

Further Mathematics Exam 1: Solutions – Multiple choice

Core : Data analysis

Question 1 Answer C

Class is a categorical variable because a category (Class 1, Class 2 or Class 3) is recorded for it. *Score on test* is a numerical variable because a number is recorded for it.

Question 2 Answer C

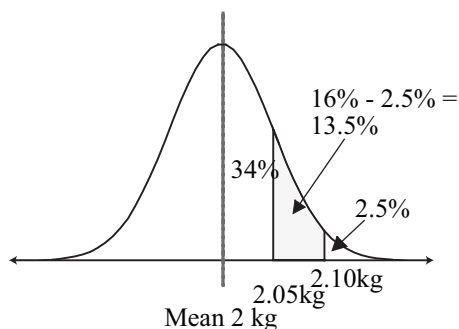
The interquartile range

= **Upper quartile** (indicated by the value of the right end of the box) – **Lower Quartile** (indicated by the value of the left end of the box)
 = 70 – 50
 = 20

Question 3 Answer D

The distribution of scores in class B is positively skewed because the boxplot is stretched to the right; the right whisker is longer than the left and the median line is towards the left of the box.

Question 4 Answer C



2.05kg is one standard deviation above the mean (16% of the bags will have weights more than 2.05kg) and 2.10 is two standard deviations above the mean (2.5% of the bags will have weights above this value). Between these 2.05kg and 2.10kg there will be $16\% - 2.5\% = 13.5\%$ of the bags.

Question 5 Answer B

Substituting the given values in the equation

$$z\text{-score} = \frac{\text{raw score} - \text{mean}}{\text{standard deviation}}$$

$$-1.25 = \frac{56 - 66}{\text{standard deviation}}$$

$$-1.25 = \frac{-10}{\text{standard deviation}}$$

$$\begin{aligned} \text{Standard deviation} &= \frac{-10}{-1.25} \\ &= 8 \end{aligned}$$

Question 6 Answer E

Of the 45 people in the 50⁺ age group, 23 of them are going to vote for the same party.

$$\frac{23}{45} \times \frac{100}{1} = 51.1\%$$

Question 7 Answer E

23 out of the 45 (51.1%) in the 50⁺ age group are going to vote for the same party.

18 out of the 42 (42.8%) of the 30 - < 50 age group are going to vote for the same party

20 out of the 33 (60.6%) of the 18 - < 30 age group are going to vote for the same party. This is more than the 50⁺ age group.

Question 8 Answer D

The right median point will be (9, 4) and the gradient of the 3-median regression line will have the same gradient as the line joining the left (2, 10) and right median points.

$$\text{Gradient} = \frac{4 - 10}{9 - 2} = \frac{-6}{7} \approx -0.857 \approx -0.9$$

Question 9 Answer E

The regression line underestimates the y -value for $x = 5$ as it is below this point.

There are the same number of points above and below the line hence the same number of positive and negative residuals.

There is a residual of -2.5 but the largest is around $+2$.

The gradient of the regression line is approx.

$$\frac{-6}{8} \approx -0.67 \text{ a value } > -1$$

Question 10 Answer B

Mar.	April	May	June	July
29	26	25	22	29

$\frac{29 + 26 + 25 + 22}{4} = 25.5$	$\frac{26 + 25 + 22 + 29}{4} = 25.5$
--------------------------------------	--------------------------------------

Centering these two values:

$$\frac{25.5 + 25.5}{2} = 25.5$$

Question 11 Answer D

3	4	Yearly average
279	173	224
283	181	231

The sales figures are divided by their yearly average and then these two values are averaged.

$$\frac{\frac{279}{224} + \frac{283}{231}}{2} = 1.2353 \text{ correct to four decimal places.}$$

Question 12 Answer A

To deseasonalise data the actual data value is divided by its seasonal index.

$$\frac{190}{0.8440} = 225.1 \text{ an increase}$$

$$\frac{254}{1.1427} = 222.3 \text{ a decrease}$$

Question 13 Answer D

A $\frac{1}{y}$ transformation compresses the y scale and makes the large y -values small and the small y -values large.



Further Mathematics Exam 1: Solutions – Multiple choice

Module 1: Number Patterns

Question 1 Answer D

Arithmetic sequence

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$a = 17, d = 6, n = 12$$

$$\begin{aligned} S_{12} &= \frac{12}{2}[2 \times 17 + (12-1)6] \\ &= 6(34 + 66) \\ &= 600 \end{aligned}$$

Question 2 Answer B

$$r = 1.2, t_3 = 180, a = ?, t_n > 1000, n = ?$$

Find a first

$$ar^2 = 180$$

$$a(1.2)^2 = 180$$

$$a = \frac{180}{(1.2)^2}$$

$$\therefore a = 125$$

Find n (METHOD 1)

$$t_n > 1000, n = ?$$

$$ar^{(n-1)} > 1000$$

$$125 \times 1.2^{(n-1)} > 1000$$

$$1.2^{(n-1)} > 8$$

$$n-1 > \frac{\log_{10}(8)}{\log_{10}(1.2)}$$

$$n-1 > 11.4$$

$$n > 12.4$$

$$\therefore n = 13$$

Find n (METHOD 2)

Use sequence mode

Type in $u(n) = 125(1.2)^{(n-1)}$

Plot1	Plot2	Plot3
nMin=1		
u(n)=125*1.2^(n-1)		
u(nMin)=		
u(n)=		
u(nMin)=		
u(n)=		

Then go to TABLE

n	u(n)	
8	447.9	
9	537.48	
10	644.97	
11	773.97	
12	928.76	
13	1114.51	
14	1337.4	
u(n)=1114.512556		

Scroll down until $t_n > 1000$

Read value of n

Question 3 Answer D

Sequence 1 is decreasing and non-linear with terms alternating between positive and negative. Therefore sequence is geometric with $-1 < r < 0$

Sequence 2 is decreasing and linear, therefore sequence is arithmetic with $d < 0$

Question 4 Answer D

8, 4.8, 2.88,

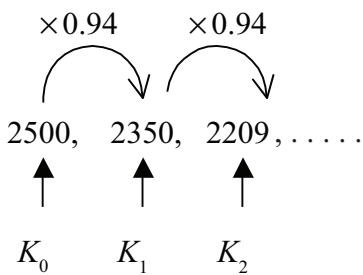
$$a = 8 \text{ and } r = \frac{2.88}{4.8} = \frac{4.8}{8} = 0.6$$

The sum of this sequence is given by

$$\begin{aligned} S_{\infty} &= \frac{a}{1-r} \\ &= \frac{8}{1-0.6} \\ &= 20 \end{aligned}$$

Question 5 Answer E

Decreasing by 6% means that $r = 1 - 0.06 = 0.94$



So the difference equation is $K_n = 0.94 \times K_{n-1}$ where $K_0 = 2500$

Question 6 Answer C

The equation $W_n = 0.88 \times W_{n-1} + 50$ where $W_1 = 200$

This can be entered in the sequence mode of the calculator

$$n_{\min} = 1$$

$$u(n) = 0.88u(n-1) + 50$$

$$u(n_{\min}) = \{200\}$$

```

Plot1 Plot2 Plot3
nMin=1
u(n)=0.88u(n-1)+50
u(nMin)=200
v(n)=
v(nMin)=
w(n)=
  
```

Then go to TABLE to and scroll down until you first see $u(n) > 400$

n	$u(n)$	
17	388.64	
18	392.01	
19	394.97	
20	397.57	
21	399.86	
22	401.88	
23	403.65	

$n=22$

This occurs at $n = 22$

Question 7 Answer E

First find t_2

$$t_3 = -4t_2 - 1$$

$$51 = -4t_2 - 1$$

$$\frac{51+1}{-4} = t_2$$

$$\therefore t_2 = -13$$

Then find t_1

$$t_2 = -4t_1 - 1$$

$$-13 = -4t_1 - 1$$

$$\frac{-13+1}{-4} = t_1$$

$$\therefore t_1 = 3$$

Question 8 Answer A

An efficient method is to use the sequence mode on your calculator and enter the equation

$$u_{\min} = 1$$

$$u(n) = 2u(n-2) + u(n-1)$$

$$u(n_{\min}) = \{5, 2\}$$

enter 2nd term first $\{t_2, t_1\}$

```

Plot1 Plot2 Plot3
nMin=1
u(n)=2u(n-2)+u(n-1)
u(nMin)=5,2
v(n)=
v(nMin)=
w(n)=
  
```

Then go to the TABLE to find $u(8) = 299$

n	$u(n)$	
5	37	
6	75	
7	149	
8	299	
9	597	
10	1195	
11	2389	

$u(n)=299$

An alternative method

Working through this algebraically

Substitute $T_1 = 2$ and $T_2 = 5$

$$T_3 = 2T_1 + T_2$$

$$T_3 = 2 \times 2 + 5$$

$$= 9$$

$$T_4 = 2T_2 + T_3$$

$$T_4 = 2 \times 5 + 9$$

$$= 19$$

$$T_5 = 2T_3 + T_4$$

$$T_5 = 2 \times 9 + 19$$

$$= 37$$

$$T_6 = 2T_4 + T_5$$

$$T_6 = 2 \times 19 + 37$$

$$= 75$$

$$T_7 = 2T_5 + T_6$$

$$T_7 = 2 \times 37 + 75$$

$$= 149$$

$$T_8 = 2T_6 + T_7$$

$$T_8 = 2 \times 75 + 149$$

$$= 299$$

Question 9 Answer BFor a Fibonacci sequence, $f_{10} + f_{11} + f_{12}$

$$\therefore f_{11} = f_{12} - f_{10}$$

$$= 555 - 212$$

$$= 343$$

Further Mathematics Exam 1: Solutions – Multiple choice

Module 2: Geometry & Trigonometry Solutions

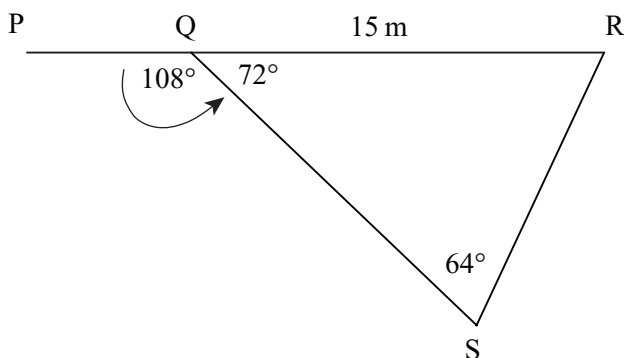
Question 1 Answer A

$$\angle QSR + \angle QRS = \angle PQS$$

$$64^\circ + \angle QRS = 108^\circ$$

$$\angle QRS = 44^\circ$$

Question 2 Answer E



$$\angle SQR = 180^\circ - 108^\circ = 72^\circ$$

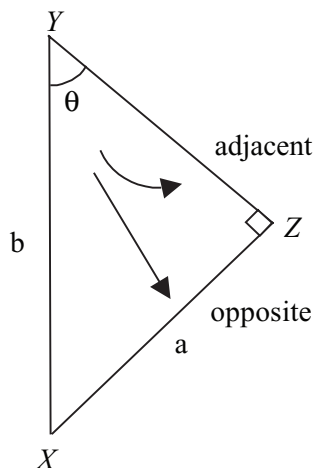
Using the sine rule

$$\frac{RS}{\sin 72^\circ} = \frac{15}{\sin 64^\circ}$$

$$RS = \frac{15 \times \sin 72^\circ}{\sin 64^\circ}$$

$$= 15.872 \text{ m}$$

Question 3 Answer E



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{XZ}{YZ}$$

Using Pythagoras

$$YZ^2 + XZ^2 = XY^2$$

$$YZ^2 = XY^2 - XZ^2$$

$$YZ^2 = b^2 - a^2$$

$$YZ = \sqrt{b^2 - a^2}$$

$$\therefore \tan \theta = \frac{a}{\sqrt{b^2 - a^2}}$$

Question 4 Answer E

Using Heron's Formula

$$s = \frac{3x + 5x + 6x}{2} = 7x$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{7x(7x-3x)(7x-5x)(7x-6x)}$$

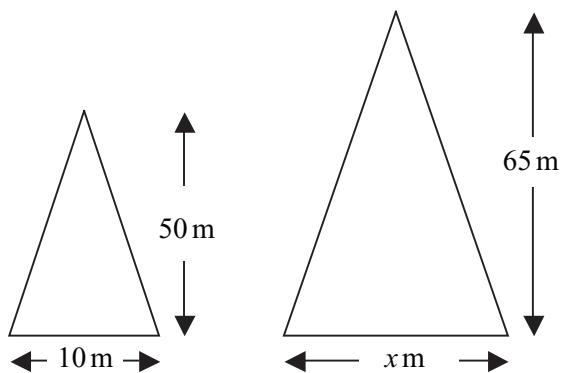
$$= \sqrt{7x(4x)(2x)(x)}$$

$$= \sqrt{56x^4}$$

$$= \sqrt{56}x^2$$

Question 5 Answer B

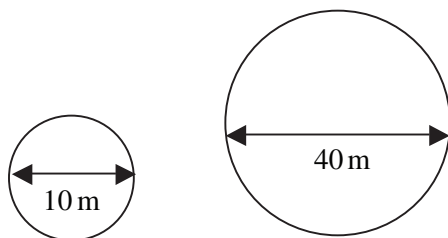
Using similar triangles



$$\frac{x}{10} = \frac{65}{50}$$

$$x = \frac{65 \times 10}{50}$$

$$x = 13 \text{ metres}$$

Question 6 Answer C

Length ratio is 1 : 4

Area ratio is $1^2 : 4^2$

$$= 1 : 16$$

Therefore area is 16 times larger.

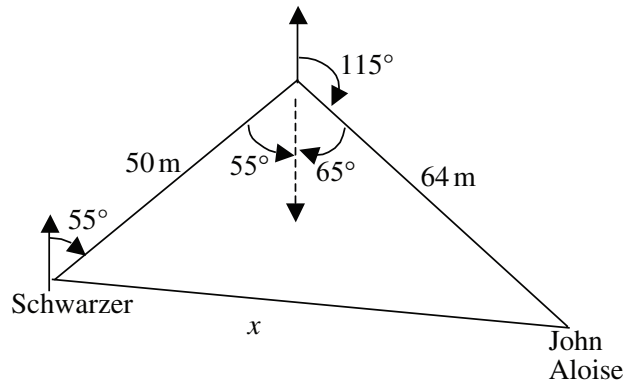
Question 7 Answer D

$$\text{Area of Trapezium} = \frac{1}{2}(a+b)h$$

$$240 = \frac{1}{2}(20+28)h$$

$$240 = 24h$$

$$\therefore h = \frac{240}{24} = 10 \text{ cm}$$

Question 8 Answer E

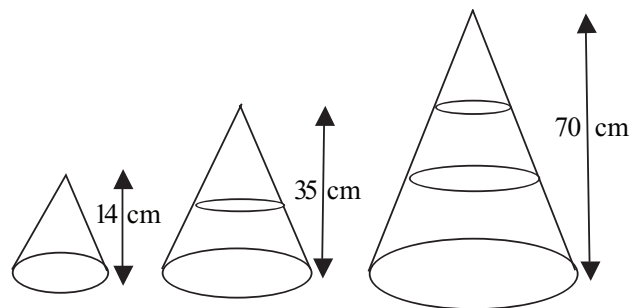
Using cosine rule

$$x^2 = 50^2 + 64^2 - 2(50)(64)\cos 120^\circ$$

$$x^2 = 9796$$

$$x = 98.97$$

$$x \approx 99 \text{ m}$$

Question 9 Answer A

Length ratio is 14 : 35 : 70

$$= 2 : 5 : 10$$

Area ratio is $2^2 : 5^2 : 10^2$

The ratio of the olive : bottle green is

$$25 - 4 : 100 - 25$$

$$21 : 75$$

$$= 7 : 25$$

Further Mathematics Exam 1: Solutions – Multiple choice

Module 3: Graphs and Relations

Question 1 Answer C

The vertical line has no gradient. Every point on this line has the same x value. Since the point $(5, 2)$ is on the line then the equation is given by $x = 5$.

Question 2 Answer D

Transform the equation to make y the subject

$$3x + 5y = 30$$

$$5y = 30 - 3x$$

$$y = 6 - \frac{3}{5}x$$

This means that the gradient is $-\frac{3}{5}$, so the graph is decreasing

i.e. as x increases, y decreases or conversely as x decreases, y increases $\therefore$ A is true

When $x = 0$, $y = 6 \therefore$ B is true

When $y = 0$, $x = 10 \therefore$ C is true

The gradient is $-\frac{3}{5}$, not $-\frac{5}{3}$ so D is false

Substituting $x = 5$ in equation gives $y = 3 \therefore$ E is true

Question 3 Answer B

Let s = the cost of one scarf and

h = the cost of one hat

$$3s + 2h = 60 \quad - \quad (1)$$

$$2s + 5h = 73 \quad - \quad (2)$$

To eliminate s

$$3 \times (2) - 2 \times (1)$$

$$6s + 15h = 219$$

$$6s + 4h = 120$$

$$11h = 99$$

$$\therefore h = \$9$$

Question 4 Answer D

\$22 is charged for 2 hours.

Therefore Betty paid $\$48 - \$22 = \$26$.

\$26 is charged for 3 to 4 hours of parking.

Question 5 Answer D

For 0 – 20 minutes

$$m \approx \frac{180 - 20}{20} = 8^\circ\text{C} / \text{min}$$

For 20 – 40 minutes

$$m \approx \frac{160 - 180}{20} = -1^\circ\text{C} / \text{min}$$

For 40 – 60 minutes

$$m \approx \frac{165 - 160}{20} = 0.25^\circ\text{C} / \text{min}$$

For 60 – 80 minutes

$$m \approx \frac{50 - 165}{20} = -5.75^\circ\text{C} / \text{min}$$

For 80 – 100 minutes

$$m \approx \frac{25 - 50}{20} = -1.25^\circ\text{C} / \text{min}$$

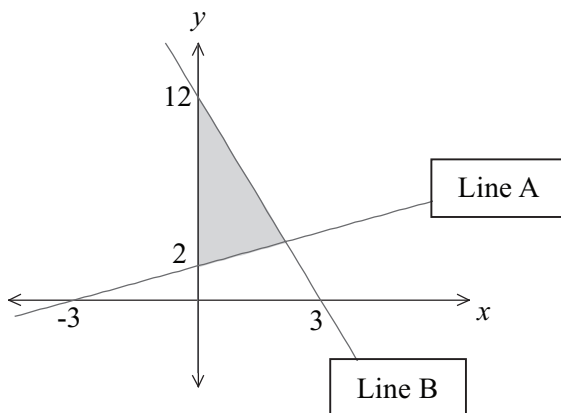
The steepest negative gradient occurs between 60 to 80 minutes.

Question 6 Answer D

Vertex	$2x + 4y$
A(0, 8)	$2 \times 0 + 4 \times 8 = 32$
B(4, 6)	$2 \times 4 + 4 \times 6 = 32$
C(6, 4)	$2 \times 6 + 4 \times 4 = 28$
D(8, 0)	$2 \times 8 + 4 \times 0 = 16$
E(0, 6)	$2 \times 0 + 4 \times 6 = 24$

The maximum occurs along the boundary line joining points A and B.

Question 7 Answer A



Line A

Passes through points $(-3, 0)$ and $(0, 2)$

$$\text{Gradient} = \frac{2}{3}, \text{y-intercept} = 2$$

$$\therefore \text{equation is } y = \frac{2}{3}x + 2$$

$$3y = 2x + 6$$

$$3y - 2x = 6$$

Test point $(0, 3)$ is in the required region

$$3 \times 3 - 2 \times 0 \geq 6$$

$$\therefore 3y - 2x \geq 6$$

defines required region.

This eliminates D

Line B

Passes through points $(0, 12)$ and $(3, 0)$

$$\text{Gradient} = -\frac{12}{3} = -4, \text{y-intercept} = 12$$

$$\therefore \text{equation is } y = -4x + 12$$

$$y + 4x = 12$$

Test point $(0, 3)$ is in the required region

$$3 + 4 \times 0 \leq 12$$

$$\therefore y + 4x \leq 12$$

defines required region.

This eliminates B and C.

Since the feasible region shows all x values to be positive then $x \geq 0$. This eliminates E.

Question 8 Answer E

Permanent Staff

For a 12 hour day each of the permanent staff will be paid $12 \times \$8 = \96 per day

If x permanent staff are working for the day then a total of $\$96x$ is paid.

Casual Staff

For a 12 hour day each of the casual staff will be paid $12 \times \$10 = \120 a day

If y casual staff are working for the day then a total of $\$120y$ is paid.

Total Pay

This gives a total of $96x + 120y$ dollars per day.

The supermarket can afford to pay *at most* 3000 per day i.e. less than or equal to \$3000

$$\text{So } 96x + 120y \leq 3000$$

Question 9 Answer B

The graph of y against x^3 is linear and passes through the origin so the equation of the relation is $y = kx^3$ where k represents the gradient of the graph.

$$\text{Gradient} = \frac{16}{8} = 2 \quad \therefore k = 2$$

The equation of the relation is $y = 2x^3$.

The graph of y against x produces a non-linear graph. This eliminates A and E.

$$\text{When } x = 2, y = 2 \times 2^3 = 2 \times 8 = 16$$

This means the point $(2, 16)$ is on the graph.

Further Mathematics Exam 1: Solutions – Multiple choice

Module 4: Business-related mathematics

Question 1 Answer A

$$\text{Monthly interest} = \frac{6.2}{12}\%$$

Using the simple interest formula:

$$\begin{aligned}\text{Interest earned} &= \frac{Prt}{100} = \frac{\$8000 \times \frac{6.2}{12} \times 3}{100} \\ &= \$124\end{aligned}$$

Question 2 Answer B

$$\begin{aligned}\text{Depreciation} &= \$40\,000 - \$8200 \\ &= \$31\,800 \text{ in six years.}\end{aligned}$$

Flat rate depreciation means that the depreciation is the same each year.

$$\begin{aligned}\text{Depreciation per year} &= \frac{\$31\,800}{6} \\ &= \$5300\end{aligned}$$

As a percentage of the purchase price this is

$$\frac{5300}{40000} \times \frac{100}{1} = 13.25\% \approx 13.3\%$$

Question 3 Answer C

Stamp duty

$$\begin{aligned}&= \$2560 + \frac{6}{100} \times (\$376\,000 - \$115\,000) \\ &= \$2560 + \$15\,660 \\ &= \$18\,220\end{aligned}$$

Question 4 Answer C

\$1210 is the interest earned per month on the investment of \$220 000.

$$\begin{aligned}\text{Interest earned per year} &= 12 \times \$1210 \\ &= \$14\,520\end{aligned}$$

As a percentage this is

$$\frac{14\,520}{220\,000} \times \frac{100}{1} = 6.6\%$$

Question 5 Answer D

Using the compound interest formula the amount in the account: $A = PR^n$

where

$$P = \$15\,000$$

$$R = 1 + \frac{\frac{6.25}{12}}{100} = 1.005208 \dots \text{leave on calculator}$$

$$n = 3 \times 12 = 36 \text{ months}$$

$$\begin{aligned}A &= 15\,000 \times (1.005208\dots)^{36} \\ &\approx 18\,084.652\end{aligned}$$

So the interest earned

$$\begin{aligned}&= \$18\,084.65 - \$15\,000 \\ &= \$3084.65 \text{ to the nearest cent.}\end{aligned}$$

Question 6 Answer C

$$\begin{aligned}\text{Amount owed} &= \$17\,000 - \$1700 \\ &= \$15\,300\end{aligned}$$

Interest that will accumulate over three years

$$= \frac{\$15\,300 \times 8.5 \times 3}{100} = \$3901.50$$

$$\begin{aligned}\text{Total owing} &= \$15\,300 + \$3901.50 \\ &= \$19\,201.50\end{aligned}$$

$$\begin{aligned}\text{Monthly payment} &= \frac{\$19\,201.50}{36} = \$533.38 \text{ to} \\ &\text{the nearest cent.}\end{aligned}$$

Question 7 Answer D

Using the formula.

Effective rate of interest

$$\approx \frac{2n}{n+1} \text{ flat rate of interest}$$

Effective rate of interest

$$= \frac{2 \times 36}{36+1} \times 8.5$$

$$= \frac{72}{37} \times 8.5$$

$$= 16.54\%$$

$$\approx 17\%$$

Question 8 Answer A

Use the TVM solver to find the monthly payment:

```
N=300
I%=6.75
PV=280000
PMT=-1934.5522...
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

The monthly payment is \$1934.55

From this we can deduct the interest earned on the loan in the first month:

$$I = \frac{\$280\,000 \times \frac{6.75}{12} \times 1}{100} = \$1575$$

The amount of principal repaid in the first month is \$359.55 $\approx$ \$360

Alternatively:

Use the 'sum of the principal paid over a specified period' calculator on the FINANCE menu:

```
CALC VARS
6: TVM_FV
7: nPV(
8: irr(
9: bal(
10: ΣPrn(
A: ΣInt(
B: Nom(
```

```
ΣPrn(1,1)
-359.5522706
```

Question 9 Answer D

Firstly find the principal remaining after 10 years.

```
N=120
I%=6.75
PV=280000
PMT=-1934.5522...
FV=-218615.8419
P/Y=12
C/Y=12
PMT:END BEGIN
```

Deduct from the principal remaining the \$100 000 and make this the present value (PV) and the future value (FV) equal to 0.

```
N=120
I%=6.75
PV=118615.8419
PMT=-1934.5522...
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

Now SOLVE for N.

```
N=75.4036633
I%=6.75
PV=118615.8419
PMT=-1934.5522...
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

It will take 76 more months to repay the loan; a total of $10 \times 12 + 76 = 196$ months instead of 300 months (25 years).

The term is reduced by 104 months
= 8 years, 8 months.

Further Mathematics Exam 1: Solutions – Multiple choice

Module 5: Networks and decision mathematics

Question 1 Answer C

Euler's formula for planar graphs is

$$v + f = e + 2$$

where v is the number of vertices, f is the number of 'faces' or regions and e is the number of edges.

Substituting:

$$\begin{aligned} 7 + f &= 10 + 2 \\ f &= 5 \end{aligned}$$

Question 2 Answer B

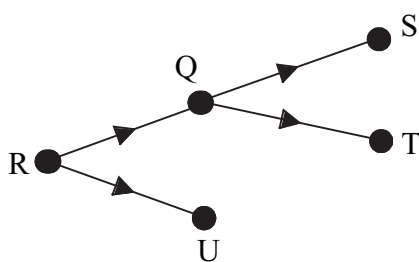
An Eulerian circuit exists when the degree of all the vertices is even. In this graph all the vertices are of even degree except C and D (both degree 3).

An edge joining vertex C to vertex D would make all vertices of even degree and so an Eulerian circuit would exist.

Question 3 Answer A

There is an edge going from vertex R to vertices Q and U

From vertex Q there are edges going to vertices S and T but there are no edges going from vertex U.



So S, Q, T and U can be reached from R but not vertex P.

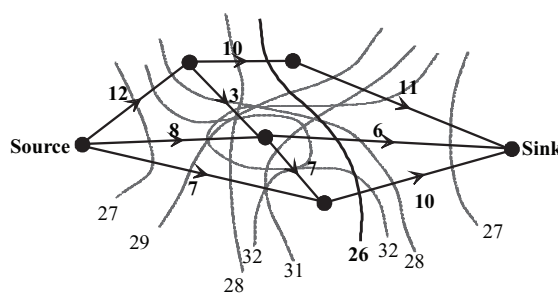
Question 4 Answer D

A spanning tree is a subset of the original graph which connects all the vertices but contains no circuits.

All the answer graphs are connected, contain no circuits and contain all the vertices but the graph in Answer D has an extra edge going from vertex 4 to vertex 1 which means it is not a subset of the original graph.

Question 5 Answer B

The maximum flow from source to sink is the value of the minimum cut



All possible cuts are shown and the minimum is 26.

Question 6 Answer E

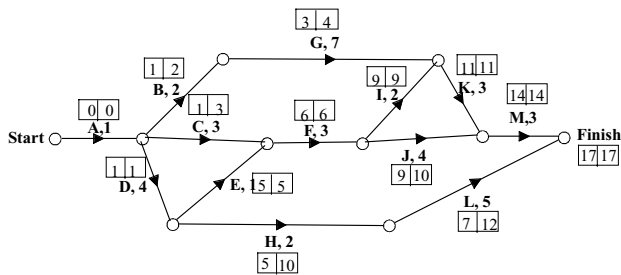
A Hamiltonian circuit is a path that visits every vertex (outlet) exactly once, starting and ending at the same vertex (the factory).

Question 7 Answer D

The critical path of a project is the sequence of activities that is the longest throughout the network.

Activities on the critical path have the same *earliest start time* and *latest start time*. (These are the figures in the rectangle on the diagram below.)

A forward and backward scan will identify the critical path.



The critical path is **ADEFIKM**

Question 8 Answer E

The slack time is the difference between the *earliest start time* and *latest start time*. For activity **L** this is $12 - 7 = 5$ days

Question 9 Answer D

The dominance vector for M (found by adding the values in each of the rows of M) is

$$\begin{bmatrix} 3 \\ 3 \\ 1 \\ 1 \\ 2 \end{bmatrix} \text{ which gives the number of first-order wins}$$

for the teams but does not give a ranking for the teams.

The dominance vector for $M + M^2$ is

$$\begin{bmatrix} 3 \\ 3 \\ 1 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 4 \\ 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 2 \\ 3 \\ 6 \end{bmatrix} \begin{matrix} P \\ Q \\ R \\ S \\ T \end{matrix}$$

Team P had the highest number of wins (first and second order) followed by Q then T, then S and last was R

Further Mathematics Exam 1: Solutions – Multiple choice

Module 6 : Matrices

Question 1 Answer B

$$\begin{aligned}
 A^2 &= \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \times 1 + 2 \times 0 & 1 \times 2 + 2 \times -1 \\ 0 \times 1 + -1 \times 0 & 0 \times 2 + -1 \times -1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
 \end{aligned}$$

Alternatively use the calculator.

```

MATRIX[A] 2 ×2
[[ 1  2 ]
 [ 0 -1 ]]

```

```

[A]^2
[[ 1  0 ]
 [ 0  1 ]]

```

```

[A]^-1*[B]
[[ 1  1 ]
 [ 1 -1 ]]

```

Question 2 Answer A

The element in the first row, second column of MN

$$\begin{aligned}
 &= 1^{\text{st}} \text{ row of } M \times 2^{\text{nd}} \text{ column of } N \\
 &= 2 \times -1 + 1 \times 3 + -1 \times 2 \\
 &= -1
 \end{aligned}$$

Question 3 Answer C

The inverse will not exist if the determinant is zero.

$$\text{ie. } x \times 8 - -1 \times 4 = 0$$

$$8x + 4 = 0$$

$$8x = -4$$

$$x = -\frac{1}{2}$$

Question 4 Answer E

Re-arranging the equations to 'standard form'.

$$x - 3y = -16$$

$$-8x + y = -10$$

gives the equations

$$\begin{bmatrix} 1 & -3 \\ -8 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -16 \\ -10 \end{bmatrix}$$

Question 5 Answer D

$$\begin{aligned}
 M &= \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \\
 &= \frac{1}{3 \times 1 - 2 \times -1} \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \\
 &= \frac{1}{5} \begin{bmatrix} 5 & 5 \\ 5 & -5 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}
 \end{aligned}$$

Alternatively use the calculator.

```

MATRIX[A] 2 ×2
[[ 3 -1 ]
 [ 2  1 ]]

```

```

MATRIX[B] 2 ×2
[[ 2  4 ]
 [ 3  1 ]]

```

Question 6 Answer D

The points matrix is a 5×2 matrix so this eliminated answers A, B and E because the multiplications are not possible. Answer C will not give a total for each of the customers.

Question 7 Answer B

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \times P = \begin{bmatrix} 3 & 3 \\ -2 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So P is equal to the inverse of $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

$$P = \frac{1}{1 \times 3 - 1 \times 2} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{1}{1} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

Question 8 Answer C

Of those who watched the first episode 76% watched the second episode and 24% watched another show. Of those who did not watch the first episode 15% watched the second episode and 85% watched another show.

The transition matrix, T , will be formed from the table:

		1 st episode	
		<i>Watch</i>	<i>Other</i>
2 nd episode	<i>Watch</i>	0.76	0.15
	<i>Other</i>	0.24	0.85

Question 9 Answer D

The initial state matrix will be $\begin{bmatrix} 38 \\ 62 \end{bmatrix}$ using the percentages of the available audience initially watching the show.

If the transition matrix is T then after five weeks the expected percentage of the available audience still watching the television show will be given by the element in the first row of the matrix

$$T^5 \times \begin{bmatrix} 38 \\ 62 \end{bmatrix}$$