Year 2004

VCE
Further Mathematics
Trial Examination 1

Suggested Solutions

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Question 1 D Use your graphics calculator. Press stat → edit and type in the values in the given table. Points scored in L1 and number of students in L2. Press stat → calc → 1 : 1 – Var Stats → Enter Type L1, L2 ENTER Read off the mean as $\bar{x} = 6$	**Question 2 E** Use the same screen from the calculator. The sample standard deviation $S_X = 2.25$ $\bar{x} \pm S_x$ $= 6 \pm 2.25$ $= 3.75 \rightarrow 8.25$ the number of scores that lie within 3.75 and 8.25 is the number of students who got a score of 4, 5, 6, 7 or 8. That is, 3 + 7 + 7 + 6 + 5 = 28
Question 3 D From the same screen on the calculator, the upper quartile is $Q_3 = 8$	**Question 4 D** This is bivariate data where time is the dependent variable. The relationship between the two variables is most easily seen in a scatter plot.
Question 5 C 95% of scores lie within ± 2 standard deviations of the mean. That is, between 45 ± 2 × 13. That is, from 19 to 71. 5% of results lie outside this range. 2.5% less than 19 and 2.5% greater than 71. Therefore, 2.5% lie above 71.	**Question 6 D** $r^2 = 0.49$ $\Rightarrow r = \pm\sqrt{0.49} = \pm 0.7$ But from the graph we can see that the correlation is negative. Therefore, $r = -0.7$
Question 7 E This box plot is negatively skewed because the median is up towards the right hand end of the box plot. 25% of the results lie within each whisker. The percentage of results in the box = 50% and the percentage of results in both whiskers together = 50%. 12 is the upper quartile so 25% + 50% = 75% of results are less than this value.	**Question 8 D** Use your graphics calculator and enter the data using stat – edit. Then use stat – calc – linear regression. L2, L1 if you entered number of units sold (y) in list 2. Press ENTER. This gives $y = 15.66 + 0.56x$

Question 9 A Using the same screen, read $r = 0.51$	**Question 10 E** The given graph is $y = \frac{1}{x}$ If we plot y against $\frac{1}{x}$,we will get a straight line.
Question 11 A Taking the values for 2000, 2001 and 2002 gives 23, 21 and 38. The median of 21, 23, 38 is 23 Therefore, sales = 23 × 100,000 = $2,300,000	**Question 12 A** The sum of the seasonal indices = 4. Therefore, seasonal index for winter = 4 – (1.3 + 1.5 + 0.7) = 0.5
	Question 13 D Seasonally adjusted value $= \frac{1.2}{0.7}$ = $1.7 million

Question 1 B
Arithmetic sequence
$a = -9$
$d = 4$
$n = 20$
$t_n = a + (n-1)d$
$t_n = -9 + (20-1) \times 4$
$t_n = -9 + 19 \times 4$
$t_n = 67$

Question 2 B
S_∞ of geometric sequence $= \dfrac{a}{1-r}$
$a = 12$
$r = \dfrac{1}{3}$
$S_\infty = \dfrac{12}{1-\frac{1}{3}} = \dfrac{12}{\frac{2}{3}} = 18$

Question 3 D
Geometric sequence
$a = 100$
$r = 0.8$
$n = 12$
$t_n = ar^{n-1}$
$t_{12} = 100 \times 0.8^{11}$
$t_{12} = 8.6$

Question 4 E
A straight line graph represents an arithmetic sequence. Because the graph is decreasing, the common difference is negative.

Question 5 E
$ar^2 = 27 \quad (1)$
$ar^5 = 8 \quad (2)$
$(2) \div (1) \rightarrow r^3 = \dfrac{8}{27}$
$\Rightarrow r = \sqrt[3]{\dfrac{8}{27}} = \dfrac{\sqrt[3]{8}}{\sqrt[3]{27}} = \dfrac{2}{3}$
Substituting in (1)
$\Rightarrow a \times \dfrac{4}{9} = 27$
$\Rightarrow a = 27 \div \dfrac{4}{9} = 60\dfrac{3}{4}$

Question 6 D

Bananas		Oranges		Apples
5	:	4		
		3	:	2

Make number of oranges = 12 for both ratios.

5 × 3	:	4 × 3		
		3 × 4	:	2 × 4
15	:	12	:	8

so the ratio of Bananas to Apples is 15 : 8

Question 7 C

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_5 = \frac{5}{2}[2a+4d] = 75$

$\Rightarrow 5a + 10d = 75 \quad (1)$

$S_{10} = \frac{10}{2}[2a+9d] = 175$

$\Rightarrow 10a + 45d = 175 \quad (2)$

$(1) \times 2 \to 10a + 20d = 150 \quad (3)$

$(2) - (3) \to 25d = 25$

$\Rightarrow d = 1$

Substituting in (1)

$5a = 65$

$\Rightarrow a = 13$

$S_{15} = \frac{15}{2}[2 \times 13 + 14 \times 1]$

$\Rightarrow S_{15} = \frac{15}{2} \times 40 = 300$

Question 8 C

This is a geometric sequence with

$a = 2, r = -3$

$t_n = ar^{n-1} = 13122$

$\Rightarrow 2 \times (-3)^{n-1} = 13122$

$\Rightarrow (-3)^{n-1} = 6561$

Use $y =$ on the graphics calculator

and enter (-3) $\wedge X$

Press second table and read the value of X

that gives $y = 6561$

This is $X = 8$

$\Rightarrow n - 1 = 8$

$\Rightarrow n = 9$

Question 9 D

Geometric sequence

$a = 6$

$r = 1 - 0.05 = 0.95$

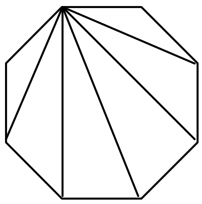
$t_n = ar^{n-1}$

$t_6 = 6 \times (0.95)^5$

$t_6 = 4.6$

Question 1 D

The sum of the angles in each triangle = 180°



Therefore, the sum of the angles in 6 triangles

$= 6 \times 180 = 1080^{\circ}$

In the regular octagon, there are 8 equal angles. Therefore, the size of each angle

$= \dfrac{1080}{8} = 135^{\circ}$

Question 2 E

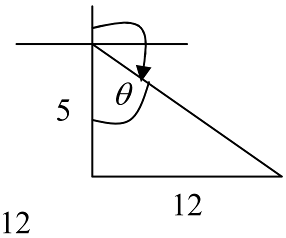
Volume of cylinder A = $\pi r^2 h$

Volume of cylinder B = $\pi(3r)^2 \times 2h = 18\pi r^2 h$

Ratio $V_A : V_B = \pi r^2 h : 18\pi r^2 h$

$\Rightarrow$ Ratio is 1 : 18

Question 3 B



$\tan\theta = \dfrac{12}{5}$

$\theta = \tan^{-1}\left(\dfrac{12}{5}\right) = 67.38^{\circ}$

True bearing $= 90^{\circ} + (90 - 67.38)^{\circ}$

$= 112.62^{\circ}$

which is closest to 113°

Question 4 C

Remaining angle of triangle

$= 180 - (80 + 38)$

$= 62^{\circ}$

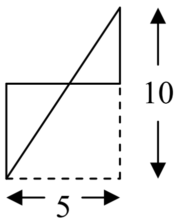
Using Sine Rule

$\dfrac{x}{\sin 62^{\circ}} = \dfrac{12}{\sin 38^{\circ}}$

$\Rightarrow x = \dfrac{12\sin 62^{\circ}}{\sin 38^{\circ}}$

$\Rightarrow x = 17.2$

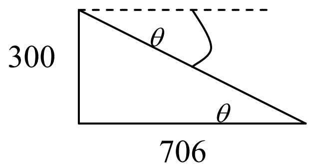
Question 5 B



$x^2 = 5^2 + 10^2 = 125$

$x = \sqrt{125} = 11.18$

Question 6 A

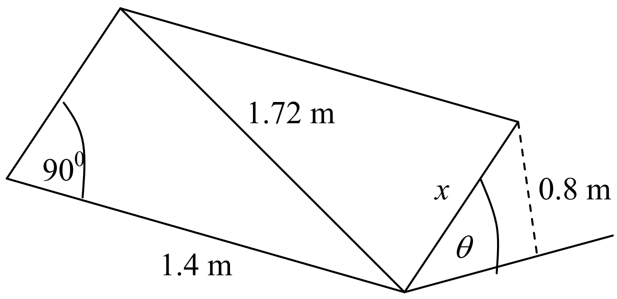


θ is the angle of depression.

$\tan\theta = \dfrac{300}{706}$

$\theta = \tan^{-1}\left(\dfrac{300}{706}\right) = 23^{\circ}$

Question 7 C



Using Pythagoras' Theorem

$x = \sqrt{1.72^2 - 1.4^2} = 0.999$

$\sin\theta = \dfrac{0.8}{0.999}$

$\Rightarrow \theta = \sin^{-1}\left(\dfrac{0.8}{0.999}\right)$

$\Rightarrow \theta = 53°$

Question 8 B

Using the Cosine Rule

$2^2 = 5^2 + 4^2 - 2 \times 5 \times 4\cos\theta$

$4 = 25 + 16 - 40\cos\theta$

$\Rightarrow 40\cos\theta = 41 - 4$

$\Rightarrow 40\cos\theta = 37$

$\Rightarrow \cos\theta = \dfrac{37}{40}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{37}{40}\right)$

$\Rightarrow \theta = 22.3°$

Question 9 C

Triangles ABE and CDE are similar (AAA)
Therefore, their sides are in the same ratio.
Let DE = x cm
Therefore, AE = $25 - x$

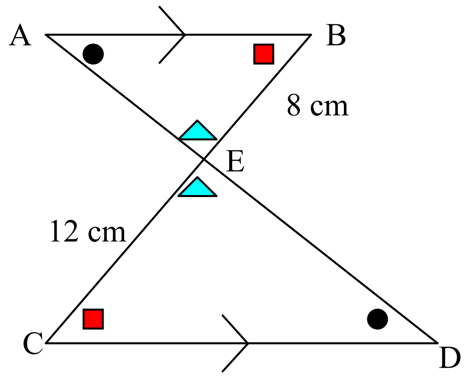
$\Rightarrow \dfrac{8}{12} = \dfrac{25 - x}{x}$

$\Rightarrow 8x = 12(25 - x)$

$\Rightarrow 8x = 300 - 12x$

$\Rightarrow 20x = 300$

$\Rightarrow x = \dfrac{300}{20} = 15$



Question 1 E Highest value of the graph is 600. This is the greatest distance from his house. Therefore, the shop is 600 m from his house.	**Question 2 D** It takes 18 minutes to reach the shop. He returns from the shop in half this time. That is, he takes 9 minutes to return home. He leaves the shop after 20 minutes. So time to reach home = 20 + 9 = 29 minutes.
Question 3 A He travelled 100 m in 6 minutes. Therefore, speed = $\frac{100}{6} = 16\frac{2}{3}$ m/min	**Question 4 B** The break even point is where the two graphs intersect.
Question 5 C $60 < 8x + 4 \le 120$ $\Rightarrow 60 - 4 < 8x + 4 - 4 \le 120 - 4$ $\Rightarrow 56 < 8x \le 116$ $\Rightarrow \frac{56}{8} < \frac{8x}{8} \le \frac{116}{8}$ $\Rightarrow 7 < x \le 14.5$	**Question 6 A** A straight line graph has the form $y = mx + c$ c is the y intercept = 1 so $y = mx + 1$ m is the gradient of the line joining the points $(0,1)$ and $(-\frac{1}{2}, 0)$ $m = \frac{1-0}{0 - -\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$ $\Rightarrow y = 2x + 1$

Question 7 A

$x \geq 0$

$y \geq 0$ $\therefore$ not E

$x + 2y \leq 8$

$2y \leq -x + 8$

$y \leq -\frac{1}{2}x + 4$

This line joins the points (0,4) and (8,0)
Less than this line. Therefore, not C or D.

$2x - y \leq 6$

$-y \leq -2x + 6$

$y \geq 2x - 6$

This line joins the points (0,-6) and (3,0)
Greater than this line. Therefore, A

Question 8 D

$x + 2y = 8 \quad (1)$

$2x - y = 6 \quad (2)$

$(1) \times 2$

$\Rightarrow 2x + 4y = 16 \quad (1a)$

$\quad 2x - y = 6 \quad (2)$

$(1a) - (2)$

$\Rightarrow 5y = 10$

$\Rightarrow y = 2$

Question 9 A

x is at least double y

$x \geq 2y$

z is no greater than one third x

$z \leq \frac{1}{3}x$

If $x \geq 2y$

then $2y \leq x$

so $y \leq \frac{1}{2}x$

If $z \leq \frac{1}{3}x$

then $\frac{1}{3}x \geq z$

so $x \geq 3z$

Therefore, A is not true.

Module 4 Business-related mathematics. Suggested solutions.

Question 1 E $I = \frac{PRT}{100}$ $I = \frac{5000 \times 10 \times 4}{100} = \$2000$ Value of investment $= 5000 + 2000$ $= \$7000$	**Question 2 D** Depreciation = 2400 Fraction depreciation = $\frac{2400}{10500}$ % depreciation = $\frac{2400}{10500} \times 100 = 22.86\%$
Question 3 C $A = 23460\left(1 + \frac{0.075}{2}\right)^4$ $\Rightarrow A = \$27{,}181.94$	**Question 4 C** Amount depreciation $= 20{,}000 \times \frac{20}{100} \times 5$ $= \$20{,}000$ Value of car $= 30{,}000 - 20{,}000$ $= \$10{,}000$
Question 5 C Effective interest rate $= \frac{2n}{n+1} \times$ flat rate $= \frac{2 \times 36}{36+1} \times 12$ $= 23.35\%$	**Question 6 D** $18{,}463 = P\left(1 + \frac{0.1}{4}\right)^{20}$ $\Rightarrow P = \frac{18{,}463}{\left(1 + \frac{0.1}{4}\right)^{20}}$ $\Rightarrow P = \$11{,}267.43$

Module 4 Business-related mathematics. Suggested solutions.

Question 7 E	**Question 8 A**
Minimum amount for February is the amount in the account from 1 Feb to 19 Feb 19 inclusive. This equals the amount in the account after 24 Jan = \$15,631.46	Minimum amount in the account in December = \$14,247.19 Interest $= \frac{0.05}{12} \times 14247.19 = \$59.36$

Question 9 B

Cash deposit $= \frac{10}{100} \times 3500 = \$350$

Interest paid on balance of $3500 - 350 = 3150$

Interest $= 3150 \times \frac{8.5}{100} \times 3 = 803.25$

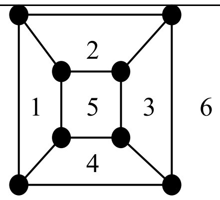
Amount to be paid back $= 3150 + 803.25 = 3953.25$

Number of repayments $= 3 \times 12 = 36$

Value of each repayment $= 3953.25 \div 36 = \$109.82$

Question 1 D
A Hamiltonian Path passes through each vertex once only, starting and finishing at different vertices. This cannot be done with diagram D.

Question 2 C

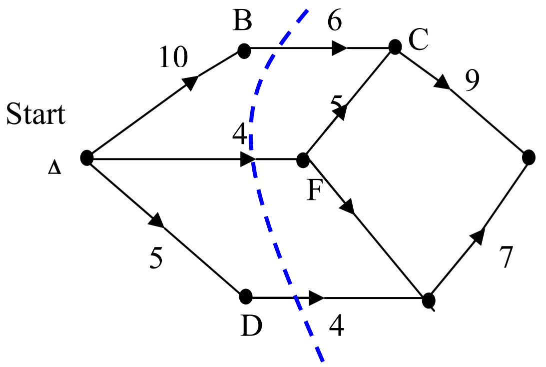


Number of faces = 6

Question 3 B
2 + 2 + 6 + 3 = 13

Question 4 E
For an Eulerian circuit to exist, all vertices must be of even degree. Since D and A are of odd degree, adding an edge joining A to D would make both these vertices even.

Question 5 B



Maximum flow = minimum cut
Minimum cut = 6 + 4 + 4 = 14
Therefore, maximum flow = 14

Question 6 D
DE can hold 4 and the maximum amount of 4 can flow along here when maximum flow is occurring.

Question 7 C

From A to A = 0
From A to B = 2
From A to C = 0
So first row is 0 2 0 therefore, not B

From B to A = 1
From B to B = 0
From B to C = 1
So second row is 1 0 1 therefore, C

Question 8 E

Circle the smallest value in each row

$$\begin{array}{cccc} 5 & 4 & 3 & 6 \\ 9 & 10 & 6 & 9 \\ 10 & 13 & 8 & 7 \\ 6 & 9 & 10 & 10 \end{array}$$

Subtract this circled value from each value in its row.

$$\begin{array}{cccc} 2 & 1 & 0 & 3 \\ 3 & 4 & 0 & 3 \\ 3 & 6 & 1 & 0 \\ 0 & 3 & 4 & 4 \end{array}$$

Circle the smallest value in each column. Subtract this value from each value in its column.

$$\begin{array}{cccc} 2 & 0 & 0 & 3 \\ 3 & 3 & 0 & 3 \\ 3 & 5 & 1 & 0 \\ 0 & 2 & 4 & 4 \end{array}$$

Take a row and column where only one zero occurs. (See the lines drawn above)
⇒Jake makes the Racing Bike
⇒Ida makes the Super Athlete Bike
⇒Greg makes the Mountain bike
Therefore, Helen makes the Non-geared Bike.
Therefore, E

Question 9 C

Earliest time G can start is when C and F are both finished. C finishes in 8 hours. F finishes in 4 + 3 + 2 = 9 hours. Therefore G must wait to start until F finishes. So, the earliest G can start is 9 hours from the beginning.

End of suggested solutions
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