

2008

VCE Further Mathematics Trial Examination 1

Suggested Solutions

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Core: Data analysis

Question 1 C $\frac{10}{30} \times \frac{100}{1} = 33.3\%$	Question 2 C The median is 34 The lower quartile is 22 The upper quartile is 62 The interquartile range is $62 - 22 = 40$
Question 3 B This graph has an outlier from 90 to 100, and it has a tail to the left. This means it is negatively skewed with an outlier.	Question 4 D $8 + 10 + 2 = 20$
Question 5 E Both variables are numerical $\therefore$ scatter plot. One variable is time so the special type of scatter plot is a time series plot. Stem and leaf and box plot are used when one variable is numerical and one is categorical. A bar graph is used when both variables are categorical.	Question 6 D 68% would lie between $600 + 80$ and $600 - 80$ $\therefore$ not A The % greater than 520 would have to be greater than 50% since the mean is 600 $\therefore$ not B 440 is 2 standard deviations below the mean so 2.5% are less than 440 $\therefore$ not C 680 is 1 standard deviation above the mean so about 16% are greater than 680 $\therefore$ not E 360 is 3 standard deviations below the mean so about 0.15% are less than 360 $\therefore$ D
Question 7 B $Z = \frac{x - \bar{x}}{s}$ $-2 = \frac{60 - \bar{x}}{5}$ $-10 = 60 - \bar{x}$ $\bar{x} = 70$	Question 8 A When the residual plot has no clear pattern, then a linear relationship exists.

Core: Data analysis

Question 9 E $r^2 = 0.04$ $r = \pm 0.2$ But r is negative because the gradient of the regression line is negative $\therefore r = -0.2$	Question 10 A English mark = $7.25 - 0.18 \times \text{Maths mark}$ $7 = 7.25 - 0.18 \times M$ $-0.25 = -0.18M$ $M = \frac{-0.25}{-0.18} = 1.4$
Question 11 D There is only a seasonal pattern here, the graph following a similar pattern in the 4 quarters of each year.	Question 12 A Coordinates of median of lower 4 points is (2.5,40) Coordinates of median of upper 4 points is (10.5,35) $m = \frac{35 - 40}{10.5 - 2.5} = -0.625$
Question 13 D Using May, June, July and August $\text{Mean} = \frac{16 + 15 + 14 + 13}{4} = 14.5$ Using June, July, August and September $\text{Mean} = \frac{15 + 14 + 13 + 12}{4} = 13.5$ Centring these two means that would be either side of July, August and September 4 – mean smoothed value centred at July $= \frac{14.5 + 13.5}{2} = 14$	

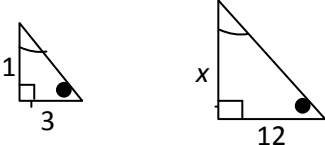
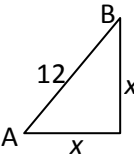
Module 1 Number patterns

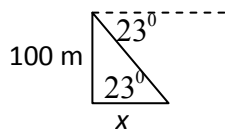
Question 1 D Each term in the sequence is found by adding 6 to the term before it. Hence it is an arithmetic sequence with a common difference of 6	Question 2 B This is a geometric sequence with a common ratio of $-\frac{1}{2}$ $a = 8, n = 12$ $t_n = ar^{n-1}$ $t_{12} = 8\left(-\frac{1}{2}\right)^{11} = -\frac{1}{256}$
Question 3 C $t_7 = 3 \times t_6 + 4$ $8017 = 3 \times t_6 + 4$ $8013 = 3 \times t_6$ $2671 = t_6$ $t_6 = 3 \times t_5 + 4$ $2671 = 3 \times t_5 + 4$ $2667 = 3 \times t_5$ $889 = t_5$	Question 4 C $t_4 = t_{1+3} = t_{1+2} + t_{1+1} + t_1 = t_3 + t_2 + t_1 = 1 + 1 + 1 = 3$ $t_5 = t_{2+3} = t_{2+2} + t_{2+1} + t_2 = t_4 + t_3 + t_2 = 3 + 1 + 1 = 5$
Question 5 C $4 + 6 + 8 + \dots$ Sum of Arithmetic Sequence $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{10} = \frac{10}{2}[8 + 9 \times 2] = 5 \times 26 = 130\text{m}$	Question 6 E Value at beginning of first year, 2008 = 32,000 Value at beginning of second year = $32,000(0.85)$ Value at beginning of third year = $32,000(0.85)^2$ Following the pattern, Value at beginning of fifth year = $32,000(0.85)^4$ = \$16704.20 This is closest to \$16704

Module 1 Number patterns

Question 7 D Arithmetic Sequence. $a = 85$ $d = -7$ $t_n = a + (n-1)d = 85 + (n-1)(-7)$ Use sequence mode on graphics calculator and press $y =$ $n(min) = 1$ $\mu(n) = 85 + (n-1)(-7)$ $\mu(n \text{ min}) = 85$ Press second table and scroll down to get -48 This corresponds to $n = 20$	Question 8 A $\frac{72}{x} = \frac{x}{8}$ $x^2 = 576$ $x = \pm\sqrt{576} = \pm 24$ Common ratio = $\frac{\pm 24}{8} = \pm 3$
Question 9 E An arithmetic sequence is linear $\therefore$ not A The geometric sequence must have a negative common ratio because the terms go from positive to negative to positive $\therefore$ not B or D Because the terms are getting closer to the x axis as n increases, then the common ratio must be a fraction.	

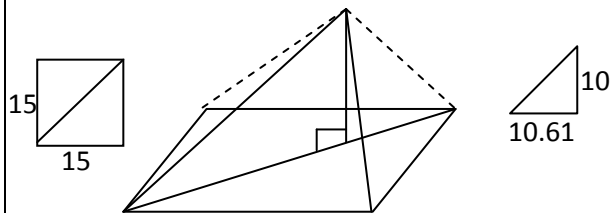
Module 2 Geometry and trigonometry

Question 1 D $\tan \theta = \frac{8}{4} = 2$ $\theta = \tan^{-1}(2) = 63^{\circ}$	Question 2 D $\angle SOA = 90 - 25 = 65$ $\text{True bearing of } OA = 180 + 65 = 245^{\circ} \text{ T}$
Question 3 B  The triangles are similar, AAA $\therefore$ sides are in the same ratio $\frac{x}{1} = \frac{12}{3}$ $x = 4 \text{ m.}$	Question 4 D  $x^2 + x^2 = 144$ $2x^2 = 144$ $x^2 = 72$ $x = \sqrt{72} = 8.485$ $\text{Perimeter} = 8.485 \times 2 + 12$ $\text{Perimeter} = 28.97$
Question 5 C $\text{Area} = \frac{1}{2}bc \sin A$ $\text{Area} = \frac{1}{2} \times 6 \times 6 \times \sin 50^{\circ}$ $\text{Area} = 13.8$	Question 6 E $\angle BAC = 180 - (135 + 26) = 19^{\circ}$ $\frac{AC}{\sin 135^{\circ}} = \frac{16}{\sin 19^{\circ}}$ $AC = \frac{16 \times \sin 135^{\circ}}{\sin 19^{\circ}} = 34.8$

Module 2 Geometry and trigonometry**Question 7 D**

$$\tan 23^\circ = \frac{100}{x}$$

$$x = \frac{100}{\tan 23^\circ} = 235.6 \text{ m}$$

Question 8 B

Length of diagonal of square base = $\sqrt{15^2 + 15^2}$

Length of diagonal of square base = 21.213

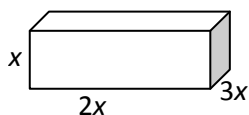
Half length of diagonal of square base = 10.61

Length of sloping sides, $x = \sqrt{10.61^2 + 10^2}$

Length of sloping sides, $x = 14.58$

Wire = 4 sloping sides + 4 sides to square base

Wire = $4 \times 14.58 + 4 \times 15 = 118.32$

Question 9 E

$$\text{Surface area} = 2x^2 \times 2 + 6x^2 \times 2 + 3x^2 \times 2$$

$$\text{Surface area} = 4x^2 + 12x^2 + 6x^2 = 22x^2 = 7942$$

$$x^2 = \frac{7942}{22} = 361$$

$$x = 19$$

$$\text{Width} = 3x = 57$$

Module 3 Graphs and relations

Question 1 B Points are (3,0) and (0,-4) $m = \frac{-4-0}{0-3} = \frac{-4}{-3} = \frac{4}{3}$	Question 2 E When $x = 3800$, y is between 4000 and 4500 $\therefore \\$4250$
Question 3 B Original means $t = 0$ $P = 0 + 50 = 50$	Question 4 C When Meg stops for lunch her speed would be 0 $\therefore$ not A or D Meg rides home faster than she goes at first, $\therefore$ not B The distance travelled is not 0 when Meg stops for lunch $\therefore$ not E
Question 5 A The equation of the line is $y = x - 3$ Area under or on this line is $y \leq x - 3$ This is the same as $y - x \leq -3$ This is the same as $-y + x \geq +3$ This is the same as $x - y \geq 3$	Question 6 B Using points (0,6) and (4,5) $m = \frac{5-6}{4-0} = -\frac{1}{4}$ Equation of this part of the function is $y = -\frac{1}{4}x + 6 \quad 0 \leq x \leq 4$ Using points (14,0) and (4,5) $m = \frac{5-0}{4-14} = -\frac{5}{10} = -\frac{1}{2}$ Equation of this part of the function is $y = -\frac{1}{2}x + c$ When $x = 14$, $y = 0$ $0 = -7 + c$ $c = 7$ $y = -\frac{1}{2}x + 7 \quad 4 < x \leq 14$

Module 3 Graphs and relations

Question 7 C When $x^2 = 16$, $y = 4$ $\therefore y = \frac{1}{4}x^2$	Question 8 D In A, one of the lines has a gradient of 2 and the other line has a gradient of -2, $\therefore$ not parallel In B, one of the lines has a gradient of 3 and the other line has a gradient of -3, $\therefore$ not parallel In C, both lines are the identical same line, not two separate lines. In A, one of the lines has a gradient of 2 and the other line has a gradient of -2, $\therefore$ not parallel In E, one of the lines has a gradient of $-\frac{1}{2}$ and the other line has a gradient of $-\frac{1}{7}$, $\therefore$ not parallel
Question 9 D Move line $Z = 0$ or $3x - y = 0$ keeping the movement parallel to the line $Z = 0$. Moving to the left passes through P last. Moving to the right passes through S last. P would give the minimum and S would give the maximum.	In D, both lines have the same gradient of 3 $\therefore$ parallel.

Module 4 Business-related mathematics

Question 1 A $0.95^5 = 0.77$ $50000 - 0.77$ is just a little less than 50000 $\therefore$ not C $50000 + 0.77$ is just a little more than 50000 $\therefore$ not D $50000 \times (1.05)^5$ is larger than 50000 $\therefore$ not B $0.95^5 = 0.77$ $50000 - (1.05)^5$ is just a little less than 49000 $\therefore$ not E	Question 2 E $\text{GST} = \frac{118.75}{950} \times \frac{100}{1} = 12.5\%$
Question 3 D Total Amount Paid = $66 \times 30 + 396 = 2376$ Interest = $2376 - 1200 = \\$1176$	Question 4 C $A = P \left(1 + \frac{r}{100} \right)^n$ $A = 9400 \left(1 + \frac{7.2}{100} \right)^2 = 10802.33$ $I = 10802.33 - 9400 = \\$1402.33$
Question 5 E Amount repaid = $139 \times 6 = 834$ Interest = $834 - 800 = 34$ $R = \frac{100I}{PT} = \frac{100 \times 34}{800 \times 0.5} = 8.5$ Effective interest rate = $\frac{2 \times 6}{6 + 1} \times 8.5 = 14.6\%$	Question 6 C Let amount invested = x $I = \frac{PRT}{100}$ $740 - x = \frac{x \times 9.6 \times 5}{100}$ $74000 - 100x = 48x$ $74000 = 148x$ $x = \frac{74000}{148} = \\500
Question 7 E Annual Depreciation = $\frac{13500 - 920}{10} = 1258$ %Depreciation = $\frac{1258}{13500} \times 100 = 9.3\%$	

Module 4 Business-related mathematics**Question 8 A**

Use TVM solver

$$N = 120$$

$$I = 9.2$$

$$PV = 250000$$

$$PMT =$$

$$FV = -250000$$

$$P/Y = 12$$

$$C/Y = 12$$

End

This gives $PMT = 1916.67$

$$\text{Total interest repayments} = 120 \times 1916.67$$

$$\text{Total interest repayments} = 230000.40$$

Total amount paid for property

$$= 230000.40 + 250000$$

$$= 480000.40$$

$$\text{Profit} = 485000 - 480000.40 = \$4999.60$$

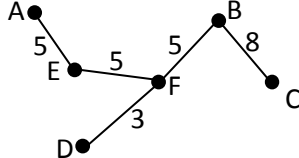
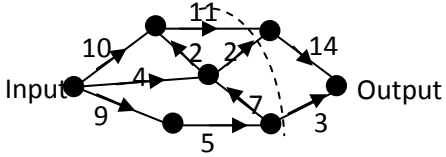
Question 9 BLet original deposit be x

$$\text{After 5 years} = x \times (1.08)^5$$

$$\text{After 7 years} = x \times (1.08)^5 \times (1.06)^2 = 12000$$

$$x = \frac{12000}{(1.08)^5 \times (1.06)^2} = \$7268.60$$

Module 5 Networks and decision mathematics

Question 1 C This is the only graph where you cannot get to the bottom left point from any other point.	Question 2 D Number of edges = $\frac{n \times (n-1)}{2} = \frac{20 \times 19}{2} = 190$
Question 3 D Sum of vertices = $6 \times 3 + 6 = 24$ Number of edges = $\frac{24}{2} = 12$	Question 4 A P and R have more than 2 odd vertices $\therefore$ no Euler path and no Euler circuit. T has all even vertices $\therefore$ it has an Euler circuit. Q and S have 2 odd vertices $\therefore$ they have an Euler path and no Euler circuit.
Question 5 B Only E goes to E so the number in the number in the 5 th row and 5 th column must be 1. A to A is 0, so the number in the first row and first column is 0	Question 6 D  $3 + 5 + 5 + 5 + 8 = 26$
Question 7 C $11 + 2 + 5 = 18$	Question 8 A  Maximum flow = minimum cut $= 11 + 2 + 3 = 16$
Question 9 E The critical path is ADFI $\therefore$ not A C can be delayed for 5 hours without delaying the finishing time $\therefore$ not B C can be delayed for 5 hours without delaying the finishing time so delaying it by 6 hours will only delay the project by 1 hour $\therefore$ not C E can be delayed for 1 hour without delaying the finishing time so delaying it by 6 hours will only delay the project by 5 hours $\therefore$ not D Statement E is true.	

Module 6 Matrices

Question 1 A Use calculator or $-3\begin{bmatrix} 2 & 6 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} -6 & -18 \\ 12 & -3 \end{bmatrix}$ $\begin{bmatrix} -6 & -18 \\ 12 & -3 \end{bmatrix} - \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -9 & -14 \\ 11 & -5 \end{bmatrix}$	Question 2 C If the determinant = 0 then there is no unique solution. For A, determinant = $12 - -12 = 24$ For B, determinant = $-12 - 12 = -24$ For D, determinant = $-32 - -10 = 22$ For E, determinant = $3 - -12 = 15$ For C, determinant = $24 - 24 = 0$																
Question 3 D $X = \begin{bmatrix} -2 & 0 \\ 4 & -3 \end{bmatrix} - \begin{bmatrix} a & b \\ -b & a \end{bmatrix} = \begin{bmatrix} -2-a & -b \\ 4+b & -3-a \end{bmatrix}$	Question 4 D From <table><tr><td></td><td>A</td><td>B</td><td>C</td></tr><tr><td>To A</td><td>$\begin{bmatrix} 0.5 & 0.25 & 0.1 \end{bmatrix}$</td><td></td><td></td></tr><tr><td>To B</td><td>$\begin{bmatrix} 0.2 & 0.6 & 0.35 \end{bmatrix}$</td><td></td><td></td></tr><tr><td>To C</td><td>$\begin{bmatrix} 0.3 & 0.15 & 0.55 \end{bmatrix}$</td><td></td><td></td></tr></table>		A	B	C	To A	$\begin{bmatrix} 0.5 & 0.25 & 0.1 \end{bmatrix}$			To B	$\begin{bmatrix} 0.2 & 0.6 & 0.35 \end{bmatrix}$			To C	$\begin{bmatrix} 0.3 & 0.15 & 0.55 \end{bmatrix}$		
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Question 5 E Steady State = $T^{30}S_0$ $= \begin{bmatrix} 0.4 & 0.8 \\ 0.6 & 0.2 \end{bmatrix}^{30} \begin{bmatrix} 200 \\ 300 \end{bmatrix}$ Use calculator to get $\begin{bmatrix} 286 \\ 214 \end{bmatrix}$	Question 6 A $\det A$ $= 3p \times s - -2q \times 4r$ $= 3ps + 8qr$ $\frac{1}{\det A} = \frac{1}{3ps + 8qr}$ $A^{-1} = \frac{1}{3ps + 8qr} \begin{bmatrix} s & 2q \\ -4r & 3p \end{bmatrix}$																
Question 7 C $2x + 2y + 2z = 1 \Rightarrow x + y + z = 0.5$ $1x + 0y + 1z = -1$ $0x - 5y + 4z = 5$ Matrix is $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & -5 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.5 \\ -1 \\ 5 \end{bmatrix}$	Question 8 E Columns must add to 1 $\therefore$ not A, B or C 0.1 must be in the umbrella column and row.																
Question 9 E B and C must be of the same order which would have to be 4×2 A multiplied by a 4×2 matrix would have to have 4 columns, so it must be a 3×4 matrix.																	

End of suggested solutions 2008 Further Mathematics VCE Trial Examination 1

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