

**THE
HEFFERNAN
GROUP**

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FURTHER MATHEMATICS

WRITTEN TRIAL EXAMINATION 2

2007

Reading Time: 15 minutes
Writing time: 1 hour 30 minutes

Instructions to students

This exam consists of Section A and Section B.
Section A contains a set of extended answer questions from the core, 'Data Analysis'.
Section A is compulsory and is worth 15 marks.
Section B begins on page 7 and consists of 6 modules. You should choose 3 of these modules and answer every question in each of your chosen modules. Each of the modules is worth 15 marks.
Section B is worth 45 marks.
There is a total of 60 marks available for this exam.
The marks allocated to each of the four questions are indicated throughout.
Students may bring one bound reference into the exam.
An approved graphics or CAS calculator may be used in the exam.
Formula sheets can be found on pages 33 and 34 of this exam.
Unless otherwise stated the diagrams in this exam are not drawn to scale.

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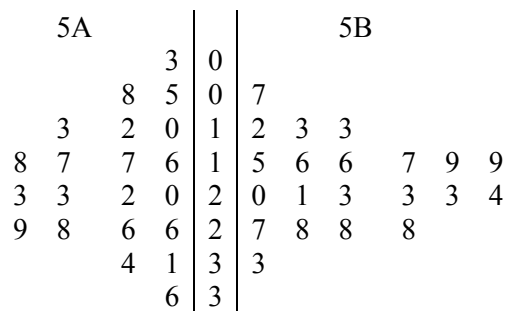
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SECTION A**Core**

This section is compulsory.

All students in Years 4 – 6 at Southvale Primary School entered a statewide mathematics competition.

The results, out of forty, for the two Year 5 classes are displayed on the back-to-back ordered stemplot below.

**Question 1**

- a. Calculate the standard deviation of the results in 5B and enter your answer in the table below.
Express your answer correct to one decimal place.

Class	5A	5B
Mean	19.9	20.2
Standard deviation	9.3	

1 mark

- b. Use the back-to-back stemplot to find and name two properties of the distribution of results of 5A and 5B that are the same.

2 marks

Sam is a student in 5A and his result was 28.

- c.** Find Sam's standardized result or z score, in relation to the other students in 5A. Express your answer correct to 1 decimal place.

1 mark

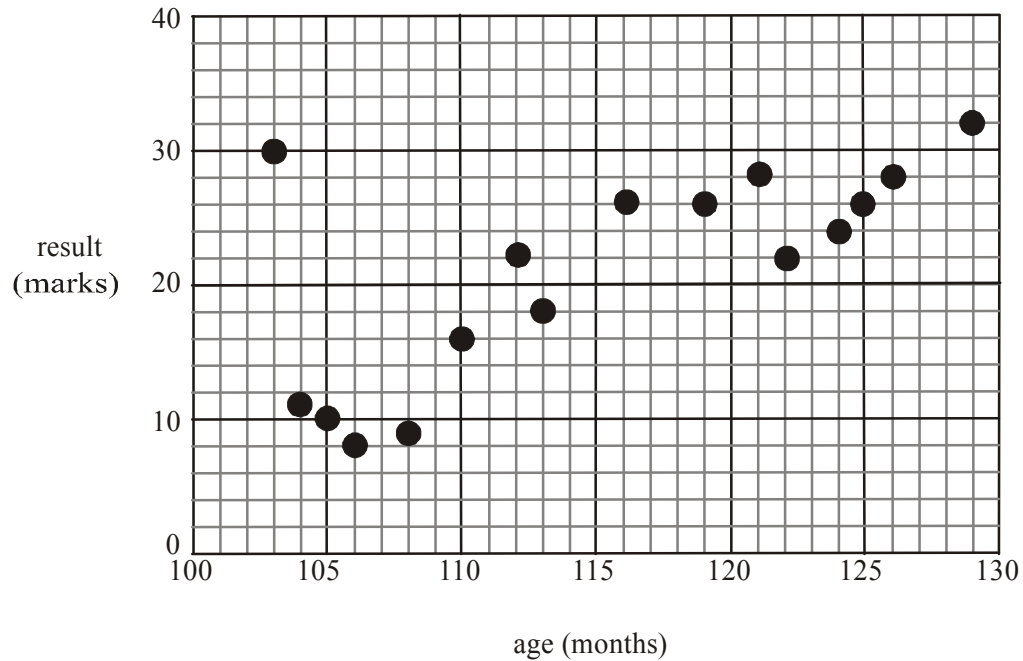
Lucy is in 6A and scored a standardized result of -1 . The results in 6A were normally distributed.

- d.** Find the approximate percentage of students in 6A who gained a higher result than Lucy.

1 mark

Question 2

In one of the Year 4 classes at Southvale Primary School, the teacher prepared a scatterplot comparing the result (marks) and the age (months) of each of his sixteen students.



- a. Fit a three median line to this scatterplot. Indicate clearly the three points that you used in making this three median line.

2 marks

- b. Find the gradient of this three median line.

1 mark

- c. Use your three median line to find the increase in the result that came from an increase of one month in age.
Express your answer correct to one decimal place.

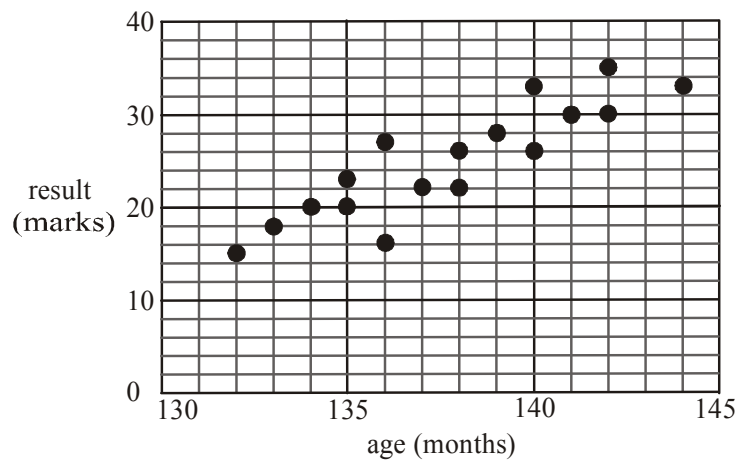
1 mark

Question 3

The age (months) and results (marks) of the seventeen students in 6B are shown in the table below.

age (months)	result (mark)
132	15
134	20
142	35
138	22
136	27
136	16
140	26
139	28
142	30
135	20
137	22
140	33
141	30
133	18
135	23
138	26
144	33

The teacher of 6B constructed a scatterplot of this data which is shown below.



- a. Write down two reasons why it is appropriate to fit a least squares regression line to this data.

2 marks

- b.** Find the equation of the least squares regression line for this data. Express the coefficients correct to one decimal place.

$$\text{result} = \boxed{} + \boxed{} \times \text{age}$$

2 marks

- c. i.** Find the coefficient of determination of this data. Express your answer correct to four decimal places.

- ii.** Explain what the coefficient of determination tells us in terms of the variables; result and age.

1 + 1 = 2 marks
Total 15 marks

SECTION B**Module 1: Number patterns**

If you choose this module all questions must be answered.

Question 1

At a community market, Paula sells jars of jam at her stall. At one such market Paula brought 84 jars of jam to sell and sold 14 each hour.

- a.** How many jars of jam remained after three hours of selling?

1 mark

- b.** After how many hours will Paula run out of jars of jam to sell?

1 mark

- c.** What type of sequence is formed by the number of jars of jam Paula has left to sell at the start of each hour?

1 mark

- d.** The number of jars of jam Paula has left to sell at the start of each hour of selling can be described by the difference equation

$$J_{n+1} = J_n + d \text{ where } J_1 = 84.$$

What is the value of d ?

1 mark

Question 2

Paula also sells chutney at her stall. Paula made 10kg of chutney for the first market she attended. She made 11kg for the second and 12.1kg for the third.

The amount of chutney Paula makes for each successive market follows a geometric sequence.

- a. Show that the common ratio $r = 1.1$.

1 mark

- b. How much chutney does Paula make for the ninth market? Express your answer correct to 2 decimal places.

1 mark

- c. Write an expression for the amount of chutney A_n , in kg, Paula makes for the n^{th} market that she attends.

1 mark

- d. At which market did the amount of chutney Paula made for that market, first exceed 30kg?

1 mark

- e. Paula operates her stall at the market every month. How much more chutney will Paula make in the second year than in the first year of running her stall. Express your answer correct to 2 decimal places.

2 marks

Question 3

Paula's most popular item on the stall is her fudge.

For each market each month, Paula makes 15% more fudge than she had to sell at the previous market.

Each month Paula keeps 200g of the fudge she makes for her family.

The amount of fudge F_n ; in grams, Paula has to sell at the n^{th} market is modelled by the equation

$$F_{n+1} = rF_n + d \text{ where } F_1 = 1\,600$$

- a. Find the values of r and d .

2 marks

- b. At which market will the amount of fudge Paula makes first exceed 3kg?

1 mark

- c. If Paula didn't keep 200g of the fudge for her family but took it to sell at the stall, what sort of sequence would be formed?

1 mark

- d. If the amount of fudge Paula made was the **same** as the amount she had to sell at the previous market, and she continued to keep 200g for her family, and had 1 600g to sell at her first market, which would be the first market at which Paula had no fudge to sell?

1 mark

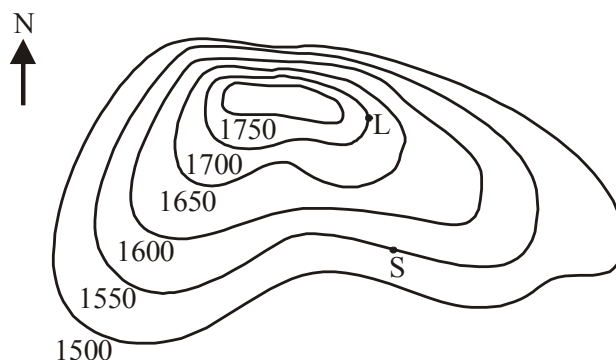
Total 15 marks

Module 2: Geometry and trigonometry

If you choose this module all questions must be answered.

Question 1

The contour map below shows a peak in an Alpine National Park. The contour intervals are located 50m apart.



A lookout is located at point L ; located at height 1700 m and a shelter is located at point S .

- a. Write down the difference in height, in metres, between the lookout and the shelter

1 mark

- b. Describe the direction of the steepest part of the land in relation to the peak.

1 mark

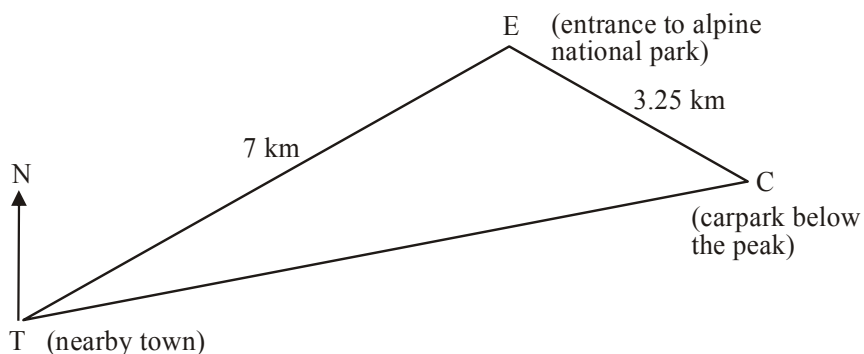
- c. The horizontal distance between L and S is 900 metres. Find the average slope between L and S . Give an exact answer.

1 mark

Question 2

The diagram below shows the flat lands below the peak.

The carpark at the base of the peak is located at C , the entrance to the Alpine National Park is located at E and a nearby town is located at T .



The bearing of E from T is 055° and the bearing of the carpark from T is 080° .

The distance from T to E is 7km and the distance from E to C is 3.25km. Also, $\angle ECT$ is an acute angle.

- a.** Explain why angle ETC is 25° .

1 mark

- b.** How far south of the entrance to the Alpine National Park is the town? Express your answer in kilometers correct to 2 decimal places.

1 mark

- c.** Find angle ECT . Express your answer to the nearest degree.

1 mark

- d.** Using your answer to part **c.**, find the bearing of the entrance to the Alpine National Park at E from the carpark at C .

1 mark

- e.** Find the distance CT . Express your answer in km correct to 1 decimal place.

2 marks

Question 3

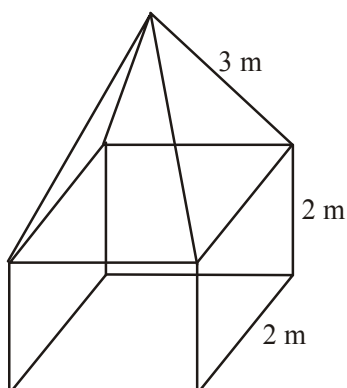
The horizontal distance between the nearby town and the peak is 8km and the vertical height of the peak above the nearby town is 300m.

What is the angle of elevation of the peak from the nearby town? Express your answer to the nearest minute.

1 mark

Question 4

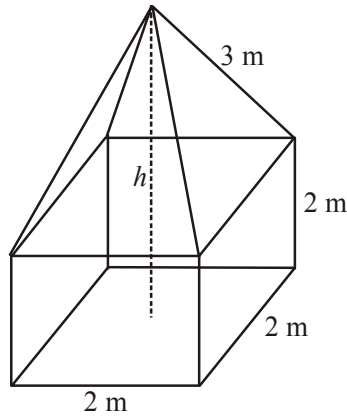
A trig marker is located at the top of the peak. It has a metal frame in the shape of a cube with a square pyramid on top. The diagram below shows the trig marker together with its dimensions.



- a.** Find the surface area, excluding the base, of the trig marker. Express your answer in square metres correct to 2 decimal places.

2 marks

- b. Find the height, h of the trig marker, as indicated in the diagram below. Express your answer in metres correct to 2 decimal places.



2 marks

- c. Hence find the volume of the trig marker. Express your answer in cubic metres correct to 2 decimal places.

1 mark

Total 15 marks

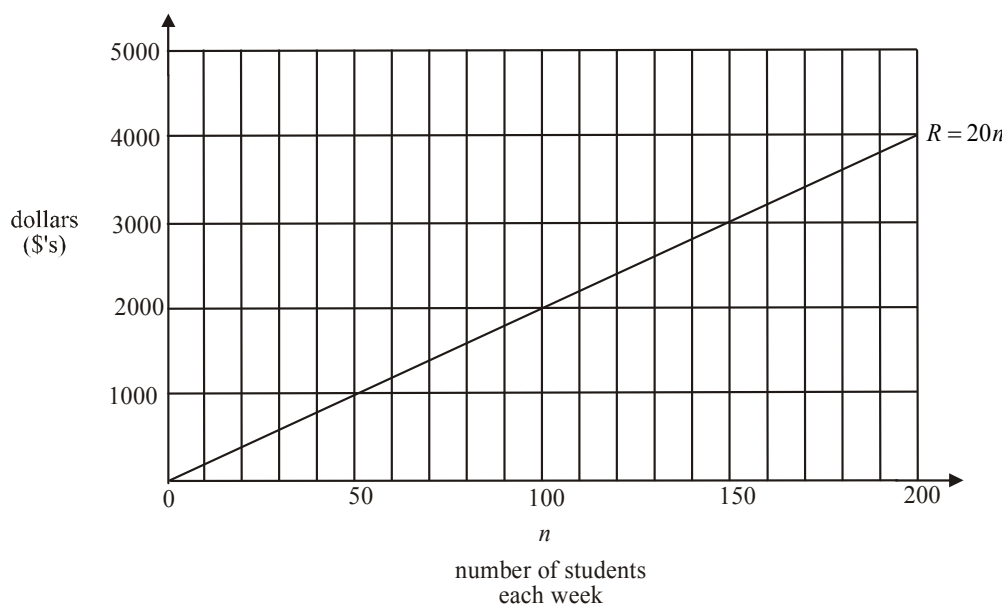
Module 3: Graphs and relations

If you choose this module all questions must be answered

Question 1

At a dance school, the weekly revenue R , in dollars, received for having n students attend classes that week is given by $R = 20n$. The maximum number of students that can be taught at the school in a week is 200.

The graph of R versus n is shown.



- a. What is the average weekly revenue received by the dance school for each student who attends a class?

1 mark

The cost, C , in dollars, associated with having n students attending classes in a week is given by

$$C = 10n + 1000.$$

- b. On the same set of axes as the revenue function above, sketch the graph of this cost function.

1 mark

- c. Either graphically or algebraically, find the number of students needed to attend classes in a week for the dance school to break even.

1 mark

- d. i. Using the equation for revenue and the equation for costs, write an expression for P , the profit in dollars, made each week by the dance school in terms of n .

- ii. Write an inequation involving n for which a profit occurs (as opposed to a loss).

1 + 1 = 2 marks

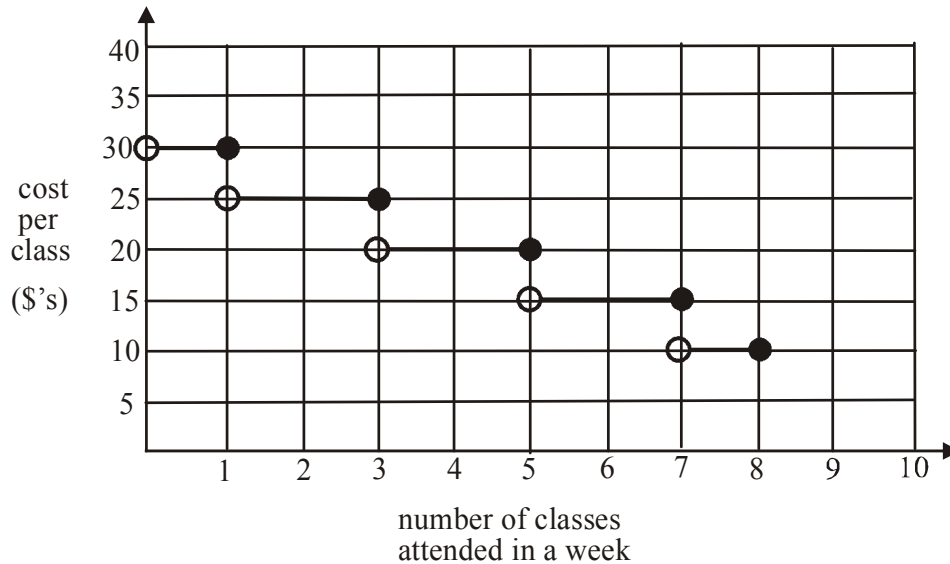
- e. What is the maximum weekly profit that the dance school can make?

1 mark

Question 2

Students can do up to 8 different dance classes in a week.

The graph below shows the cost per class associated with doing up to 8 different classes in a week.



- a. How much would it cost per week for a student to attend 4 classes?

1 mark

- b. How many classes per week would you be attending if you paid the maximum total amount possible per week for classes?

1 mark

- c. The number of different classes offered is to be increased to 10 and the cost per class is to be \$8 for attending 9 or 10 classes per week. Draw this information on the graph.

1 mark

Question 3

On Mondays, because of staff difficulties, the number of classes has to be reorganized.

Let x be the number of junior classes run on a Monday.

Let y be the number of senior classes run on a Monday.

There are 9 students in each junior class and 3 in each senior class. There is a minimum of 45 students required to attend on Mondays for classes to proceed.

Junior classes run for 1 hour and senior classes run for 2 hours. On Mondays there is a maximum of 10 teaching hours available.

The inequalities that describe the situation on Mondays are given by:

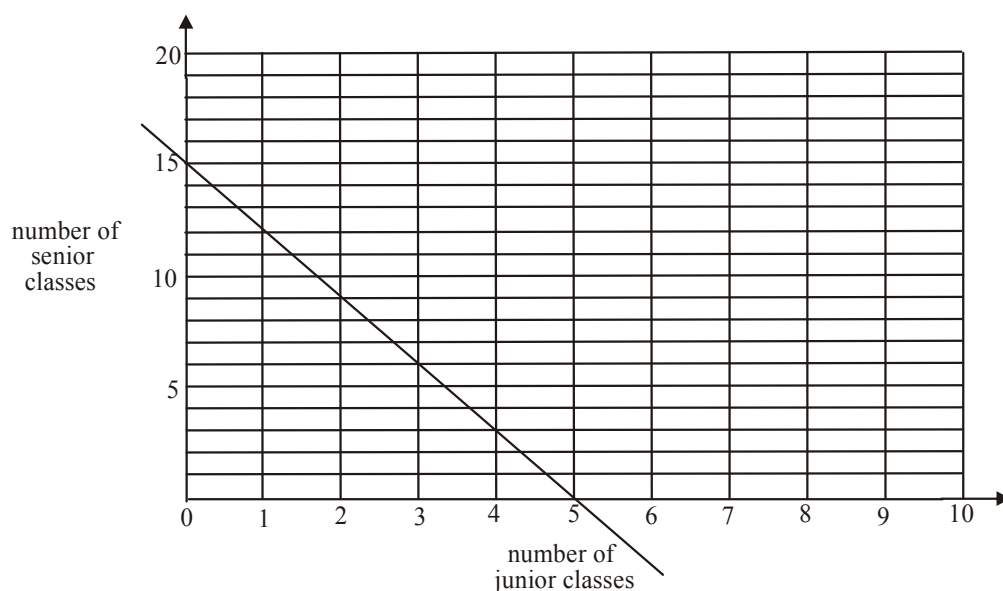
$$x \geq 0$$

$$y \geq 0$$

$$9x + 3y \geq 45$$

$$x + 2y \leq 10$$

On the set of axes below the boundary of one of these inequalities is shown.



- a. On the set of axes above, sketch the boundary of the other inequalities and hence clearly show the feasible region for this problem. Indicate clearly whether the boundary of the feasible region is included.

3 marks

- b.** In the entire feasible region, how many points are there that represents possible numbers of junior and senior classes that could be held on a Monday?

1 mark

- c.** If junior classes make a profit of \$12 and senior classes make a profit of \$8, write down an expression for P , the profit in dollars made on Mondays by the dance school.

1 mark

- d.** What is the maximum profit that could be made on a Monday by the dance school?

1 mark
Total 15 marks

Module 4: Business-related mathematics

If you choose this module all questions must be answered.

Question 1

- a.** In 1980, Dawn bought an investment property for \$85 000. Assuming that the average annual inflation since 1980 has been 5% and assuming no other factors other than inflation affected the price of the property, what would its value have been in 2005? Express your answer to the nearest dollar.

1 mark

- b.** In 2005, Dawn sold this investment property to her granddaughter Julie. The capital gain that Dawn made on the sale was calculated to be \$120 000. She must pay capital gains tax on this at the rate of 42% per dollar. How much capital gains tax does Dawn pay?

1 mark

- c.** With proceeds from the sale of her investment property, Dawn invested \$200 000 in a perpetuity which earned annual interest of 6%.

- i.** How much would Dawn receive each month from this perpetuity?

- ii.** How much in total would Dawn need to invest in this perpetuity to receive a monthly payment of \$2 500?

1 + 1 = 2 marks

Question 2

In order to purchase Dawn's investment property, Julie and her partner Juan, sell three of their investments.

- a.** Julie invested \$5 000 in a bank account which earned simple interest at the rate of 5.75% per annum. Julie has had this account for four years. What is the total value of this investment?

2 marks

- b.** Julie had \$18 000 invested in an account that earned annual interest of 5.4% compounding monthly. Julie has had this account for 6 years. What is the total value of this investment?

Express your answer to the nearest cent.

1 mark

- c.** Juan invested \$3 000 in an account initially, and then invested an additional \$800 on the last day of each quarter just after interest of 6.4% per annum; paid quarterly, had been added to the account.

Juan has had this account for nine years.

What is the total value of this investment?

1 mark

After having sold three of their investments Julie and Juan still need to borrow \$140 000 to purchase the property. They take out a reducing balance loan for this amount. The interest rate on the loan is 7.2% per annum compounding monthly.

- d. What monthly payment do they need to make in order to reduce the balance of their loan to \$90 000 after 4 years?

1 mark

- e. After 4 years when they reduce their loan balance to \$90 000, the amount that Julie and Juan can afford to repay each month will be reduced to \$900 per month. How long after this will it take them to pay out the loan? Express your answer in years and correct to 1 decimal place.

1 mark

Question 3

Juan owns his own business and has equipment used in the business which was purchased for \$35 000. For taxation purposes, this equipment is to be depreciated over 6 years so that its book value at that time will be \$11 000.

- a.** Find the annual depreciation rate if the equipment is depreciated using flat rate depreciation?

2 marks

- b.** Find the annual depreciation rate if the equipment is depreciated using reducing balance depreciation. Express your answer correct to one decimal place.

2 marks

- c.** One year after the equipment was purchased, the difference between the book value of the equipment when depreciated by flat rate method compared to reducing balance method was \$ x . Find the value of x .

1 mark

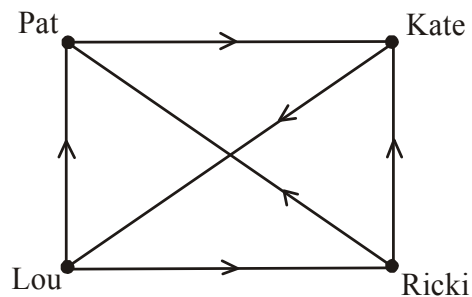
Total 15 marks

Module 5: Networks and decision mathematics

If you choose this module all questions must be answered.

Question 1

In a netball squad, players are ranked according to fitness. Each player competes against each of the other players in a fitness activity after which one player is judged the winner and the other is the loser. The results of these competitions between the four goalers in the squad; Pat, Kate, Ricki and Lou, are shown in the one-step dominance matrix and in the directed graph below. The directed edge between Pat and Kate indicates that Pat defeated Kate.



	P	K	R	L
P	0	x	0	0
K	0	0	0	1
R	1	1	0	0
L	1	y	1	0

one-step
dominance matrix

- a. Two of the entries in the one-step dominance matrix are given by x and y .

- i. Explain why $x = 1$.

- ii. Explain why $y = 0$.

1 + 1 = 2 marks

- b. Complete the table below which shows the one-step dominance score for each of the goalers in the squad in this fitness competition.

Goaler	One-step dominance
Pat	1
Kate	1
Ricki	
Lou	2

1 mark

The matrix below shows the two-step dominances between the four players. Two of the entries a and b have not been completed.

	P	K	R	L
P	0	0	0	a
K	1	0	1	0
R	0	1	0	1
L	1	b	0	0

- c. Explain why $a = 1$.

1 mark

- d. Find the value of b .

1 mark

- e. Using the information on one-step and two-step dominances, find who is the top ranked goaler and the second top ranked goaler in this fitness competition.

2 marks

Question 2

During the season the coaching panel scores players on their ability to play in a particular position during a game.

A score of 0 indicates perfect play in that position and a score of 10 indicates that a lot further development is required to play in that position.

The average of these scores for four particular players in four specific positions is shown in the table below.

	Wing Attack	Centre	Wing Defence	Goal Attack
Claire	3	2	2	5
Michelle	2	1	3	1
Sue	1	3	2	4
Kylie	5	4	3	1

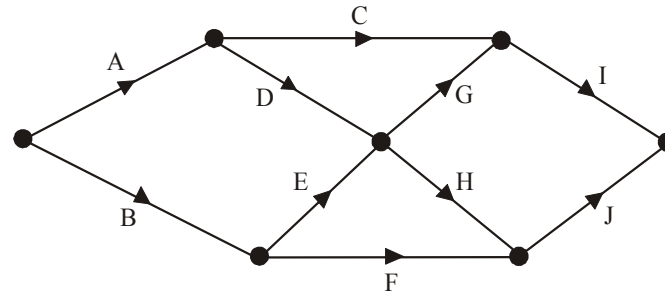
Use the Hungarian algorithm to find the allocation of a player to a position so that the performance of the team is optimized.

3 marks

Question 3

The netball squad want to compete in state championships and their coaching squad draws up a list of team activities that they need to commit to in order to achieve their aim.

The network showing these team activities is shown below.



The earliest start times and latest start times for two of the activities are shown in the table below.

Activity	Earliest start time	Latest start time
<i>H</i>	7	7
<i>J</i>	10	10

The critical path for this project includes four activities; two of which are activities *A* and *D*. The minimum time for the project to be completed is 12 weeks. The time taken for completing each of the activities must be in whole weeks.

- a.** How long does activity *J* take?

1 mark

- b.** What is the maximum duration of activities *G* and *I* combined?

1 mark

- c. If activities A , C and I are each of equal duration, write down the duration that A could have.

1 mark

- d. Assume that activities E and F each have a duration of 3 weeks.

- i. What is the latest starting time for activity E ?

- ii. What could be the duration of activity B given that it does not lie on the critical path?

1 + 1 = 2 marks

Total 15 marks

Module 6: Matrices

If you choose this module all questions must be answered.

Question 1

At a holiday resort the profit made on the rental for one night of one, two and three bedroom apartments as well as penthouse suites is described by the matrix P where

$$P = \begin{bmatrix} 80 \\ 100 \\ 90 \\ 120 \end{bmatrix} \begin{matrix} \text{1 bed} \\ \text{2 bed} \\ \text{3 bed} \\ \text{penthouse} \end{matrix}$$

On a particular night at the resort, the number of one, two and three bedroom apartments as well as penthouse suites that were rented out is described by the matrix R where

$$R = \begin{bmatrix} 7 & 6 & 2 & 4 \end{bmatrix} \begin{matrix} \text{1 bed} & \text{2 bed} & \text{3 bed} & \text{penthouse} \end{matrix}$$

- a. What is the order of matrix P ?

1 mark

- b. Explain which matrix product PR or RP gives the total profit made by the resort on rentals for that particular night.

1 mark

- c. Find the total profit made by the resort on rentals for that particular night.

1 mark

Question 2

In tropical North Queensland there are three resorts; A , B and C located in and around a town centre.

Tourist data suggests that most resort holiday makers to the area return to the resort they stayed in last time, but some prefer to try one of the other two resorts.

Specifically:

82% of guests who stayed at resort A last time will return to resort A the following year.

8% of guests who stayed at resort A last time will stay at resort B the following year.

10% of guests who stayed at resort A last time will stay at resort C the following year.

78% of guests who stayed at resort B last time will return to resort B the following year.

12% of guests who stayed at resort B last time will stay at resort A the following year.

10% of guests who stayed at resort B last time will stay at resort C the following year.

88% of guests who stayed at resort C last time will return to resort C the following year.

5% of guests who stayed at resort C last time will stay at resort A the following year.

7% of guests who stayed at resort C last time will stay at resort B the following year.

- a. Enter this data in the transition matrix T below. Express all percentages as decimals.

$$T = \begin{matrix} & \begin{matrix} \text{this year} \\ A & B & C \end{matrix} \\ \begin{matrix} A \\ B \text{ following year} \\ C \end{matrix} & \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{matrix}$$

2 marks

In the year all three resorts were first open, the total number of guests who stayed at resort A was 45 000. At resort B the total was 62 000 and at resort C it was 56 000.

- b. Enter this data in the column matrix H_0 below.

$$H_0 = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

1 mark

- c. i. By using matrices T and H_0 , write a matrix product that; when evaluated, gives us the total number of guests who stayed at resorts A , B and C respectively, in the year after they were all first open.

- ii. Evaluate the matrix product found in part i..

1 + 1 = 2 marks

- d.** Show that over the long term, the number of guests staying at resorts A , B and C each year can be given by the matrix

$$H = \begin{bmatrix} 47 & 912 \\ 40 & 997 \\ 74 & 091 \end{bmatrix}$$

Your working should include at least two relevant state matrices where the entries have been rounded to the nearest whole number.

2 marks

Question 3

Survey data from a particular holiday resort indicates that the number of guests who come each year for the scuba diving (x), the golf (y), and the swimming (z) offered at the resort, can be calculated using the equations

$$x + y - z = 580$$

$$-x + z = 800$$

$$2x + y - z = 9\,620$$

- a.** Write this system of equations as a matrix equation of the form $A \times X = B$.

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} = \begin{bmatrix} \\ \\ \end{bmatrix}$$

1 mark

- b. i.** Find the determinant of matrix A .

- ii.** Explain why matrix B does not have a determinant.

1 + 1 = 2 marks

- c.** Find the matrix X by first finding the inverse matrix A^{-1} . Hence state the number of guests who come each year to the resort for the scuba diving, the golf and the swimming.

2 marks
Total 15 marks

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \frac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r \frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: $\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \dots + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \dots = \frac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\frac{1}{2}bc \sin A$

Heron's formula: $A = \sqrt{s(s - a)(s - b)(s - c)}$ where $s = \frac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\frac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base $\times$ height

volume of a pyramid: $\frac{1}{3}$ area of base $\times$ height

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Pythagoras' theorem $c^2 = a^2 + b^2$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \frac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \frac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \frac{r}{100}$

hire purchase: effective rate of interest $\approx \frac{2n}{n+1} \times \text{flat rate}$

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2×2 matrix $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

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