The Mathematical Association of Victoria
FURTHER MATHEMATICS

Trial written examination 1
(Facts, skills and applications)

2007

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

Student's Name: .....

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A	13	13	6	3	13
B	54	27			27
					Total 40

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2007 Further Mathematics Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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Working space

MULTIPLE CHOICE ANSWER SHEET

Student Name: .....

Circle the letter that corresponds to each correct answer

Section A	Section B		
Compulsory	Answer three different modules. Show each module selected by ticking the appropriate box.		
	Module:	Module:	Module:
	❑ Number patterns ❑ Geometry and trigonometry ❑ Graphs and relations ❑ Business related mathematics ❑ Networks and decision mathematics ❑ Matrices	❑ Number patterns ❑ Geometry and trigonometry ❑ Graphs and relations ❑ Business related mathematics ❑ Networks and decision mathematics ❑ Matrices	❑ Number patterns ❑ Geometry and trigonometry ❑ Graphs and relations ❑ Business related mathematics ❑ Networks and decision mathematics ❑ Matrices
1. A B C D E	1. A B C D E	1. A B C D E	1. A B C D E
2. A B C D E	2. A B C D E	2. A B C D E	2. A B C D E
3. A B C D E	3. A B C D E	3. A B C D E	3. A B C D E
4. A B C D E	4. A B C D E	4. A B C D E	4. A B C D E
5. A B C D E	5. A B C D E	5. A B C D E	5. A B C D E
6. A B C D E	6. A B C D E	6. A B C D E	6. A B C D E
7. A B C D E	7. A B C D E	7. A B C D E	7. A B C D E
8. A B C D E	8. A B C D E	8. A B C D E	8. A B C D E
9. A B C D E	9. A B C D E	9. A B C D E	9. A B C D E
10. A B C D E			
11. A B C D E			
12. A B C D E			
13. A B C D E			

Working space

CORE: DATA ANALYSIS

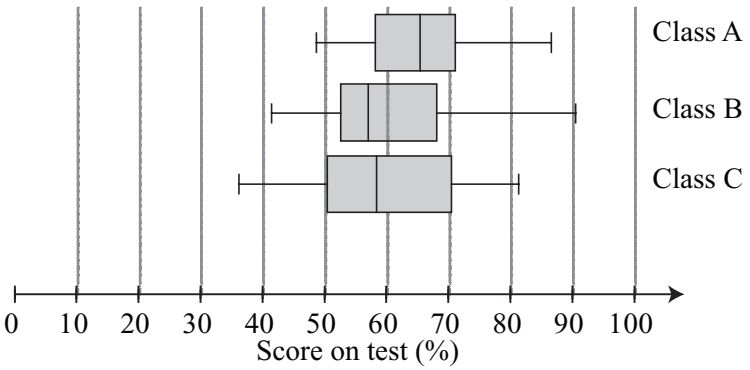
Instructions for Multiple-choice questions

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Multiple-choice questions (13 questions)

The following information relates to questions 1, 2 and 3

The graph below shows the results (percentages) for three different classes on a recent test.



Question 1

The variables ***Class*** and ***Score on test*** are respectively

A. both categorical variables
B. both numerical variables
C. a categorical and a numerical variable
D. a numerical and a categorical variable
E. neither a categorical nor numerical variable

Question 2

The interquartile range for Class C is approximately

A. from 58% to 81%
B. from 36% to 81%
C. 20%
D. 45%
E. 58%

Question 3

Which one of the following is **not** true for the data

A. Scores in class A were generally higher

B. The highest mark but the lowest median were in class B

C. More than 75% of the students obtained a score greater than 50%

D. The distribution of scores in class B was negatively skewed

E. 75% of the students in class A obtained a score that was greater than the median for classes B and C

Question 4

The weights of 2kg bags of onions are normally distributed with a mean weight of 2 kg and a standard deviation of 0.05 kg.
The percentage of bags of onions that have a weight between 2.05kg and 2.10 kg is

A. 2.5%

B. 5%

C. 13.5%

D. 16%

E. 34%

Question 5

Maria scored 56% on her recent Physics test where the scores were normally distributed around a mean mark of 66%. Maria was told that her z-score was – 1.25 for this test.
The standard deviation for this Physics test was

A. 5%

B. 8%

C. 10%

D. 12%

E. 12.5%

The following information relates to questions 6 and 7

A two-way frequency table has been used to display the results of a survey of 120 people on their voting intention at the next election. The people surveyed have been grouped into three age groups to see if age is a factor in their voting intention.

		Age group			
		18 - < 30	30 - < 50	50 $^{+}$	Total
Voting intention	Same party	20	18	23	61
	Different party	9	15	18	42
	Undecided	4	9	4	17
	Total	33	42	45	120

Question 6

The percentage of those surveyed in the 50^{+} age group that are going to vote for the same party as previously is

A. 19.2%

B. 23%

C. 37.5%

D. 37.7%

E. 51.1%

Question 7

Which one of the following statements is not a true statement based on the results of the survey?

A. More than half of the people surveyed intended to vote for the same party.

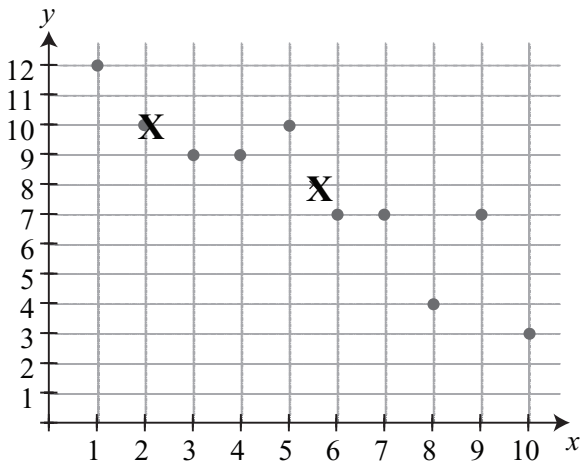
B. 35% of the people surveyed intended to vote for a different party.

C. A larger percentage of those in the 50^{+} age group compared to those in the other two age groups intended to vote for a different party.

D. The greatest percentage of undecided voters was in the 30 - < 50 age group.

E. A larger percentage of the 50^{+} age group compared to the other two age groups are going to vote for the same party.

Question 8



The left and middle median points required to construct a three-median regression line have been located, and are marked **X**, on the scatterplot above. The right median point is not marked.
The gradient of the three-median regression line, correct to one decimal place, will be

A. -1.2

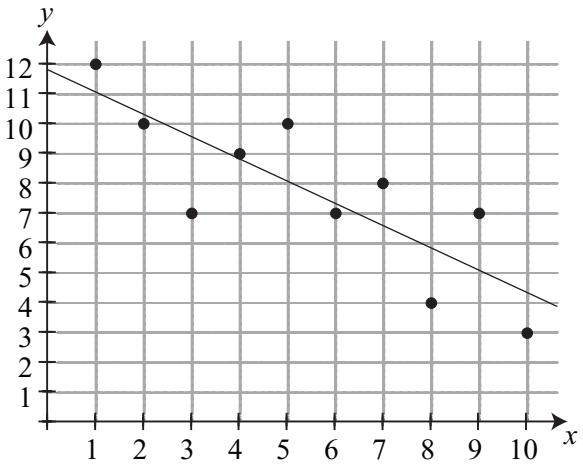
B. -1.1

C. -1.0

D. -0.9

E. -0.7

Question 9



A least squares regression line has been fitted to the scatterplot above.
Which one of the following statements is **true** for the scatterplot and the regression line?

A. The regression line overestimates the value for x = 5.

B. There are more negative residuals than positive residuals.

C. The largest residual is approximately +2.5.

D. The gradient of the regression line is less than -1.

E. The value predicted using the regression line for the point (1, 12) is the point (1, 11).

Question 10

4-point moving-mean smoothing, with centering, is being used to smooth the following time-series data.

Month	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Incidents	38	44	29	26	25	22	29	32	40	35	34	48

The smoothed number of incidents for the month of May will be

A. 25

B. 25.5

C. 26.5

D. 29

E. 32.5

The following information relates to questions 11 and 12

The following table shows the quarterly sales figures (in $1000's) for 2005 and 2006 as well as the yearly averages for these two years. The seasonal indexes, correct to four decimal places have been calculated for Quarters 1 and 2.

	Quarter				
Year	**1**	**2**	**3**	**4**	***Yearly average***
2005	190	254	279	173	***224***
2006	194	266	283	181	***231***
Seasonal Index	0.8440	1.1427			

Question 11

The seasonal index for Quarter 3, correct to four decimal places, will be

A. 1.0103
B. 1.2264
C. 1.2351
D. 1.2353
E. 1.2432

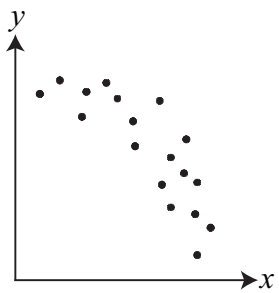
Question 12

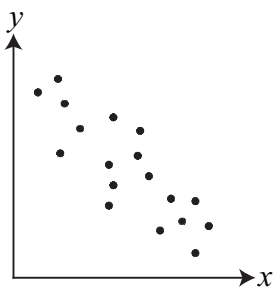
When the data is deseasonalised the sales figures for Quarters 1 and 2 of 2005 will

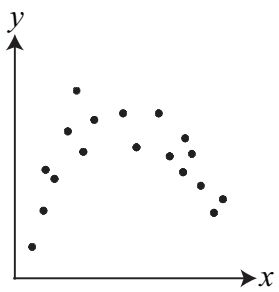
A. increase and decrease respectively
B. decrease and increase respectively
C. both increase
D. both decrease
E. remain the same

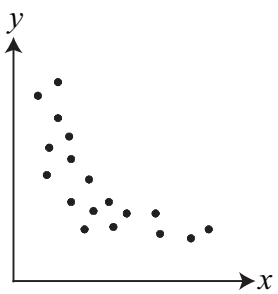
Question 13

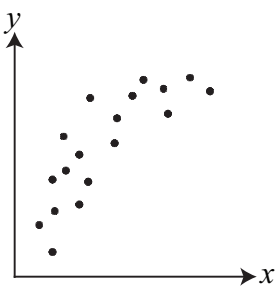
Which one of the following scatterplots could be linearised using a $\frac{1}{y}$ transformation?

A.


B.


C.


D.


E.


END OF CORE

MODULE 1: NUMBER PATTERNS

Instructions

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions.

Question 1

The sum of the first twelve terms of the sequence 17, 23, 29, 35,. . . is

A. 77
B. 83
C. 564
D. 600
E. 1620

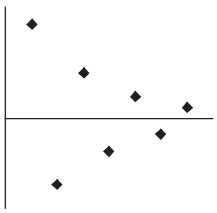
Question 2

A geometric sequence has a common ratio of 1.2 and the third term has the value 180.
The first term to exceed 1000 will be term

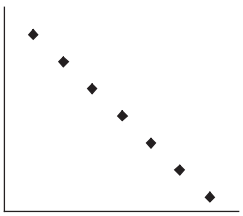
A. 12
B. 13
C. 14
D. 17
E. 18

Question 3

The first seven terms of two sequences are plotted on the graphs shown



Sequence I



Sequence II

Which of the following statements is true?

A. Both sequences are arithmetic
B. Both sequences are geometric
C. Sequence I is arithmetic and Sequence II is geometric
D. Sequence I is geometric and Sequence II is arithmetic
E. Neither sequence is arithmetic or geometric

Question 4

Adam loses 8 kg in the first month of exercise, 4.8 kg in the second month, 2.88kg in the third month and so on. If this continues, the total weight loss in kg will be

A. 13.3

B. 16.6

C. 19.9

D. 20.0

E. 23.3

Question 5

A wildlife service is monitoring the number of koalas near a suburban area for a period of n years. During this time it has found that the number of koalas is decreasing by 6% each year. There were 2500 koalas when they were first counted.
Let K_n be the number of koalas in the area n years after counting. A difference equation describing this situation is

A. $K_n = -0.06 \times K_{n-1}$ where $K_0 = 2500$

B. $K_n = K_{n-1} - 0.06$ where $K_0 = 2500$

C. $K_n = 1.06 \times K_{n-1}$ where $K_0 = 2500$

D. $K_n = K_{n-1} + 0.06$ where $K_0 = 2500$

E. $K_n = 0.94 \times K_{n-1}$ where $K_0 = 2500$

Question 6

To stop the decline in the koala population the wildlife service decides to introduce 200 koalas into a wildlife reserve. The number of koalas in the wildlife reserve at the beginning of the nth year is given by the difference equation $W_n = 0.88 \times W_{n-1} + 50$ where $W_1 = 200$
The koala population in this wildlife reserve will first exceed 400 at the start of

A. the 20^{th} year

B. the 21^{st} year

C. the 22^{nd} year

D. the 23^{rd} year

E. the 24^{th} year

Question 7

A sequence is defined by the difference equation $t_{n+1} = -4t_n - 1$. If the third term of the sequence is 51, the first term is

A. –205

B. –13

C. 13

D. 5

E. 3

Question 8

A second order difference equation is defined by $T_{n+2} = 2T_n + T_{n+1}$, where $T_1 = 2$ and $T_2 = 5$.
The value of T_8 is

A. 299

B. 296

C. 292

D. 288

E. 282

Question 9

In the Fibonacci sequence, $f_{12} = 555$ and $f_{10} = 212$
The value of the 11^{th} term is

A. 171.5

B. 343

C. 383.5

D. 424

E. 767

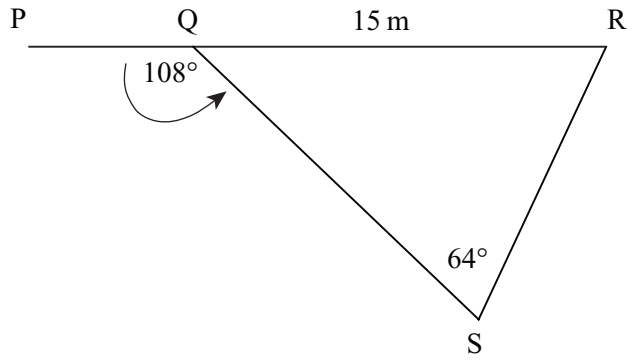
END OF MODULE 1

MODULE 2: GEOMETRY AND TRIGONOMETRY

Instructions

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions.

The following information relates to Questions 1 and 2



Question 1

The size of $\angle QRS$ is exactly

A. 44°

B. 72°

C. 64°

D. 79°

E. 136°

Question 2

Given that the length of QR is 15 m, the length of RS is

A. $15 \tan 64°$

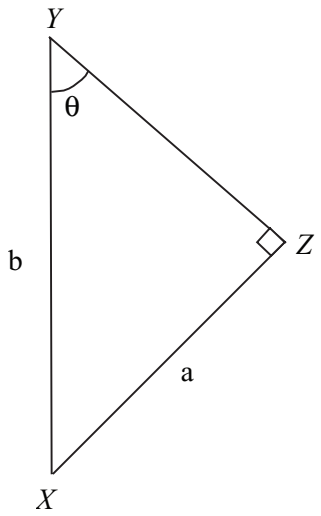
B. $\dfrac{15}{\tan 64°}$

C. $\dfrac{15 \sin 44°}{\sin 64°}$

D. $\dfrac{15 \sin 64°}{\sin 72°}$

E. $\dfrac{15 \sin 72°}{\sin 64°}$

Question 3



In the right-angled ΔXYZ, the length of $XZ = a$, the length of $XY = b$ and $\angle XYZ = \theta$
For the triangle, $\tan\theta$ is given by

A. $\frac{a}{b}$

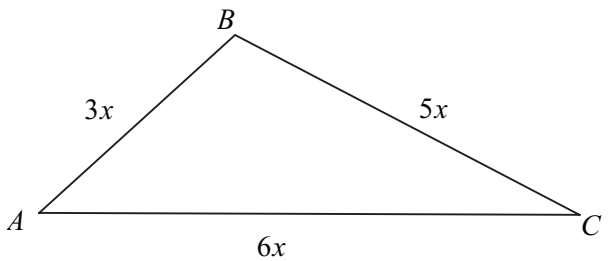
B. $\frac{a}{\sqrt{b^2 + a^2}}$

C. $\frac{\sqrt{b^2 - a^2}}{a}$

D. $\frac{b}{\sqrt{b^2 - a^2}}$

E. $\frac{a}{\sqrt{b^2 - a^2}}$

Question 4



In the ΔABC, the length of $AB = 3x$, the length of $BC = 5x$ and the length of $AC = 6x$.
The area of ΔABC is given by

A. $9x^2$

B. $15x^2$

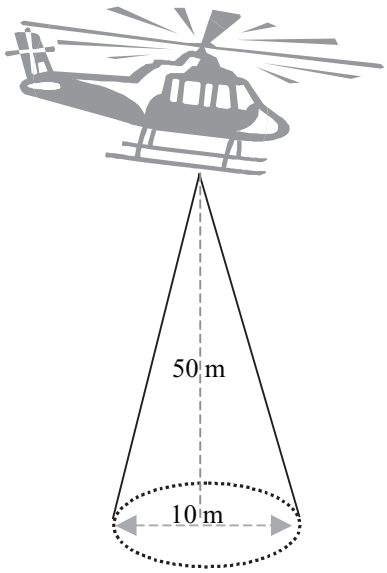
C. $\sqrt{14}x^2$

D. $\sqrt{15}x^2$

E. $\sqrt{56}x^2$

The following information relates to Questions 5 and 6

A search and rescue helicopter shines a light down from a vertical height of 50 metres as shown below. The circular area of light it creates on the ground has a diameter of 10 metres.



Question 5

The helicopter is elevated an additional 15 metres away from the ground. The diameter of the circular area of light on the ground is now closest to

A. 8 m

B. 13 m

C. 15 m

D. 20 m

E. 25 m

Question 6

The helicopter moves to a height so that the diameter of the circular area increases from 10 metres to 40 metres.

The area of the circular light is now

A. four times what it was before.

B. eight times what it was before.

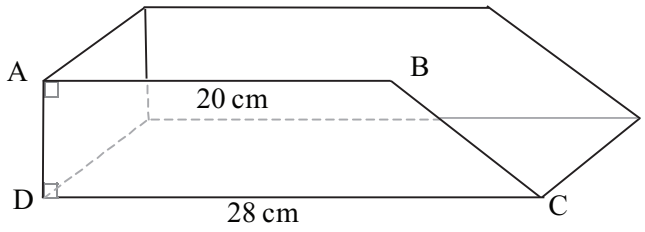
C. sixteen times what is was before.

D. thirty-two times what is was before.

E. sixty-four times what it was before.

Question 7

An open paper tray is made of a rectangular base and three sides as shown.



The area of side ABCD is 240 square centimetres.
The height of the tray AD is closest to

A. 5 cm

B. 8 cm

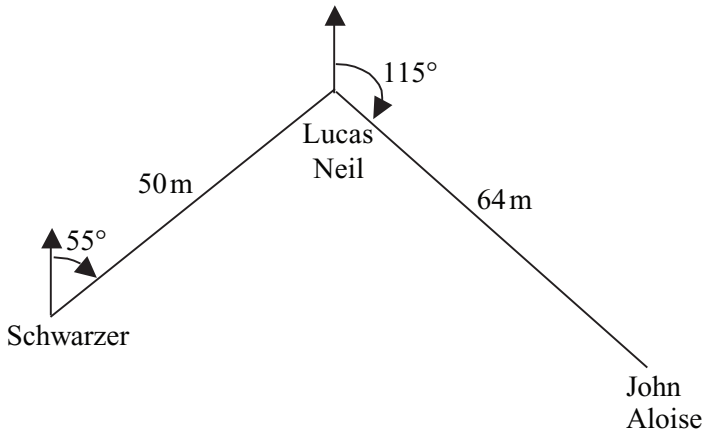
C. 9 cm

D. 10 cm

E. 12 cm

Question 8

Goal keeper Schwarzer kicks a soccer ball to Lucas Neil 50 metres away at a bearing of 055°T. Lucas Neil then kicks the ball to John Aloise 64 metres away on a bearing of 115°T.



The distance between John Aloise and Schwarzer is closest to

A. 58 m

B. 62 m

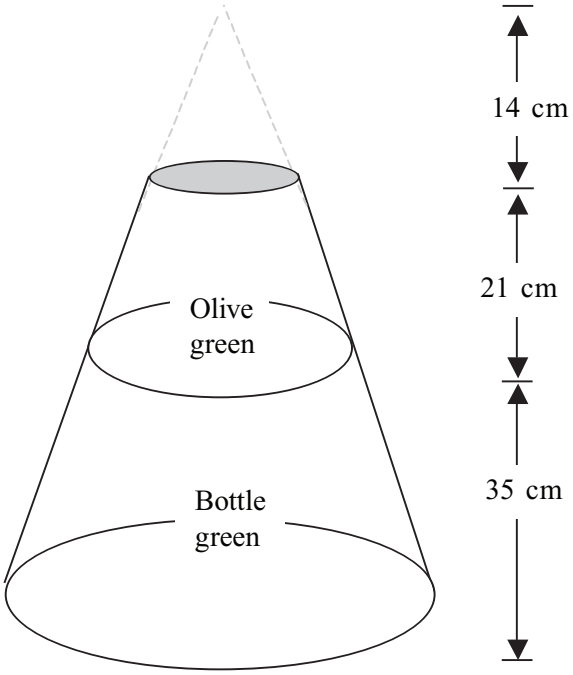
C. 73 m

D. 81 m

E. 99 m

Question 9

A skirt is to be made of two shades of green material, olive green along the top section and bottle green along the bottom section.
The shape of the skirt is a truncated cone as shown



The ratio representing the area of olive green material to the area of bottle green material is

A. 7: 25
B. 3: 5
C. 9: 25
D. 1: 2
E. 1: 4

END OF MODULE 2

MODULE 3: GRAPHS AND RELATIONS

Instructions

Before answering these questions you must **shade** the Graphs and Relations box on the answer sheet for multiple-choice questions.

Question 1

The graph shown has equation

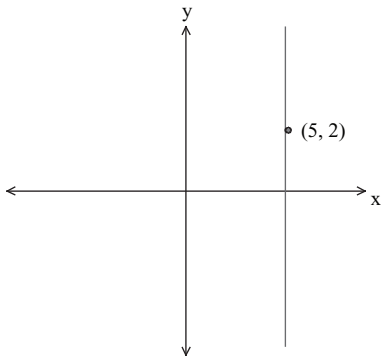
A. $y = 5$

B. $y = 2$

C. $x = 5$

D. $x = 2$

E. $5x + 2y = 0$



Question 2

For the straight line with equation $3x + 5y = 30$, which of the following statements is false?

A. as x decreases, y increases

B. the y-intercept is 6

C. the x-intercept is 10

D. it has a gradient of $-\frac{5}{3}$

E. the line passes through the point (5, 3)

Question 3

At the craftmarket, Helga sold 3 scarves and 2 hats for $60. She later sold 5 hats and 2 scarves for $73. From this information, one hat costs

A. $6.50

B. $9.00

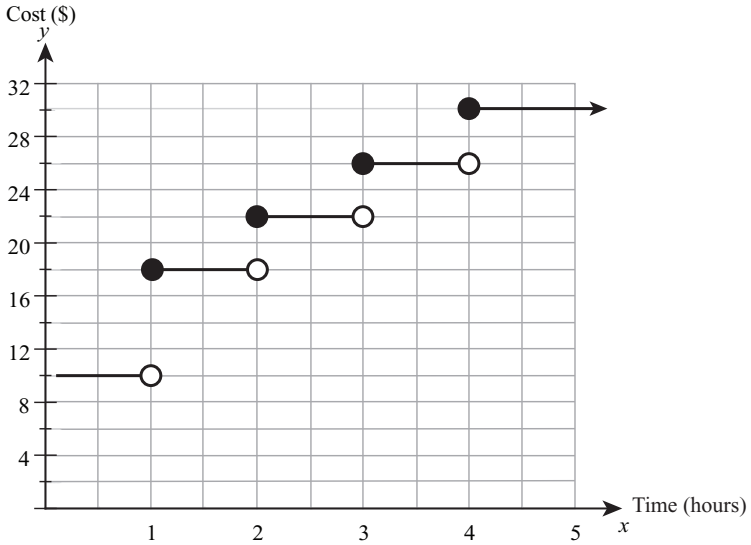
C. $14.00

D. $16.50

E. $20.25

Question 4

The graph below shows the cost of parking in a Melbourne city carpark.



John and Betty parked their car in the same carpark. John paid $\$x$ and Betty paid $\$y$.
The sum of their parking costs $x + y = \$48$.

If John parked his car for 2 hours then Betty parked her car for

A. less than one hour

B. one hour or more but less than to two hours

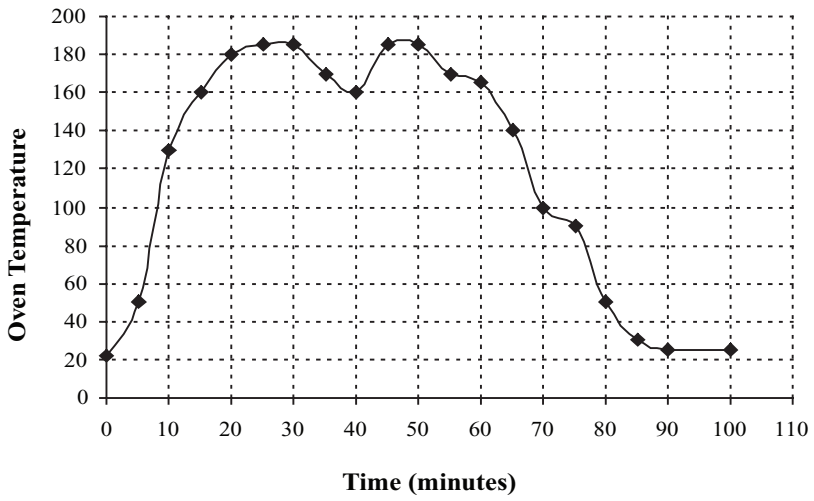
C. two hours or more but less than three hours

D. three hours or more but less than four hours

E. more than four hours

Question 5

The graph below shows the temperature change (in degrees Celsius) in an oven over a period of 100 minutes.



The period of greatest temperature decrease was

A. 0 – 20 minutes

B. 20 – 40 minutes

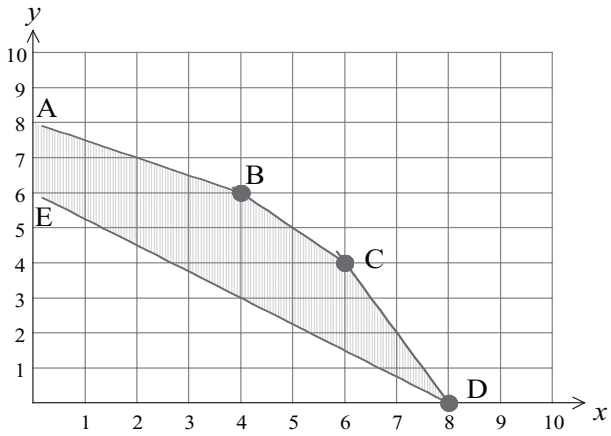
C. 40 – 60 minutes

D. 60 – 80 minutes

E. 80 – 100 minutes

Question 6

The feasible region for a particular linear programming problem is shaded on the following graph. All the relevant vertices are labelled.



The maximum value of the expression $2x + 4y$ occurs at

A. point A

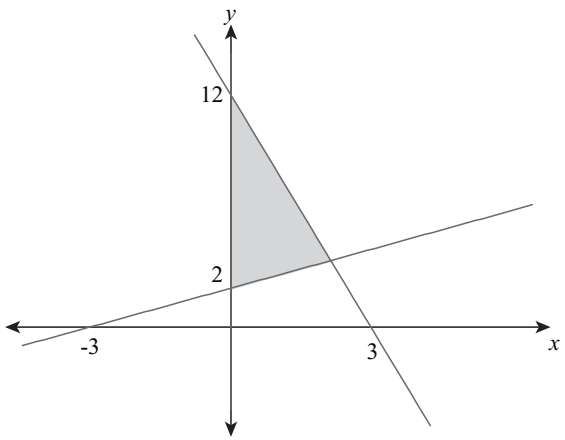
B. point B

C. point A and B

D. the points on the line joining points A and B

E. the points on the line joining points B and C

Question 7



The shaded region shown in the graph above (with boundaries included) is described by

A. $y + 4x \leq 12$
$3y - 2x \geq 6$
$x \geq 0$

B. $y - 4x \leq 12$
$3y - 2x \geq 6$
$x \geq 0$

C. $y + 4x \geq 12$
$3y - 2x \geq 6$
$x \geq 0$

D. $y - 4x \leq 12$
$3y - 2x \leq 6$
$x \geq 0$

E. $y + 4x \leq 12$
$3y - 2x \geq 6$
$x \leq 0$

Question 8

The local supermarket uses permanent staff and casual staff.

The permanent staff work for $8 per hour and the casual staff work for $10 an hour. The supermarket can afford to pay their staff at most $3000 in a 12 hour working day.

Let x be the number of permanent staff, and y be the number of casual staff that work at the local supermarket per day.

The constraint imposed by staff pay is expressed by the inequality

A. $8x + 10y \leq 3000$

B. $8x + 10y \geq 3000$

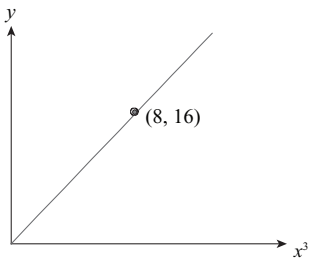
C. $12x + 12y \leq 3000$

D. $96x + 120y \geq 3000$

E. $96x + 120y \leq 3000$

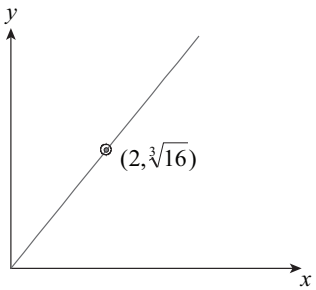
Question 9

The graph of y versus x^3 is shown

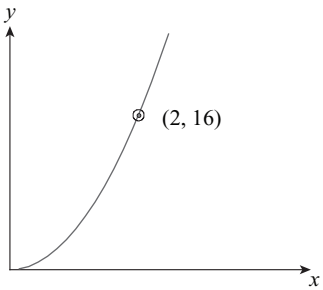


The graph of y versus x is:-

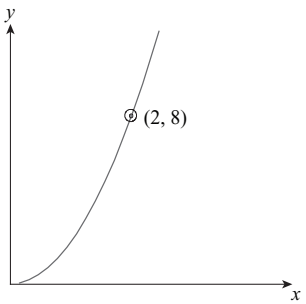
A.



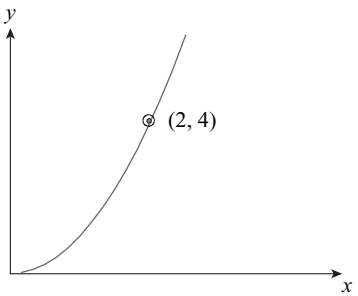
B.



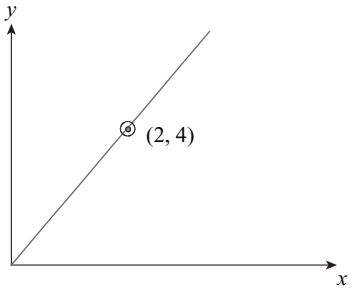
C.



D.



E.



END OF MODULE 3

MODULE 4: BUSINESS-RELATED MATHEMATICS

Multiple-choice questions (9 questions)

Question 1

Jaime has deposited her $8000 savings in a 3-month term deposit that pays 6.2% p.a. interest. If the interest is credited at the end of the term then the amount of interest earned, to the nearest dollar, will be

A. $124
B. $125
C. $165
D. $378
E. $8125

Question 2

A machine bought for $40 000 is valued at $8200 after 6 years. The flat rate of depreciation for this machine, as a percentage correct to one decimal place, is

A. 12.6%
B. 13.3%
C. 16.7%
D. 20.5%
E. 25.5%

Question 3

Selena has bought a property for $376 000 and is required to pay Stamp Duty on this purchase. The price of the property means that the duty payable can be calculated using the following:

$2560 plus 6% of the dutiable value in excess of $115 000

The amount of stamp duty that Selena has to pay is

A. $9460
B. $15 660
C. $18 220
D. $22 560
E. $25 120

Question 4

Marion receives $1210 each month as a perpetuity on an investment of $220 000. The rate of interest per annum being credited on this investment is

A. 5.5%
B. 6.2%
C. 6.6%
D. 7.0%
E. 7.2%

Question 5

Rajiv has invested his savings of $15 000 in an account that pays 6.25% p.a. interest compounding each month. After three years the amount of interest that he has earned on this investment, to the nearest cent, is

A. $235.60

B. $2812.50

C. $2991.94

D. $3084.65

E. $3112.76

The following information relates to questions 6 and 7.

A hire-purchase plan for the purchase of a $17 000 kitchen requires the purchaser to pay a deposit of $1700 and then the remainder, plus interest, in 36 equal monthly instalments.

Question 6

If the credit agency providing the plan charges a flat rate of 8.5% p.a. interest then the monthly payments, to the nearest cent, will be

A. $498.14

B. $518.76

C. $533.38

D. $545.42

E. $547.95

Question 7

The effective rate of interest for this hire-purchase plan, to the nearest percentage, is

A. 5 %

B. 13 %

C. 15 %

D. 17 %

E. 18 %

The following information relates to questions 8 and 9

Mike has negotiated a housing loan of $280 000 which he will repay over 25 years. The loan attracts an interest rate of 6.75% p.a., compounding monthly, and he will make monthly repayments. Each repayment consists of an amount of principal and an amount of interest.

Question 8

The amount of principal, correct to the nearest dollar, that is repaid in the first repayment is

A. $360

B. $412

C. $1523

D. $1575

E. $1935

Question 9

After ten years, Mike pays $100 000 off the principal of the loan but continues repaying the same amount each month. The term of the loan will be reduced by

A. 5 years 3 months

B. 6 years 4 months

C. 7 years 2 months

D. 8 years 8 months

E. 9 years 3 months

END OF MODULE 4

MODULE 5: NETWORKS AND DECISION MATHEMATICS

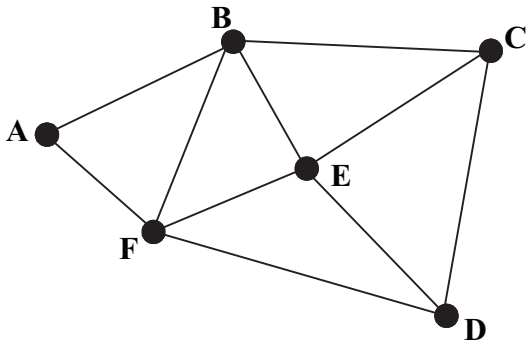
Multiple choice questions (9 questions)

Question 1

A planar graph has 7 vertices and 10 edges. The graph divides the plane into

A. 2 regions
B. 3 regions
C. 5 regions
D. 8 regions
E. 15 regions

Question 2



An Eulerian circuit is possible on the graph above if an additional edge joins two of the vertices. The edge joins vertices

A. **A** to **E**
B. **C** to **D**
C. **A** to **C**
D. **A** to **D**
E. **F** to **D**

Question 3

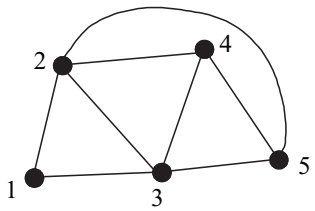
The connectivity matrix below has been constructed for a directed network graph

From \ To	P	Q	R	S	T	U
P	0	0	1	1	0	0
Q	0	0	0	1	1	0
R	0	1	0	0	0	1
S	0	0	0	0	1	0
T	0	0	0	1	0	0
U	0	0	0	0	0	0

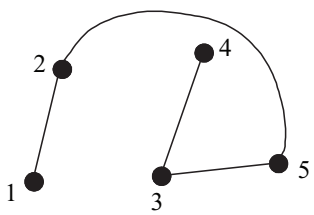
Which one of the following vertices cannot be reached from vertex R?

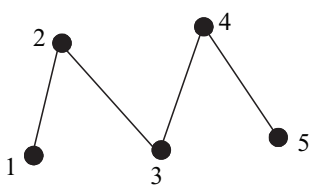
A. P
B. Q
C. S
D. T
E. U

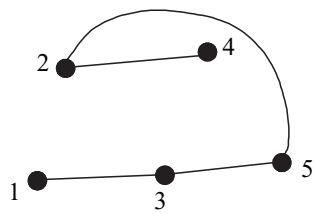
Question 4

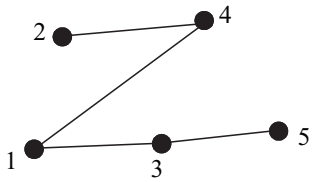


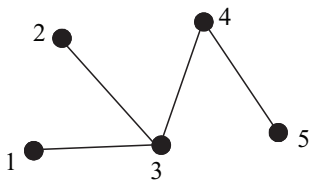
Which one of the following is **not** a spanning tree of the graph above?

A.


B.


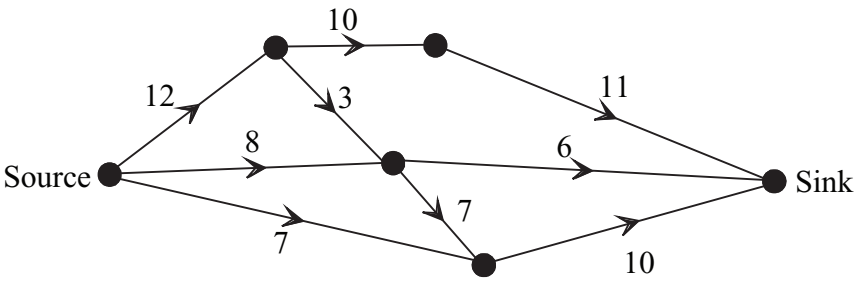
C.


D.


E.


Question 5

On the directed graph below, the values on the edges give the maximum flow between nodes in the direction of the arrows.



The maximum flow, from source to sink, for this graph is

A. 25

B. 26

C. 27

D. 28

E. 31

Question 6

A manufacturer wants to deliver goods to each of his outlets; the delivery van making one trip starting and ending at the factory and visiting each of the outlets once only.
The route that the delivery van takes could be described as

A. a minimum length spanning tree

B. an Eulerian path

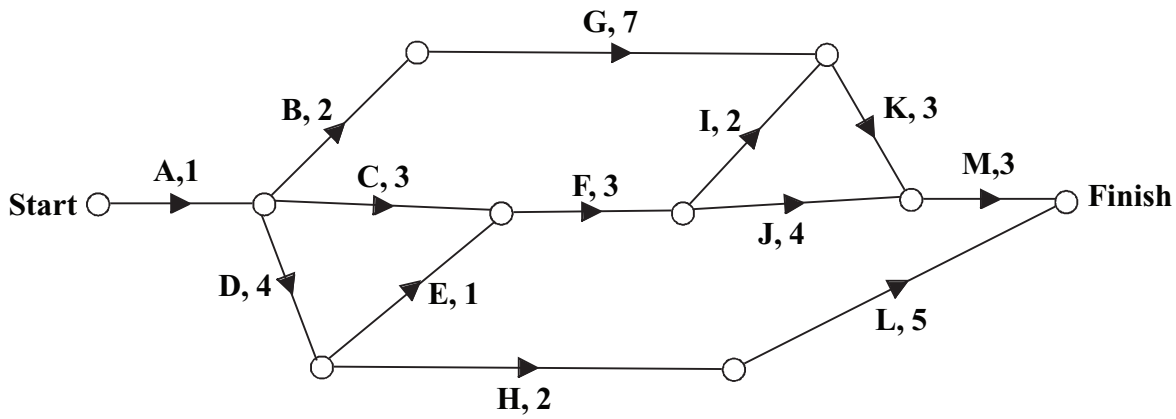
C. an Eulerian circuit

D. a Hamiltonian path

E. a Hamiltonian circuit

The following information relates to questions 7 and 8

The graph below represents the activities (**A** to **M**) and the duration of the activities, in days, for a project.



Question 7

The critical path for the project will be

A. **ACFJM**

B. **ABGKM**

C. **ADHL**

D. **ADEFIKM**

E. **ADEFJM**

Question 8

The slack time, in days, for activity **L** will be

A. 0

B. 1

C. 2

D. 3

E. 5

Question 9

Five teams, P, Q, R, S and T have played in a round-robin tournament where each team plays each other team once only. The results of the games are represented by the dominance matrix M, below, where a '1' in the matrix represents a win and a '0' represents a loss.

$$M = \begin{array}{c} \\ P \\ Q \\ R \\ S \\ T \end{array} \begin{array}{c} \text{Defeated} \\ \begin{array}{ccccc} P & Q & R & S & T \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

The placings, first to last, of the teams could not be determined from the matrix M so the second-order dominance matrix, M^2, was calculated. This is given below

$$M^2 = \begin{array}{c} \\ P \\ Q \\ R \\ S \\ T \end{array} \begin{array}{c} \text{Defeated} \\ \begin{array}{ccccc} P & Q & R & S & T \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 2 & 2 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

Using M and M^2 to rank the teams, the ranking, first to last, will be

A. PQRST

B. PQTRS

C. QPTSR

D. PQTSR

E. PTQST

END OF MODULE 5

MODULE 6: MATRICES

Multiple choice questions (9 questions)

Question 1

If A is the matrix $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ then A^2 is the matrix

A. $\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

E. Cannot be determined

Question 2

If the matrix $M = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 0 & 2 \\ 1 & -2 & 5 \end{bmatrix}$ is multiplied by the matrix $N = \begin{bmatrix} 1 & -1 \\ 3 & 3 \\ -2 & 2 \end{bmatrix}$ then the value of the element in the first row, second column of MN is

A. -1

B. 0

C. 1

D. 3

E. 7

Question 3

The inverse of the matrix $\begin{bmatrix} \boldsymbol{x} & 4 \\ -1 & 8 \end{bmatrix}$ will not exist if $\boldsymbol{x}$ is equal to

A. -4

B. -2

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

E. 2

Question 4

Matrices are to be used to solve the system of simultaneous linear equations

$x - 3y + 16 = 0$

and $y = 8x - 10$

The matrix equation to solve this system is

A. $\begin{bmatrix} 1 & -3 \\ 1 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 10 \end{bmatrix}$

B. $\begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 1 & -8 \end{bmatrix} = \begin{bmatrix} 16 \\ -10 \end{bmatrix}$

C. $\begin{bmatrix} 1 & -3 \\ 8 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -16 \\ -10 \end{bmatrix}$

D. $\begin{bmatrix} 1 & -3 \\ -8 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -16 \\ -10 \end{bmatrix}$

E. $\begin{bmatrix} 1 & -3 \\ -8 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -16 \\ -10 \end{bmatrix}$

Question 5

If $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} \times M = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$ then M is the matrix

A. $\begin{bmatrix} -1 & 5 \\ 1 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 3 & 11 \\ 7 & 9 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

E. $\frac{1}{5}\begin{bmatrix} -6 & 14 \\ 1 & 6 \end{bmatrix}$

Question 6

A customer reward scheme for a shop has had a '*bonanza*' week where double bonus points were offered on sales in that week.
The total bonus points prior to *bonanza* week and the points accumulated in *bonanza* week are given for five customers in the following table.

Customer	Total points prior to *bonanza* week	Points accumulated in *bonanza* week
Dave W.	105	21
Theo N.	34	12
Janine P.	93	10
Vinh N.	68	15
Mario P.	72	5

The shop proprietor wants to use matrices to double the points accumulated in *bonanza* week and add them to the total points accumulated prior to bonanza week giving a total number of points for each customer.
A matrix product that will do this for the proprietor is

A. $\begin{bmatrix}1\\2\end{bmatrix}\times\begin{bmatrix}105 & 21\\34 & 12\\93 & 10\\68 & 15\\72 & 5\end{bmatrix}$

B. $\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}\times\begin{bmatrix}105 & 21\\34 & 12\\93 & 10\\68 & 15\\72 & 5\end{bmatrix}$

C. $\begin{bmatrix}105 & 21\\34 & 12\\93 & 10\\68 & 15\\72 & 5\end{bmatrix}\times\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$

D. $\begin{bmatrix}105 & 21\\34 & 12\\93 & 10\\68 & 15\\72 & 5\end{bmatrix}\times\begin{bmatrix}1\\2\end{bmatrix}$

E. $\begin{bmatrix}1 & 2\end{bmatrix}\times\begin{bmatrix}105 & 21\\34 & 12\\93 & 10\\68 & 15\\72 & 5\end{bmatrix}$

Question 7

If $\begin{bmatrix}2 & 3\\-2 & -1\end{bmatrix}+\begin{bmatrix}1 & 2\\1 & 3\end{bmatrix}\times P=\begin{bmatrix}3 & 3\\-2 & 0\end{bmatrix}$ then the matrix P will be

A. $\begin{bmatrix}1 & 2\\1 & 3\end{bmatrix}$

B. $\begin{bmatrix}3 & -2\\-1 & 1\end{bmatrix}$

C. $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

D. $\begin{bmatrix}-3 & 2\\1 & -1\end{bmatrix}$

E. $\frac{1}{4}\begin{bmatrix}-1 & -3\\2 & 2\end{bmatrix}$

The following information relates to questions 8 and 9

After an intense advertising campaign 38% of the available television audience watched the first episode of a television show.
Of those who watched the first episode 76% watched the second episode and of those who did not watch the first episode 15% watched the second episode.

Question 8

A transition matrix, *T*, that could be used to describe this situation is

A. $\begin{bmatrix} 0.76 \\ 0.15 \end{bmatrix}$

B. $\begin{bmatrix} 0.76 \\ 0.24 \end{bmatrix} + \begin{bmatrix} 0.85 \\ 0.15 \end{bmatrix}$

C. $\begin{bmatrix} 0.76 & 0.15 \\ 0.24 & 0.85 \end{bmatrix}$

D. $\begin{bmatrix} 0.76 & 0.85 \\ 0.24 & 0.15 \end{bmatrix}$

E. $\begin{bmatrix} 0.76 & 0.15 \\ 0.85 & 0.24 \end{bmatrix}$

Question 9

Assuming that the choice of television program in a particular time slot depends entirely on the choice of program in the previous week, and *T* is the transition matrix for this situation, the percentage of the available television audience that is watching this particular television show after five weeks can be found using

A. $T^5 \times \begin{bmatrix} 76 \\ 24 \end{bmatrix}$

B. $[76 \quad 24] \times T^5$

C. $\left[T \times \begin{bmatrix} 38 \\ 62 \end{bmatrix} \right]^5$

D. $T^5 \times \begin{bmatrix} 38 \\ 62 \end{bmatrix}$

E. $T \times \begin{bmatrix} 38 & 0 \\ 0 & 62 \end{bmatrix}^5$

END OF MODULE 6

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

April 2006

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: $\text{seasonal index} = \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n-1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET