

FURTHER MATHEMATICS EXAM 2: SOLUTIONS

Core

Question 1

a.

	Country	Totals
	Independent Variable	
No. of Medals		
Dependant Variable		
Totals		

[A1]

b. Any two of dotplots, barchart or pictograms as the independent variable is categorical (do not accept histograms) [A1]

c. For India: 11 Bronze out of a total of 50 medals is 22%. The balance is silver medals (17 silver or 34%). [A1]

For South Africa: 11 Gold, 12 Silver and 12 Bronze. These approximate to a third of the column each or 33%. [A1]

Question 2

a. From r value of 0.99

Coefficient of Determination $r^2 = 0.99^2 = 0.98$

Therefore: "We can conclude from this that **98** % of the variation in the total number of medals can be explained by the variation in **number of gold medals**. The other **2**% variation in number of gold medals is due to other factors".

b. For the gradient

$$b = \frac{r \times s_y}{s_x}$$

$$= \frac{+0.99 \times 62.8}{23.9}$$

$$= 2.601338$$

$$= 2.60$$

For y-intercept

$$a = \bar{y} - b\bar{x}$$

$$= 63.4 - 2.60 \times 22.0$$

$$= 6.20$$

c.i

$$z = \frac{\text{score} - \bar{x}}{s_x}$$

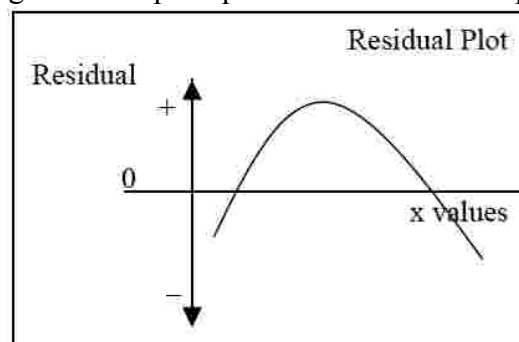
$$= \frac{84 - 22}{23.9}$$

$$= 2.59$$

c.ii A z-score of +1 means the actual score is one standard deviation above the mean. [A1]

Question 3

a. The general shape expected for the residual plot is



b. Log of Angle is the best transformation as it compresses the x -values and has the highest value of coefficient of determination (r^2). [A1]

c. y^2 or square of distance as it will stretch the y values in the direction of the y -axis. [A1]

d. For Angle = 37°

$$\begin{aligned} \text{Distance} &= 46.41 \times \log(\text{Angle}^\circ) - 4.55 \\ &= 46.41 \times \log(37^\circ) - 4.55 \\ &= 46.41 \times 1.568 - 4.55 \\ &= 68.2 \text{ metres} \end{aligned}$$

[A1]

For Distance = 71.0 metres

$$\text{Distance} = 46.41 \times \log(\text{Angle}^\circ) - 4.55$$

$$71.0 = 46.41 \times \log(\text{Angle}^\circ) - 4.55$$

[A1]

$$71.0 + 4.55 = 46.41 \times \log(\text{Angle}^\circ)$$

$$\log(\text{Angle}^\circ) = \frac{75.55}{46.41}$$

$$\text{Angle} = 10^{1.6279} = 42.5^\circ$$

Module 1: Number patterns

Question 1

a. This is an arithmetic sequence where the common difference is 2600.

Method 1 – Iteration

$$\begin{aligned} A_1 &= 64400 \\ A_2 &= 64400 + 2600 \\ &= 67000 \\ A_3 &= 67000 + 2600 \\ &= 69600 \end{aligned}$$

Method 2 – Algebra

$$\begin{aligned} a &= 64400 \\ d &= +2600 \\ n &= 3 \\ t_n &= a + (n-1)d \\ &= 64400 + (3-1) \times 2600 \\ &= 64400 + 5200 \\ &= 69600 \end{aligned}$$

On the third morning its expected an attendance of 69600. [A1]

b. Using the Arithmetic series formula for

$$\begin{aligned} a &= 64400 \\ d &= +2600 \\ n &= 7 \\ S_n &= \frac{n}{2} [2a + (n-1)d] \\ &= \frac{7}{2} [2 \times 64400 + (7-1) \times 2600] \quad [\text{M1}] \\ &= 3.5 [128800 + 7800] \\ &= 505400 \end{aligned}$$

The total for the seven days of the morning session is 505 000 people. [A1]

c.i.

For geometric pattern

$$\begin{aligned} \text{common ratio} &= \frac{t_2}{t_1} = \frac{t_3}{t_2} \\ &= \frac{67620}{64400} = \frac{71001}{67620} \\ &= 1.05 = 1.05 \quad [\text{A1}] \end{aligned}$$

There is a common ratio of 1.05

c.ii. This is a geometric sequence where

$$\begin{aligned} a &= 1.05 \quad (\text{common ratio}) \\ b &= 0 \quad (\text{common difference}) \\ c &= 64400 \quad (\text{first term}) \quad [\text{A1}] \end{aligned}$$

d.i. For geometric sequence

$$\begin{aligned} a &= 64400 \\ r &= 1.05 \\ n &= 7 \quad (\text{final evening}) \\ t_n &= ar^{n-1} \\ t_7 &= 64400 \times 1.05^{7-1} \quad [\text{M1}] \\ &= 86302.16 \end{aligned}$$

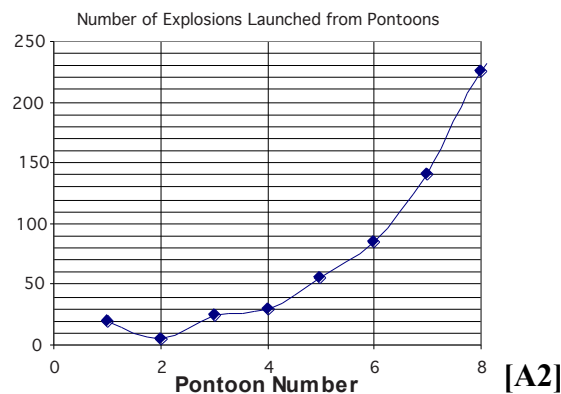
This is below the capacity of the MCG of 88000. [A1]

d.ii. For geometric series

$$\begin{aligned} a &= 64400 \\ r &= 1.05 \\ n &= 7 \quad (\text{final evening}) \\ S_n &= \frac{a(r^n - 1)}{r - 1} \\ S_7 &= \frac{64400(1.05^7 - 1)}{1.05 - 1} \quad [\text{M1}] \\ &= \frac{64400(0.407)}{0.05} \\ &= 524345.344 \end{aligned}$$

The total attendance to the nearest thousand is expected to be 524000. [A1]

Question 2



The sequence is

20, 5, 25, 30, 55, 85, 140, 225

Question 3

a. Create a sequence for 8 years of membership:

$$a = 10, d = 8$$

10, 18, 26, 34, 42, 50, 58, 66

[A1]

Create a sequence for 8 years of fees:

$$a = 88, d = 11$$

88, 99, 110, 121, 132, 143, 154, 165

[A1]

Total collected each year is the product of corresponding terms.

$$\begin{aligned} \text{Total} &= 10 \times 88 + 18 \times 99 + \dots + 58 \times 154 + 66 \times 165 \\ &= 42152 \end{aligned}$$

Total fee collected is \$42 152.

[A1]

b. In the 7th year, fees = $58 \times 154 = 8932$

In the 8th year, fees = $66 \times 165 = 10890$

$$\begin{aligned} \text{Percentage of total} &= 100 \times \frac{(8932 + 10890)}{42152} \\ &= 100 \times 0.470 \\ &= 47.0\% \end{aligned}$$

[A1]

Module 2: Geometry and trigonometry

Question 1

a. The figure is a regular octagon and triangle POQ is one of eight identical, isosceles triangles with a vertex at the centre of the octagon.

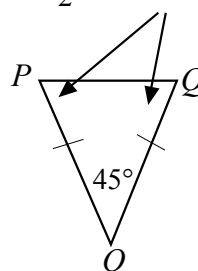
Each angle at the centre has magnitude

$$\frac{360^\circ}{8} = 45^\circ$$

[A1]

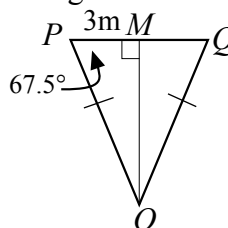
b. Triangle OPQ is an isosceles triangle so the angles OPQ and PQO are equal in size. (67.5°)

$$\frac{180^\circ - 45^\circ}{2} = 67.5^\circ$$



[A1]

c. The triangle OPM is a right-angled triangle.



$$\tan 67.5^\circ = \frac{OM}{3}$$

[M1]

$$\begin{aligned} OM &= 3 \times \tan 67.5^\circ \\ &= 7.2426\dots \\ &\approx 7.24 \text{ metres} \end{aligned}$$

[A1]

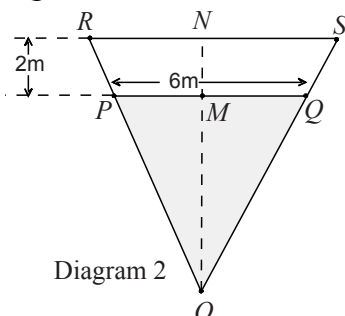
- d. Triangles ROS and POQ are similar triangles so RS and PQ are in the same ratio as ON and OM .

$$RS : PQ = ON : OM$$

[M1]

and $\frac{RS}{PQ} = \frac{ON}{OM}$ and $ON = OM + 2$

[M1]



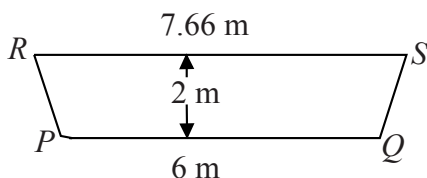
Substituting:

$$\frac{RS}{6} = \frac{9.243}{7.243}$$

$$RS = \frac{9.243 \times 6}{7.243} = 7.657 \approx 7.66\text{m}$$

[A1]

e.



Area of a trapezium = $\frac{1}{2}h(a+b)$ where a and b are the parallel sides and h is the perpendicular height
Area of $PRSQ$

$$= \frac{1}{2} \times 2 \times (7.657 + 6)$$

[M1]

$$\approx 13.66 \text{ m}^2$$

[A1]

- f. Area of the path = $8 \times 13.66 \text{ m}^2$

Volume of concrete needed

= area of path $\times$ depth

[M1]

$$= 8 \times 13.66 \times 0.15$$

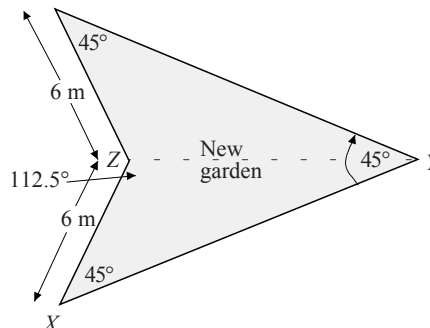
$$= 16.39 \text{ m}^3$$

$$\approx 16.4 \text{ m}^3$$

[A1]

Question 2

a.



On the diagram,

$$\angle XZY = 180^\circ - 45^\circ - 22.5^\circ$$

$$= 112.5^\circ$$

Using the sine rule:

$$\frac{XY}{\sin 112.5^\circ} = \frac{6}{\sin 22.5^\circ}$$

[M1]

$$XY = \frac{6 \sin 112.5^\circ}{\sin 22.5^\circ} = 14.485 \text{ metres}$$

[M1]

$$XY = 14.49 \text{ metres correct to two decimal places}$$

[A1]

- b. Area of triangle ZXY

$$= \frac{1}{2} \times 6 \times 14.485 \times \sin 45^\circ$$

[M1]

$$= 30.728 \text{ m}^2$$

[M1]

The area of the new garden is $2 \times 30.728 = 61.46 \text{ m}^2$
(correct to two decimal places)

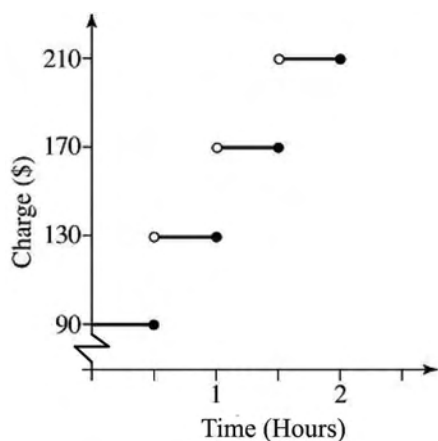
[A1]

Module 3: Graphs and relations**Question 1****a.**

Time	Charge
46 minutes	$\$50 + 2 \times \$40 = \$130$
1 hour 22 min (82min)	$\$50 + 3 \times \$40 = \$170$

[M1]

Total for the two appointments is \$300 (\$130 + \$170)
[A1]

b.

– 1 mark for correct use of open and closed ends

– 1 mark for correct \$values (90,130...)

Question 2

a. $P = \$480 + \320×4
 $= \$1\,760$ **[A1]**

b. $P = 480 + 320n$ **[A1]**

Question 3

a. Time = 6 years **[A1]**

b. Predicted value (24 years) = \$160 000 **[A1]**

Question 4

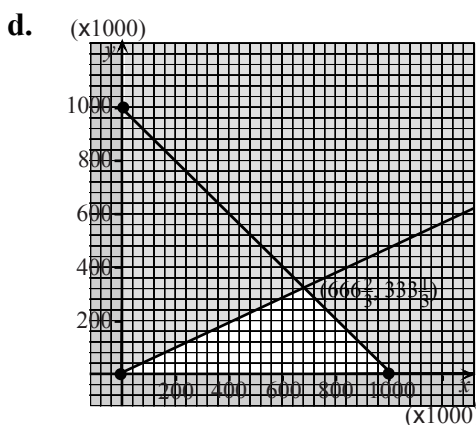
a. Let y = amount invested in shares. **[A1]**

b. The total amount constraint yields
 $x + y \leq 1\,000\,000$ **[A1]**

The investment mix constraint yields
 $x \geq 2y$ or
 $2y - x \leq 0$ **[A1]**

c. Given that bonds return 6% the profit on \$ x worth of bonds is $0.06x$. Similarly the profit on shares is $0.11y$. Therefore the objective function is

$Z = 0.06x + 0.11y$ **[A1]**



Note that the scale is in thousands of dollars.

Determine the vertices of the solution region

x axis: (1 000 000, 0)

y axis: (0, 0)

Intersection of 2 lines

$x + y = 1\,000\,000$

$2y - x = 0$

$y = 333\,333$

$x = 666\,667$ **[A1]**

To evaluate the maximum of the objective function at vertices.

At (666 667, 333 333)

$Z = 0.06(666\,667) + 0.11(333\,333)$
 $= 76\,667$

At (1 000 000, 0)

$Z = 0.06(1\,000\,000) + 0.11(0)$ **[M1]**
 $= 60\,000$

Solution is \$666 667 in to bonds and \$333 333 into shares **[A1]**

Module 4 : Business-related mathematics.**Question 1**

a. Monthly payment = $\frac{44 \times 98}{10} = \431.20 [A1]

b. A 2.3% increase corresponds to a multiplying factor of

$$1 + \frac{2.3}{100} = 1.023$$

$$\begin{aligned} \$98 \times 1.023 &= \$100.254 \\ &\approx \$100.25 \end{aligned} \quad \text{[A1]}$$

c. If m is the multiplying factor in the third year then
 $100.254 \times m = 103.06$ [M1]

$$\begin{aligned} m &= \frac{103.06}{100.254} \\ &= 1.02798\dots \\ &\approx 1.028 \end{aligned}$$

$$= 1 + \frac{2.8}{100}$$

An increase of 2.8% [A1]

Question 2

A perpetuity is the interest earned by the investment in the specified time period (1 month in this case)

Using the simple interest formula:

$$I = \frac{\$108000 \times \frac{5.8}{12} \times 1}{100} \quad \text{[M1]}$$

$$= \$522 \quad \text{[A1]}$$

Jesse would receive \$522 each month.

Question 3

a. Using the TVM solver on the calculator:

```
N=12
I%=7.2
PV=1000
PMT=520
FV=-7524.518704
P/Y=12
C/Y=12
PMT: [END] BEGIN
```

Jesse will owe \$7525 after one year. [A1]

Note: Both the present value (PV) and the payment (PMT) are positive because they are both considered 'incoming' amounts.

b. Using the TVM solver:

```
N=32.79247963
I%=7.2
PV=1000
PMT=520
FV=-20000
P/Y=12
C/Y=12
PMT: [END] BEGIN
```

After 33 months the amount he owes will exceed \$20 000. [A1]

c. Using the TVM solver:

```
N=32
I%=7.2
PV=1000
PMT=520
FV=-19495.52526
P/Y=12
C/Y=12
PMT: [END] BEGIN
```

Jesse will owe \$19 496 after 32 months. [A1]

d. Using the calculator:

```
2Int(1,32)
-1855.525259
```

Or

$$\begin{aligned} \text{Interest} &= 1000 + 520 \times 32 - 19\,496 \\ &= \$1856 \end{aligned} \quad \text{[A1]}$$

Question 4

a. The \$19 500 that is owed accumulates interest for three months at a rate of 7.2% p.a. compounding monthly.

$$7.2\% \text{ p.a.} = \frac{7.2\%}{12} = 0.6\% \text{ per month} \quad \text{[M1]}$$

Using the compound interest formula: $A = PR^n$ where

$$R = 1 + \frac{r}{100} = 1 + \frac{0.6}{100} = 1.006$$

The missing figure is 1.006 [A1]

- b. Using the TVM solver :

```

N=36
I%=7.2
PV=19853.11
PMT=-614.82346...
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN

```

Jesse will need to repay \$614.82 per month for 36 months to totally repay the loan. [A1]

- c. Using the calculator.

Monthly payments: Interest paid over the 36 months is \$2280.53

```

2Int(1,36)
-2280.534656

```

Fortnightly payments: Interest paid over the 78 fortnights is \$2248.63 [M1]

```

N=78
I%=7.2
PV=19853.11
PMT=-283.35570...
FV=0
P/Y=26
C/Y=26
PMT:END BEGIN
2Int(1,78)
-2248.634636

```

Jesse will save \$31.90 (\$32 to the nearest dollar) in interest if he makes fortnightly payments rather than monthly payments.

Alternative method for calculating interest

Monthly :

$$\text{Interest} = \$614.82 \times 36 - \$19853.11 = \$2280.41$$

Fortnightly:

$$\text{Interest} = \$283.36 \times 78 - \$19853.11 = \$2248.97$$

A saving of \$31.44; \$31 to the nearest dollar.

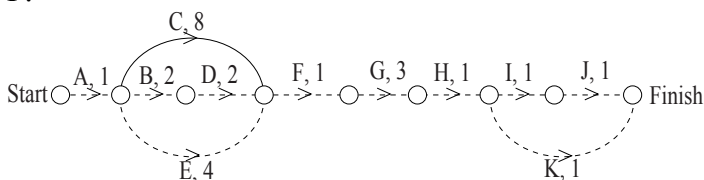
Either answer is acceptable.

[A1]

Module 5 : Networks and decision mathematics

Question 1

- a. Activity C has activity A as an immediate predecessor and is an immediate predecessor to activity F.



[A1]

- b. Activities that cannot be delayed without delaying the whole project are activities on the critical path: A, C, F, G, H, I, J [A1]

- c. The earliest completion time of the whole project is the sum of the durations of the activities on the critical path. i.e $1 + 8 + 1 + 3 + 1 + 1 + 1 = 16$ days [A1]

- d. The slack time of an activity is the difference between its latest start time and its earliest start time. For activity E this is $5 - 1 = 4$ days. [A1]

- e. The latest **finish** time for activity B is 7 days. [A1]

It is the sum of the latest start time for activity B (5 days) and the duration of B (2 days). [M1]

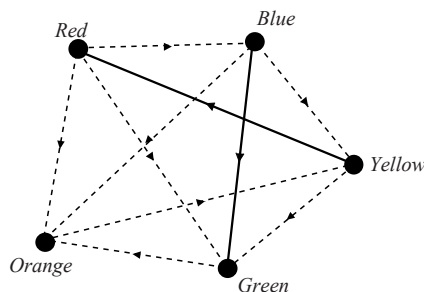
Question 2

- a. The number of games played

$$= \frac{n(n-1)}{2} = \frac{5 \times 4}{2} = 10$$

- b. The missing results are:

Blue defeated **Green** [A1]
and **Yellow** defeated **Red** [A1]



c.

	R	B	Y	G	O
Red	0	1	0	1	1
Blue	0	0	1	1	1
Yellow	1	0	0	1	0
Green	0	0	0	0	1
Orange	0	0	1	0	0

Blue defeated Green so this is a '1'

Yellow did not defeat Blue so this is a '0'

[A1][A1]

- d. The '1' in the matrix M^2 gives the second-level defeat of Blue over Red [A1]
 Blue defeated Yellow and Yellow defeated Red. [A1]

- e.i. The matrix $M + M^2$ is found by adding the corresponding elements in the two matrices.

$$M + M^2 = \begin{bmatrix} 0 & 1 & 2 & 2 & 3 \\ 1 & 0 & 2 & 2 & 2 \\ 1 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

[H1]

- ii. Adding the elements in each row gives the dominance vector ie. the sum of the level one and level two wins in the competition.

$$\begin{bmatrix} 8 \\ 7 \\ 6 \\ 2 \\ 3 \end{bmatrix}$$

[H1]

- f. The ranking of the teams (most wins to least number of wins) is:
 Red, Blue, Yellow, Orange and Green [A1]

Module 6 Matrices

Question 1

(a)

Matrix	Order of the Matrix $m \times n$	Name the element that has the value of 1 e.g. $a_{2,3}$
$A = \begin{bmatrix} 2 & 0 & 3 \\ 5 & -2 & 1 \end{bmatrix}$	2×3	$a_{2,3}$
$B = \begin{bmatrix} -4 & -1 \\ 2 & 1 \\ 3 & 0 \end{bmatrix}$	3×2	$b_{2,2}$
$C = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$	2×2	$c_{1,1}$
	[A1]	[A1]

(b)

Possible Products	Order of Matrix	Shown Workings
$A.B$	2×2	$2 \times 3 \times 3 \times 2$
$A.C$	Does not exist	$2 \times 3 \times 2 \times 2$
$B.A$	3×3	$3 \times 2 \times 2 \times 3$
$B.C$	3×2	$3 \times 2 \times 2 \times 2$
$C.A$	2×3	$2 \times 2 \times 2 \times 3$
$C.B$	Does not exist	$2 \times 2 \times 3 \times 2$

- [A1] correctly identifying those that do not exist.
 [A1] for the remaining four possible products.

- c. Using a graphics calculator, set up the two matrices A and B .

[A]

$$\begin{bmatrix} 2 & 0 & 3 \\ 5 & -2 & 1 \end{bmatrix}$$

[B]

$$\begin{bmatrix} -4 & -1 \\ 2 & 1 \\ 3 & 0 \end{bmatrix}$$

[A] [B]

$$\begin{bmatrix} 1 & -21 \\ -21 & -7 \end{bmatrix}$$

■

$$\begin{bmatrix} 2 & 0 & 3 \\ 5 & -2 & 1 \end{bmatrix} \times \begin{bmatrix} -4 & -1 \\ 2 & 1 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -21 & -7 \end{bmatrix}$$

Question 2

a.

$$\begin{bmatrix} 15 & 105 \\ 20 & 190 \end{bmatrix} \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} -195 \\ -410 \end{bmatrix} \quad [\text{A1}]$$

b. Determinant = $15 \times 190 - 20 \times 105$
 $= 750$ [A1]

c. $\begin{bmatrix} a \\ d \end{bmatrix} = \frac{1}{750} \begin{bmatrix} 190 & -105 \\ -20 & 15 \end{bmatrix} \begin{bmatrix} -195 \\ -410 \end{bmatrix}$
 $= \begin{bmatrix} 8 \\ -3 \end{bmatrix}$ [M1]

The first term, a is 8 and the common difference, d is -3 . [A1]

Question 3

a. Probability or Transition Matrix
Current Year

$$T = \begin{bmatrix} 12 & 3 & \text{Non} \\ 0.6 & 0.3 & 0.01 \\ 0.15 & 0.5 & 0.01 \\ 0.25 & 0.2 & 0.98 \end{bmatrix} \begin{matrix} 12 \\ 3 \\ \text{Non} \end{matrix} \quad \text{Next Year}$$

For columns totaling to 1.0 [M1] [A1]

b. Initial State Matrix

$$M_{2006} = \begin{bmatrix} 200 \\ 100 \\ 0 \end{bmatrix} \begin{matrix} 12 - \text{month} \\ 3 - \text{month} \\ \text{People Telemarketed} \end{matrix} \quad [\text{A1}]$$

c. i & ii

$$M_{2007} = TM_{2006}$$

$$M_{2007} = \begin{bmatrix} 0.6 & 0.3 & 0.01 \\ 0.15 & 0.5 & 0.01 \\ 0.25 & 0.2 & 0.98 \end{bmatrix} \begin{bmatrix} 200 \\ 100 \\ 0 \end{bmatrix} \quad [\text{A1}]$$

$$= \begin{bmatrix} 150 \\ 80 \\ 70 \end{bmatrix} \quad [\text{A1}]$$

d. For long term use T^n where n is large eg $n = 50$

$$M_{\text{long term}} = T^n M_{2006}$$

$$M_{\text{long term}} = \begin{bmatrix} 0.6 & 0.3 & 0.01 \\ 0.15 & 0.5 & 0.01 \\ 0.25 & 0.2 & 0.98 \end{bmatrix}^{50} \begin{bmatrix} 200 \\ 100 \\ 3500 \end{bmatrix} \quad \text{In three}$$

$$= \begin{bmatrix} 180.415 \\ 124.035 \\ 3495.549 \end{bmatrix} \quad [\text{M1}]$$

Total membership = $180.415 + 124.035$
 $= 304.45$

That is approximately 304 members. [A1]