

Further Mathematics Exam 2: Solutions

CORE: DATA ANALYSIS

Question 1

a. Mileage

A1

b. i $r = -0.902$

A1

ii. 81.35%

A1

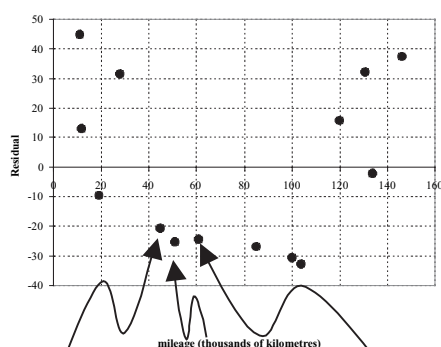
c. Price

$$= \boxed{207.82} + \boxed{-1.22} \times \text{Mileage}$$

A1

A1

d.



Residuals E(45, -21), F(51, -26) and G(61, -24)

- for correct grouping of points (ie F below E and G)

A1

- for correct positions

A1

e. Yes. The nonlinear relationship will provide a better fit because the residual plot shows a *pattern* (is not random).

A1

f. 2.16

A1

g. Strong
Negative
Linear

A1

A1

A1

h. Price = $395 - 159 \times \log(125)$
= \$61 591
= \$61 600

A1

i. 91.5% of variation in price is explained by the variation in $\log(\text{mileage})$.

A1

j. Transformation is successful because the residual plot shows a more *random* distribution of residuals.

A1

Total 15 marks

Further Mathematics Exam 2: Solutions – Multiple choice

MODULE 1: NUMBER PATTERNS

Question 1

a. $t_1 = 80, t_2 = 96, t_3 = 112$

$$t_2 - t_1 = 96 - 80 = 16$$

$$t_3 - t_2 = 112 - 96 = 16$$

Therefore there is a common difference
so the sequence is arithmetic.

b. $t_{14} = a + 13d$
 $= 80 + 13 \times 16$
 $= 288$

c. $S_n = \frac{n}{2}[2a + (n-1)d]$
 $S_{20} = 10[2 \times 80 + 19 \times 16]$
 $= 4640 \text{ kg}$

Question 2

a. $\frac{7.8}{5.2} = 1.5$

b. $t_3 = ar^2$
 $t_3 = 5.2 \times 1.5^2$
 $= 11.7 \text{ m}$

c. $t_n > 200$
 $ar^{n-1} > 200$

Method 1: Using the Calculator

In the sequence mode

Type $u(n) = 5.2 \times 1.5^{(n-1)}$

```

Plot1 Plot2 Plot3
nMin=1
u(n)=5.2*1.5^(
-1)
u(nMin)=
u(n)=
u(nMin)=
u(n)=

```

M1

A1

Then go to TABLE

And scroll down until $u(n) > 200$

n	u(n)
7	59.231
8	88.847
9	133.27
10	199.91
11	299.86
12	449.79
13	674.68

n=11

M1

A1

A1

This occurs when $n = 11$

Method 2: Using Logs

A1

$$5.2 \times 1.5^{n-1} > 200$$

$$1.5^{n-1} > \frac{200}{5.2}$$

M1

$$n-1 > \frac{\log(38.46)}{\log(1.5)}$$

$$n > 1 + 9$$

$$n > 10$$

$$\therefore n = 11$$

A1

for either method

Question 3

- a. The height of beanstalk at the end of these 3 weeks is

$$\begin{aligned} 15 + S_3 \\ = 15 + 3 + 2.55 + 2.1675 \\ = 22.72 \text{ metres} \end{aligned}$$

A1

- b. The beanstalk grows

$$\begin{aligned} S_{\infty} &= \frac{a}{1-r} \\ S_{\infty} &= \frac{3}{1-0.85} \\ &= \frac{3}{0.15} \\ &= 20 \text{ m} \end{aligned}$$

A1

The maximum height of the beanstalk is
 $20 + 15 = 35 \text{ m}$

A1**Question 4**

a. $A_2 = 0.96A_1 + 2$
 $= 0.96 \times 18 + 2$
 $= 19.28 \text{ litres}$

$$\begin{aligned} A_3 &= 0.96A_2 + 2 \\ &= 0.96 \times 19.28 + 2 \\ &= 20.51 \text{ litres} \end{aligned}$$

A1

- b. Using the sequence mode on the calculator

$$\begin{aligned} u(n) &= 0.96 u(n-1) + 2 \\ u(n \text{ min}) &= \{18\} \end{aligned}$$

```

Plot1 Plot2 Plot3
nMin=1
\U(n)=0.96u(n-1)
+2
u(nMin)=18
\U(n)=
u(nMin)=
\U(n)=

```

Go to TABLE to find a list of the first seven terms

n	u(n)
1	18
2	19.28
3	20.509
4	21.688
5	22.821
6	23.908
7	24.952

n=1

$$\begin{aligned} A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 \\ = 18 + 19.28 + 20.509 + 21.688 + 22.821 \\ + 23.908 + 24.952 = 151.158 \text{ litres} \end{aligned}$$

M1**A1**

- c. Let $A_n = 50$

$$\begin{aligned} \text{Then } A_{n+1} &= 0.96 \times 50 + 2 \\ &= 48 + 2 \\ &= 50 \end{aligned}$$

i.e. the sequence is being reduced by the same amount that is being added on. So the sequence becomes stable at 50

A1**Total 15 marks**

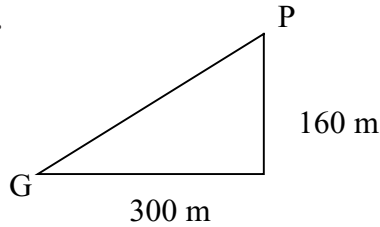
Further Mathematics Exam 2: Solutions – Multiple choice

MODULE 2: GEOMETRY & TRIGONOMETRY

Question 1

a. $480 - 240 = 240$ metres

b.



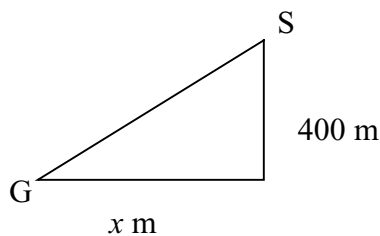
Using Pythagoras

$$GP = \sqrt{160^2 + 300^2}$$

$$\sqrt{115600}$$

$$340 \text{ m}$$

c.



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$1.3 = \frac{400}{x}$$

$$x = \frac{400}{1.3}$$

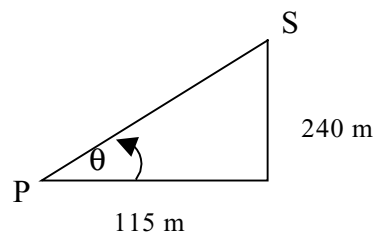
$$x = 307.692\dots$$

$$x = 308 \text{ m}$$

d. i 23×5000
 $= 115000 \text{ mm}$
 $= 115 \text{ m}$

A1

d. ii



$$\tan \theta = \frac{240}{115}$$

$$\theta = \tan^{-1}\left(\frac{240}{115}\right)$$

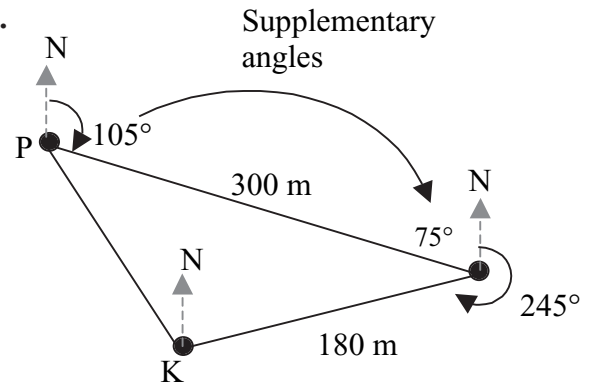
$$= 64^\circ$$

A1

A1

Question 2

a.



The supplement of 105 is 75

$$\angle PTK + 245^\circ + 75^\circ = 360^\circ$$

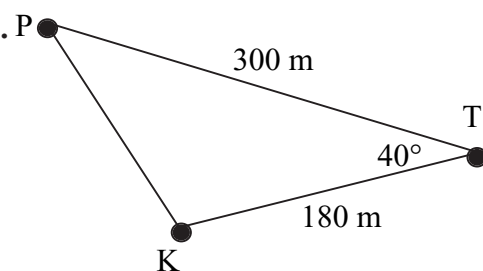
$$\angle PTK = 360^\circ - 320^\circ$$

$$\angle PTK = 40^\circ$$

M1

A1

b.



A1

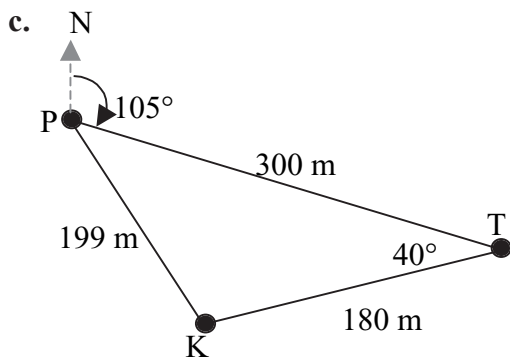
Using cosine rule

$$PK^2 = 180^2 + 300^2 - 2(180)(300)\cos 40^\circ$$

$$PK^2 = 39667.2$$

$$\therefore PK = 199 \text{ m}$$

A1



To find angle KPT

Use sine rule

$$\frac{\sin P}{180} = \frac{\sin 40^\circ}{199}$$

$$\sin P = \frac{180 \sin 40^\circ}{199}$$

$$\sin P = 0.5814...$$

$$P = \sin^{-1}(0.5814...)$$

$$P = 35.55^\circ$$

Bearing of Kiosk from Picnic area

$$= 105^\circ + 35.55^\circ$$

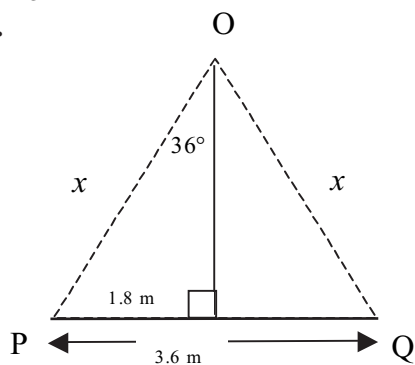
$$= 141^\circ T$$

d. $A = \frac{1}{2} \times 180 \times 300 \sin 40^\circ$
 $= 17355 \text{ m}^2$

Question 3

a. $\frac{360}{5} = 72^\circ$

b.



$$\sin 36^\circ = \frac{1.8}{x}$$

$$x = \frac{1.8}{\sin 36^\circ}$$

$$x = 3.06 \text{ cm}$$

c. $A_{POQ} = \frac{1}{2} \times 3.06 \times 3.06 \sin 72^\circ$
 $= 4.4526...$

$$\text{Area of pentagon} = 5 \times 4.4526$$

$$= 22.26 \text{ cm}^2$$

Exact answer

Area of pentagon

$$= 5 \times \frac{1}{2} \times \frac{1.8}{\sin 36^\circ} \times \frac{1.8}{\sin 36^\circ} \times \sin 72^\circ$$

$$= 22.297...$$

$$= 22.30 \text{ cm}^2$$

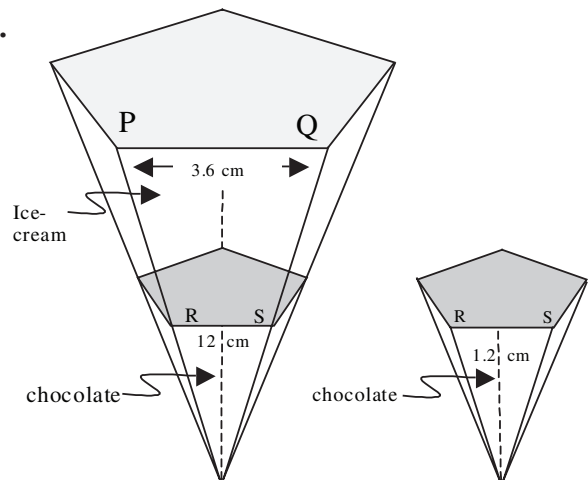
[accept answers between 22.26 and 22.30] **A1**

d.

M1

A1

A1



$$\text{Length ratio} = 3.6 : 1.2$$

$$= 3 : 1$$

A1

Volume ratio = cone : chocolate

$$\frac{3^3}{27} : 1$$

A1

So $\frac{26}{27}$ of the cone consists of ice-cream. **A1**

A1

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MODULE 3: LINEAR RELATIONS AND GRAPHS

Question 1

a. 6000

A1

b. Select two points from second segment
and use $m = \frac{R_2 - R_1}{x_2 - x_1}$ for example

$$m = \frac{150\,000 - 60\,000}{6000 - 1500} = \frac{90\,000}{4500} = 20 \quad \text{M1}$$

Substitute a point (x, R) in $R = 20x + k$

$$\text{eg } 60\,000 = 20 \times 1500 + k \quad \text{M1}$$

$$60\,000 = 30\,000 + k$$

$$\therefore k = 30\,000$$

c. i. \$40 000

A1

ii. \$94 000

A1

d. Straight line with y-intercept $(0, 60\,000)$

A1

Correct end-point $(60\,000, 102\,000)$ A1

e. Revenue = Cost

$$20x + 30\,000 = 60\,000 + 7x \quad \text{M1}$$

$$13x = 30\,000$$

$$x = 2307.69$$

2308 pairs of shoes A1

f. Profit = Revenue – Cost

$$= (20x + 30000) - (7x + 60000)$$

$$= 13x - 30000$$

$$= 13 \times 6000 - 30000$$

$$= \$48\,000 \quad \text{A1}$$

Alternatively find Revenue and Cost separately

$$\text{Revenue} = 20x + 30\,000$$

$$= 20 \times 6000 + 30\,000$$

$$= 120\,000 + 30\,000$$

$$= 150\,000$$

$$\text{Cost} = 7x + 60\,000$$

$$= 7 \times 6000 + 60\,000$$

$$= 42\,000 + 60\,000$$

$$= 102\,000$$

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

$$= 150\,000 - 102\,000$$

$$= \$48\,000$$

g. Revenue = Cost

$$40x = 30\,000 + 12x \quad \text{M1}$$

$$28x = 30\,000$$

$$x = 1071.429$$

1072 pairs of shoes A1

Question 2

a.

Week	1	2	3	4
Deficit	4000	2000	1333	1000

A1

b. $k = d \times t$

$$= 4000 \times 1$$

$$= 4000 \quad \text{A1}$$

c. If the production reaches 5000 then the deficit is zero, but

$$t = \frac{4000}{d} \text{ becomes } t = \frac{4000}{0} = \text{undefined} \quad \text{A1}$$

Total 15 marks

Further Mathematics Exam 2: Solutions – Multiple choice

MODULE 4: BUSINESS-RELATED MATHEMATICS

Question 1

- a. This is a compound interest calculation.

Substitute in the formula

$$A = P \times R^n \text{ where } P = 5000,$$

$$R = 1 + \frac{5.4}{100} = 1.0045 \text{ and} \quad \text{A1}$$

$$n = 5 \times 12 = 60 \quad \text{A1}$$

$$5000 \times 1.0045^{60} = 6545.856... \approx 6546$$

- b. Using the 'compound interest' formula for the inflated amount

$$P = \$240$$

$$R = 1 + \frac{2.8}{100} = 1.028$$

$$n = 8$$

Inflated amount =

$$240 \times 1.028^8 = 299.334... \\ \approx \$299.33$$

Use of the correct type of formula **M1**
Correct answer **A1**

- c. Using the TVM solver on the calculator set up the screen for $4 \times 12 = 48$ months, interest rate 6.5% p.a., present value ?, payment of \$1300 per month and future value of 0.

```
N=48
I%=6.5
PV=
PMT=1300
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

With the cursor on PV key $\square$:

```
N=48
I%=6.5
PV=-54817.73478
PMT=1300
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

Imogen's parents will need \$54 818 to provide \$1300 per month for 4 years.

Something written down that shows they are on the right track **M1**
Correct answer **A1**

(Note: Both marks awarded if only the correct answer is given)

- d. Using the TVM solver on the calculator set up the screen for $8 \times 12 = 96$ months, interest rate 6.5% p.a., present value -6546, payment of ? per month, future value of \$60 000.

With the cursor on PMT key $\square$:

```
N=96
I%=6.5
PV=-6546
PMT=-390.54767...
FV=60000
P/Y=12
C/Y=12
PMT:END BEGIN
```

Imogen's parents will need to save \$390.55 each month to have \$60 000 after 8 years if they start with \$6546 in the account.

Something written down that shows they are on the right track **M1**
Correct answer **A1**

Question 2

a. $\$5500 - \$4400 = \$1100$

As a percentage this is

$$\frac{1100}{5500} \times \frac{100}{1} = 20\%$$

A1

b.

HECS/HELP debt	Years of inflation	Inflated amount (\$)
\$5500	6	6491.15
\$5800	5	$5800 \times 1.028^5 = 6658.76$
\$6100	4	$6100 \times 1.028^4 = 6812.43$

$$\text{Total} = \$19\,962.34 \approx \$19\,962$$

'5' and '4' in table **A1**

One other calculation correct **A1**

Total correct **A1**

c. $\frac{4}{100} \times \$19962.34 \approx \$798.49 \approx \$798$

A1

- d. Using the TVM solver on the calculator set up the screen for $6 \times 12 = 72$ months, interest rate ? p.a., present value -4400, payment of \$0 per month and future value of \$6491.

```

N=72
I%=?
PV=-4400
PMT=0
FV=6491
P/Y=12
C/Y=12
PMT:END BEGIN

```

With the cursor on I%, key $\square$:

```

N=72
I%=6.497729643
PV=-4400
PMT=0
FV=6491
P/Y=12
C/Y=12
PMT:END BEGIN

```

The interest rate will need to be 6.498%

Something written down that shows they are on the right track **M1**

Correct answer **A1**

Further Mathematics Exam 2: Solutions – Multiple choice

MODULE 5: NETWORKS AND DECISION MATHEMATICS

Question 1

- a. Either CBAFEDC or CDEFABC

Circuit starting and ending at C **M1**

Correct circuit **A1**

- b. A Hamiltonian circuit visits each of the vertices once only and starts and ends at the same vertex. **A1**

Question 2

- a. Starting with the person who has only one task that he/she can do

Denise can only do *Equipment* which leaves *Catering co-ordinator* for Paolo, *Drive bus* for Rex, *Timekeeper* for Gina and *First Aid* for Stuart.

<i>Volunteer</i>	<i>Task</i>
Gina	Timekeeper
Rex	Drive bus
Stuart	First Aid

Two correct **A1**

All three correct **A1**

Question 3

Using the Hungarian algorithm subtract the minimum from each row.

38	30	0	0	0
3	0	32	20	11
38	32	0	19	28
22	34	0	23	48
2	0	32	25	0

Only four lines needed to cover the zeros (allocation can be made when five lines are needed to cover the zeros) so subtract the minimum from each column.

36	30	0	0	0
1	0	32	20	11
36	32	0	19	28
20	34	0	23	48
0	0	32	25	0

Still only four lines needed.

Subtract the minimum(19) from the uncovered elements and add it to the elements covered by two lines.

36	30	19	0	0
1	0	51	20	11
17	13	0	0	9
1	15	0	4	29
0	0	51	25	0

Five lines needed so start allocation.

Only one zero in rows 2 and 4.

36	30	19		0
1	0	51	20	11
17	13		0	9
1	15	0	4	29
0		51	25	

This eliminates zeros in columns 2 and 3. Continuing in this manner gives the final allocation (see table on next page).

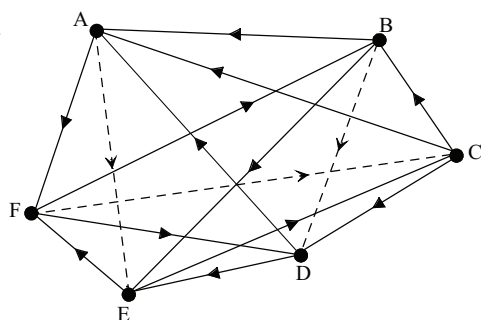
Referee	Town
Greg	F
Harry	C
Jane	E
Ken	D
Mario	B

Method, something written down. **M2**

Answer correct **A1**

Question 4

a.



All correct **A1**

b. The team from Town A defeated the team from Town F so a '1' is in that position.

The team from Town C did not play the team from Town C so a '0' is in that position.

		Defeated town					
		A	B	C	D	E	F
M = Town	A	0	0	0	0	1	1
	B	1	0	0	1	1	0
	C	1	1	0	1	0	0
	D	1	0	0	0	1	0
	E	0	0	1	0	0	1
	F	0	1	1	1	0	0

Both correct **A1**

c. A second level win is when, for example, the team from town A defeated the team from town F, who defeated the team from Town D. The team from Town A is said to have a second level win over the team from Town D.

From the graph it can be seen that **the team from Town D** did not have a second level win over the team from town A.

Alternatively the second level wins are given in the matrix M^2 . A zero appears in the 1st column only alongside A and D.

Ans 2							
		A	B	C	D	E	F
Ans 2	A	0	1	2	1	0	1
	B	1	0	1	0	2	2
	C	2	0	0	1	3	1
	D	0	0	1	0	1	2
	E	1	2	1	2	0	0
	F	3	1	0	2	2	0

A1

d. Adding the elements of the rows will give the number of first and second level wins.

A	7
B	9
C	10
D	6
E	8
F	11

The team from Town F is the overall winner.

A1

Question 5

- a.** The minimum completion time is the path that takes the longest time (the critical path) from start to finish.

In this case the critical path is BFIJ which has a total duration of 24 days.

A1

- b.** Only reducing the activities on the critical path will reduce the overall completion time.

If activity F is reduced by 3 days then a new critical path, BEHJ (22 days) will become operational. However, activity E can be reduced by one day giving both paths a completion time of 21 days.

So activity E should be reduced by 1 day and activity F by 3 days.

Reducing activity F **M1**

Correct answer **A1**

Total 15 marks

Further Mathematics Exam 2: Solutions – Multiple choice

MODULE 6 : MATRICES

Question 1

a.
$$\begin{bmatrix} 4 & 4 & 6 \\ 10 & 10 & 20 \\ 15 & 20 & 30 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6.7 \\ 18.0 \\ 29.5 \end{bmatrix}$$

A1

b. The inverse of $\begin{bmatrix} 4 & 4 & 6 \\ 10 & 10 & 20 \\ 15 & 20 & 30 \end{bmatrix}$ is needed to

solve the equations.

Using the calculator:

A1

c.
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.2 \\ 0 & -0.3 & 0.2 \\ -0.5 & 0.2 & 0 \end{bmatrix} \begin{bmatrix} 6.7 \\ 18.0 \\ 29.5 \end{bmatrix}$$

Using the calculator:

A chop costs \$0.80, a hamburger \$0.50 and a sausage \$0.25 (answers can be in cents)

A1

Question 2

a. Matrix M must have order 3×3
 $(3 \times 3) \times (3 \times 1) = (3 \times 1)$

A1

b.
$$M = \begin{bmatrix} 1.5 & 0 & 0 \\ 0 & 1.4 & 0 \\ 0 & 0 & 1.3 \end{bmatrix}$$

Correct figures on the diagonal A1

All correct A1

Question 3

a.
$$P = \begin{bmatrix} 21 & 35 & 9 \\ 12 & 18 & 25 \\ 7 & 4 & 15 \end{bmatrix} \left(\begin{bmatrix} 10.05 \\ 25.20 \\ 38.35 \end{bmatrix} - \begin{bmatrix} 6.7 \\ 18.0 \\ 29.5 \end{bmatrix} \right)$$

OR

$$P = \begin{bmatrix} 21 & 35 & 9 \\ 12 & 18 & 25 \\ 7 & 4 & 15 \end{bmatrix} \begin{bmatrix} 3.35 \\ 7.20 \\ 8.85 \end{bmatrix}$$

 3×3 matrix correct

(figures transposed from table) A1

Profit on each pack matrix correct A1

b. Using the calculator:

Budget Meats makes \$402 profit at store R, \$391.05 profit at store S and \$185 profit at store Q.

A1

Question 4**a.**

This week

 ≥ 3 1,2 0

$$T = \begin{bmatrix} 0.6 & 0.1 & 0.3 \\ 0.3 & 0.8 & 0.4 \\ 0.1 & 0.1 & 0.3 \end{bmatrix} \begin{matrix} \geq 3 \\ 1,2 \text{ next week} \\ 0 \end{matrix}$$

Correct figures in columns **M1**All correct **A1**

$$\mathbf{b. I_0 = \begin{bmatrix} 131 \\ 386 \\ 203 \end{bmatrix}}$$

A1**c.** The expected numbers are given by $T^4 \times I_0$

Using the calculator:

$$\begin{bmatrix} 181.5253 \\ 448.2939 \\ 90.1808 \end{bmatrix}$$

182 members are expected to attend at least three times, 448 members are expected to attend 1 or 2 times and 90 members are expected to not attend.

M1, A1**d.** Using a high power of T

$$\begin{bmatrix} 180 \\ 450 \\ 90 \end{bmatrix}$$

In the long term 180 members are expected to attend at least three times a week.

A1**Total 15 marks**