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Further Mathematics

2008

Trial Examination 2

Core – Data analysis

Module 2 – Geometry and trigonometry

Module 3 – Graphs and relations

Module 4 – Business-related mathematics

Instructions:

Answer all questions in the core and the three modules.

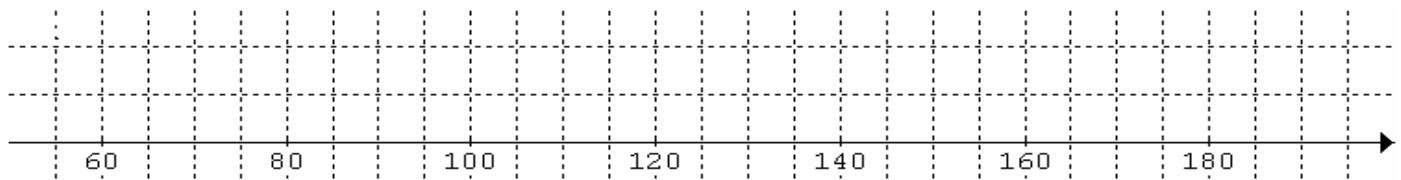
You need not give numerical answers as decimals unless instructed to do so. Alternative forms may involve, for example, π , surds or fractions.

Core – Data analysis

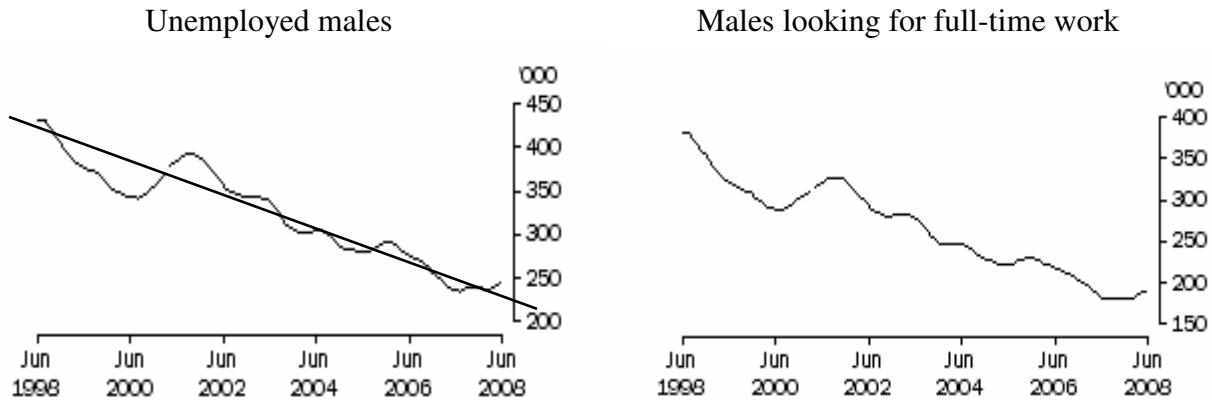
Question 1 The heights of students at Heidi Height P-12 School are **normally distributed**. The set of data is shown in the frequency table below.

Height interval (cm)	Frequency
100 to less than 105	21
105 to less than 110	24
110 to less than 115	31
115 to less than 120	44
120 to less than 125	62
125 to less than 130	88
130 to less than 135	115
135 to less than 140	129
140 to less than 145	152
145 to less than 150	121
150 to less than 155	112
155 to less than 160	101
160 to less than 165	74
165 to less than 170	56
170 to less than 175	47
175 to less than 180	23
180 to less than 185	10
	Total 1210

- a. Estimate the standard deviation of the distribution. 1 mark
- b. Estimate the height that has a standard z score of -2 . 1 mark
- c. Construct a boxplot for the set of data in the above frequency table. 2 marks



Question 2 The following UNEMPLOYED PERSONS TREND ESTIMATES data for Australia are obtained from ABS. The time series on the left shows the total number of unemployed males (**in thousands**) for the last ten years. The one on the right shows the total number of males (in thousands) looking for full-time work over the same period.



A trend line is fitted by sight to the first graph and its equation is:

$$\text{Number of thousands of unemployed males} = -20.5 \times \text{time} + 435.$$

- a. What is the range of values for time **in the above equation** for interpolation purpose? 1 mark

- b. Use the trend line to estimate the number of unemployed males by June 2010. 1 mark

- c. Fit a trend line by sight to the second graph for the number of males looking for full-time work, and determine its equation. 2 marks

- d. Use the tend lines to estimate the percentage of unemployed males looking for full-time work in June 2010. 1 mark

Question 3 A researcher seeking information on the growth of certain tree species obtains the following measurements of the heights and the diameters of 12 tree trunks 1.5 m above the ground. She wishes to determine if the diameter measurement can be used to predict tree height.

Diameter d (cm)	1.8	2.4	5.8	6.2	6.6	7.8	8.6	12.4	19.2	25.2	32.2	51.6
Height h (m)	2.0	2.9	3.6	4.0	4.9	4.0	4.5	6.4	7.5	9.3	7.4	9.7

a. Describe the ‘shape’ of the scatterplot and determine the sample correlation coefficient if a regression line is fitted.

2 marks

b. Apply $\log d$ transformation and fit a straight-line regression to the transformed data. Write down the equation of the regression.

2 marks

c. What proportion of variability is explained by the fitted model in part **b**?

1 mark

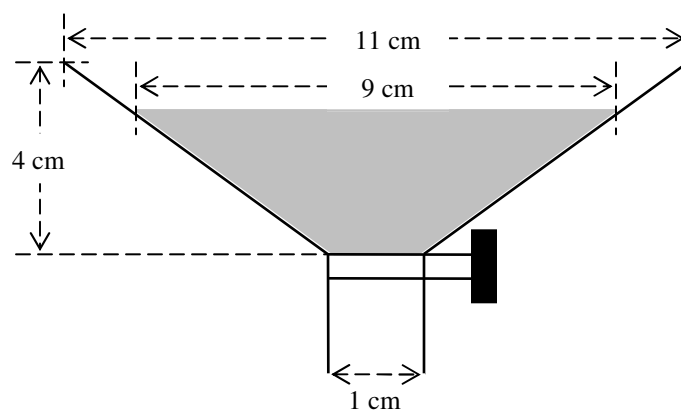
d. Predict the height of a tree belonging to the same species if its diameter measurement 1.5 m above the ground is 60 cm.

1 mark

Total 15 marks

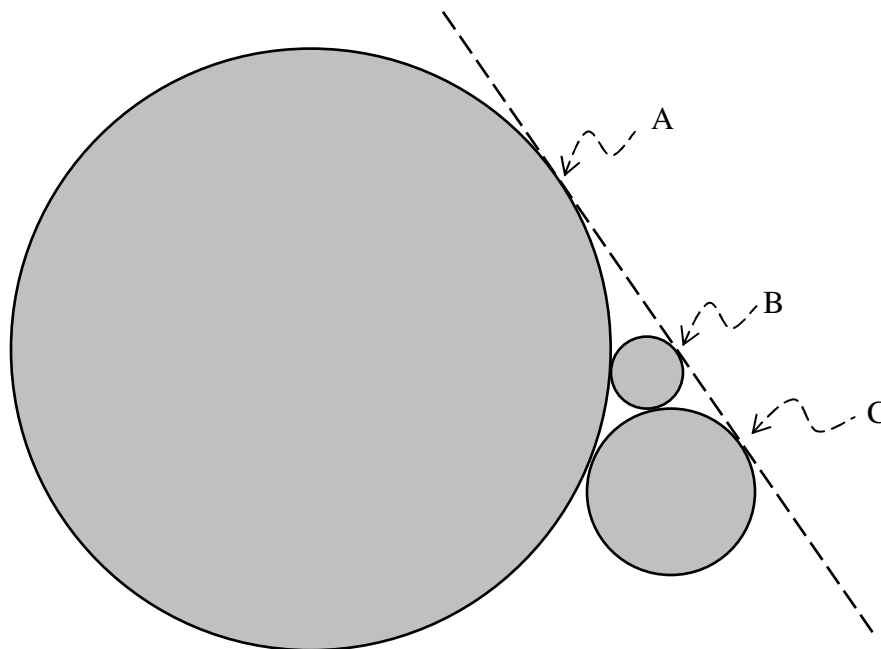
Module 2: Geometry and trigonometry

Question 1 The funnel shown below holds water to a depth of h cm.



- a. Find the area of the circular water surface in cm^2 , round to 1 decimal place. 1 mark
- b. Calculate the exact value of h . 1 mark
- c. Calculate the volume of water in the funnel in cm^3 , round to 1 decimal place. 3 marks

Question 2 A farmer bulldozed a cane field to clear three circular regions of sugarcane (see diagram below). Their radii are 4.0 m, 9.0 m and 36 m. The circles touch each other, and a tangent line can be drawn touching the circles at points A, B and C.



a. Find the area of the region bounded by straight lines joining the centres of the circles. 2 marks

b. Calculate the obtuse angle of the triangle formed by straight lines joining the centres of the circles.

1 mark

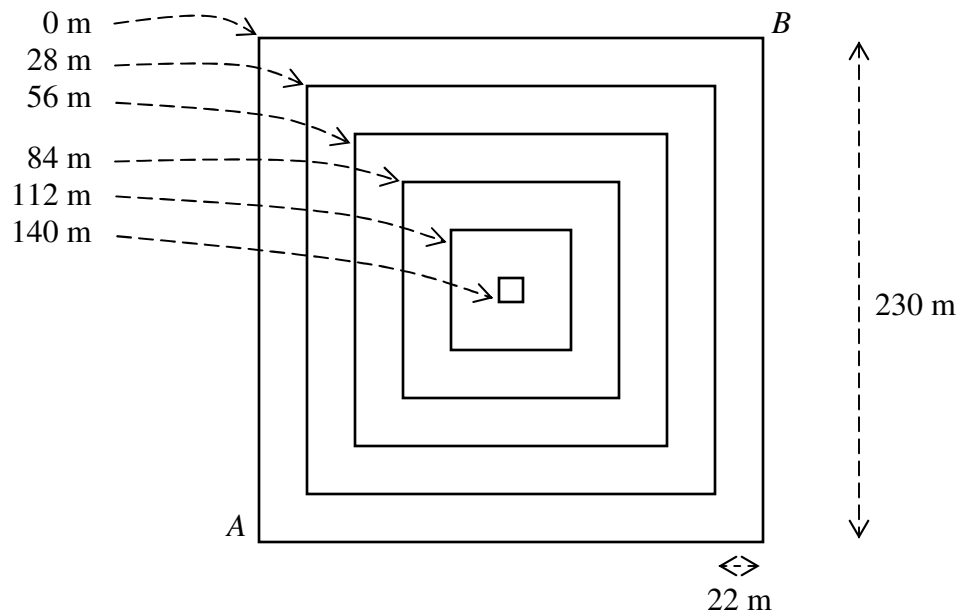
c. Calculate the distance between points A and C.

1 mark

d. Find the area of the region bounded by straight lines joining the centres of the circles when the radii of the circles are halved.

1 mark

Question 3 The following diagram shows the contour map of a square-base pyramid.



- a. Calculate the distance (round to 1 decimal place) from A to B **over the summit**. 2 marks

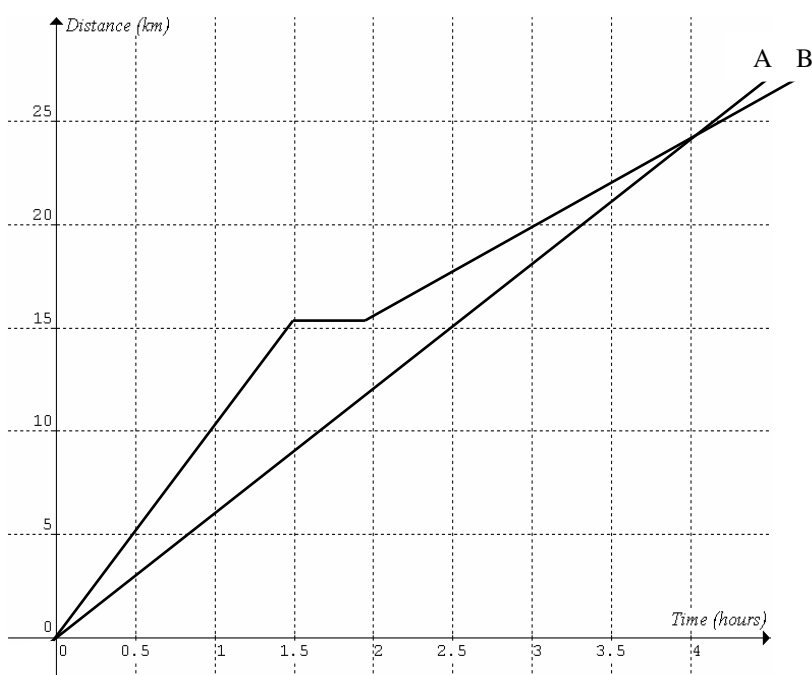
- b. Calculate the angle (nearest degree) of elevation from corner A to the summit. 1 mark

- c. The amount of clay used to make a **solid** scaled down model of the pyramid is 1.0 m^3 . Determine the area of the base of the model (round to 1 decimal place). 2 marks

Total 15 marks

Module 3: Graphs and relations

Question 1 The distance-time graphs of two joggers A and B travelling along the same path for $t \geq 0$ are shown below.



a. Calculate the speed (round to 1 decimal place in km h^{-1}) of jogger A. 1 mark

b. State the time interval during which the average speed of jogger B is equal to the speed of jogger A.

1 mark

c. If joggers A and B start from the same point, find the time and distance (round to 1 decimal place in km) that they are farthest apart.

1 mark

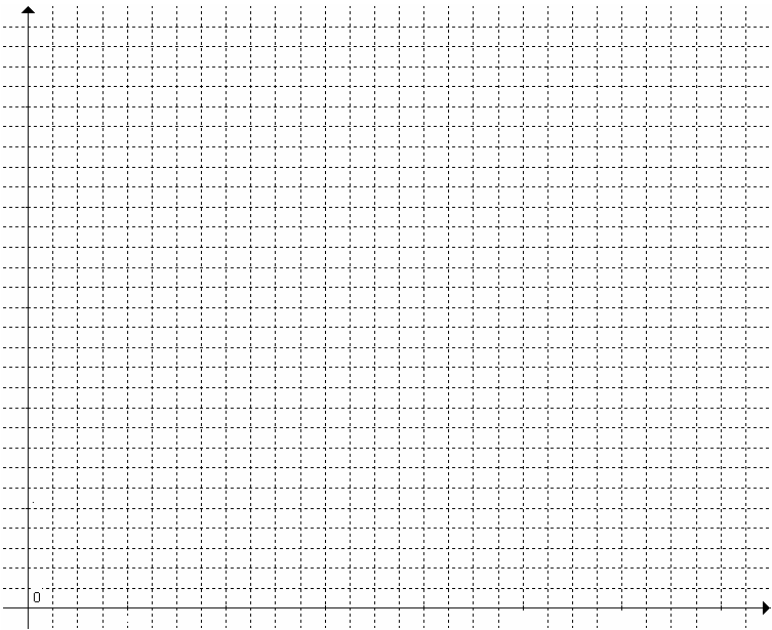
d. If jogger A was 8.0 km ahead of jogger B at $t = 0$, what would be the closest distance (round to 1 decimal place in km) between them?

1 mark

Question 2 The braking distances for different initial speeds of a car are shown in the following table.

Speed v (km h ⁻¹)	30	40	50	60	70	80	90	110
Braking distance d (m)	2.7	4.8	7.5	10.8	14.7	19.2	24.3	36.3

a. Plot the graph of d vs v^2 . Label the axes. 2 marks



b. The unit for speed is km h⁻¹ and the unit for braking distance is m. What is the unit for the slope of the graph? 1 mark

c. Find the mathematical relationship between v and d . 1 mark

d. Calculate the braking distance when the speed of the car is 100 km h⁻¹. 1 mark

Question 3 Consider the following simultaneous linear equations.

$$3x - ky + 2 = 0 \dots\dots \text{Equation (1)}$$

$$5y + 2 = 4x \dots\dots \text{Equation (2)}$$

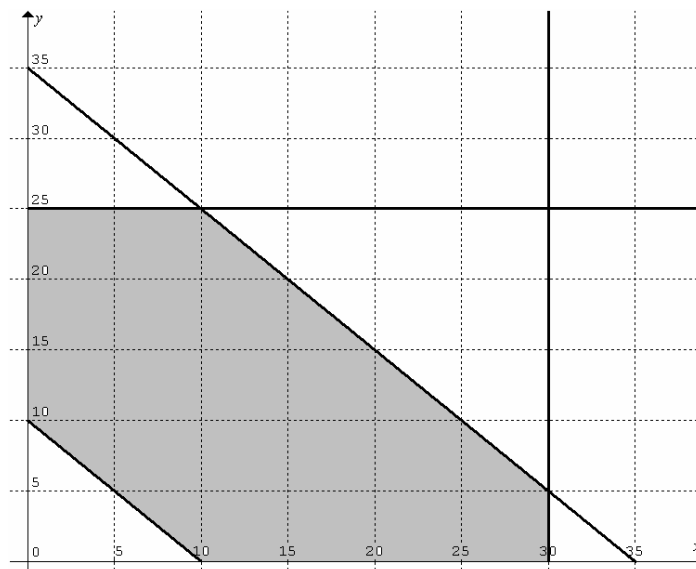
a. Algebraically solve the two simultaneous equations to find x and y when $k = 1$. Express your answers in fraction form.

1 mark

b. When $k = \frac{15}{4}$, find the relationship between x and y such that $3x - ky + 2 > 0$ **and** $5y + 2 \leq 4x$.

2 marks

Question 4 A manufacturing company produces items at its two factories A and B and delivers them to two distributors X and Y. The numbers of items from A to X is x and from A to Y is y . The function for the total delivery cost in dollars from the factories to the distributors is $\$C = 5050 - 60x - 30y$. The constraints on x and y are shown in the graph and the feasible region is shaded.



a. Accurately sketch a possible graph of the cost function on the set of axes shown above.

2 marks

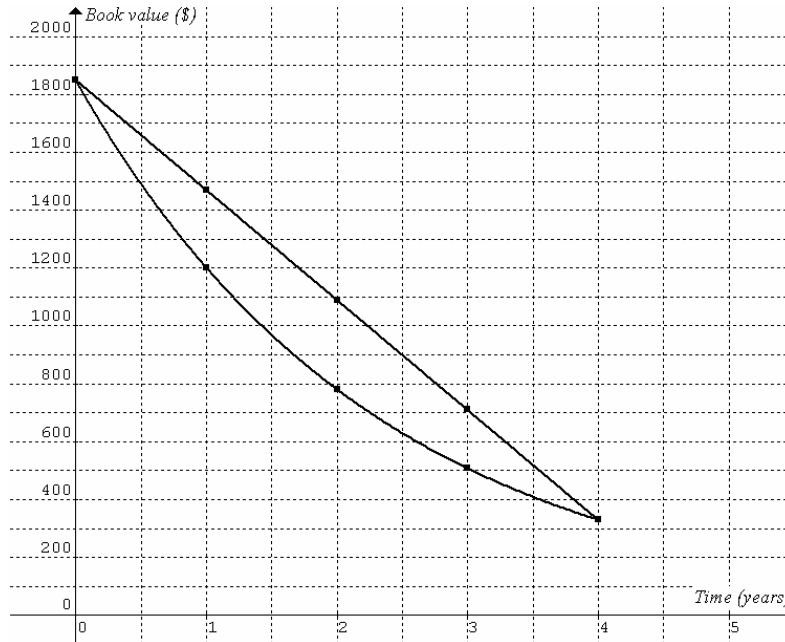
b. Find the minimum total delivery cost.

1 mark

Total 15 marks

Module 4: Business-related mathematics

Question 1 A company bought 10 identical computers for staff to use. The computers were expected to last for 4 years and their total cost was \$18,500. The scrap value of the 10 computers is \$3300. Two methods, *flat rate depreciation* and *reducing balance depreciation*, to calculate the book values of the computers were investigated. The book values of each computer by the two methods at the end of each year after purchase are plotted in the following graph.



- a. Determine the annual flat rate of depreciation used. 1 mark
- b. Determine the rate used to calculate the annual reducing balance depreciation. 2 marks
- c. i. Write down an equation for finding the total book value \$ V of the 10 computers after t years using flat rate depreciation method. 1 mark
- c. ii. Write down an equation for finding the total book value \$ V of the 10 computers after t years using reducing balance depreciation method. 1 mark

Question 2 Maria invested her spare cash of \$3800 in a 6-month term deposit at an interest rate of 8.25% p.a., then the \$3800 plus the interest in another 6-month term deposit at 8.75% p.a.

- a. Calculate the total value (round to the nearest cent) of Maria's investment at the end of the year. 2 marks

- b. The inflation rate was at 4.6% for the year. What was the **real** percentage gain (2 decimal places) in the value of Maria's investment for the year? 2 marks

Question 3 The variable interest rate for home loan in % p.a. charged by a bank for each month over 2006/07 and 2007/08 are shown in the following table. Assume equal duration for each month.

	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun
2006/07	7.57	7.82	7.92	8.02	8.27	8.27	8.52	8.52	8.52	8.77	8.77	8.87
2007/08	8.87	8.87	9.12	9.12	9.12	9.22	9.22	9.47	9.47	9.47	9.72	9.82

- a. What is the average increase in the home loan interest rate (% p.a.) charged by the bank from 2006/07 to 2007/08? 2 marks

- b. Maria took out a home loan of \$250000 for 5 years from the bank. She repaid interest only monthly for the 5-year term. How much interest in total (nearest dollar) did she pay in 2006/07 and 2007/08? 2 marks

- c. Marcus took out a monthly adjusted reducing balance home loan of the same amount from the bank at a 2-year fixed interest rate of 8.0% p.a. from the beginning of 2006/07. The monthly repayment was \$1950. How much more/less interest in total (nearest dollar) did he pay in 2006/07 and 2007/08 than Maria did? 2 marks

Total 15 marks

End of exam 2