the longest time i.e. 23 days

Question 9

Answer: C

Explanation:

Activity C is not on the critical path and Activity A is too expensive

Module 6: Matrices Solutions

Question 1

Answer: B

Explanation:

$$2a = -2, a = -1, 6b = 24, b = 4, -2c = 4, c = -2, d = -9$$

Question 2

Answer: C

Explanation:

Non-conformable matrices cannot be multiplied. $B.C.A$ involves multiplying matrices of order: $2 \times 3 \times 3 \times 2 \times 3 \times 3$ thus $C \times A$ is not possible

Question 3

Answer: D

Explanation:

The second row of matrix C multiplied by the second column of matrix B

Question 4

Answer: E

Explanation:

$$A \times B = -2I$$

$$\begin{bmatrix} 2 & 3 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} -4 & -3 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 2 \times -4 + 3 \times 2 & 2 \times -3 + 3 \times 2 \\ -2 \times -4 + -4 \times 2 & -2 \times -3 + -4 \times -2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = -2I$$

Question 5

Answer: E

Explanation:

The matrix $\frac{1}{19} \begin{bmatrix} 5 & 4 \\ -1 & 3 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 3 & -4 \\ 1 & 5 \end{bmatrix}$

Question 6

Answer: B

Explanation:

The rows in T must represent Saturday & Sunday and the columns Running & Walking if D is a column matrix with Saturday then Sunday distances.

Question 7

Answer: C

Explanation:

This relationship multiplies T^I by D .

Question 8

Answer: A

Explanation:

Note columns on T must add to 1.

Question 9

Answer: D

Explanation:

$$T^{50}N = \begin{bmatrix} 0.65 & 0.15 \\ 0.35 & 0.85 \end{bmatrix}^{50} \begin{bmatrix} 0.80 \\ 0.20 \end{bmatrix} = \begin{bmatrix} 0.30 \\ 0.70 \end{bmatrix}$$