

FURTHER MATHEMATICS EXAM 1 SOLUTIONS**Section A (Core)****Question 1 D**

Mode: The most frequent score was 60 as it occurred three times.

Mean:

$$\begin{aligned} \text{mean} &= \frac{\sum \text{scores}}{n} \\ &= \frac{8+18+19+22+\dots}{27} \\ &= \frac{1134}{27} \\ &= 42 \end{aligned}$$

Median: the middle score

Middle score

$$\begin{aligned} &= \frac{n+1}{2} \\ &= \frac{27+1}{2} = 14^{\text{th}} \text{ score} \\ 14^{\text{th}} \text{ score is } 45 \end{aligned}$$

Alternatively, use graphic calculator with data in list L₁.

```
1-Var Stats      1-Var Stats
x̄=42             ↑Sx=15.21133181
Σx=1134          σx=14.92698278
Σx²=53644        n=27
Sx=15.21133181   minX=8
σx=14.92698278   Q1=30
↓n=27            ↓Med=45
```

Thus mean, median and mode in that order is 42, 45 and 60.

Question 2 B

Using graphics calculator

```
1-Var Stats
x̄=56
Σx=560
Σx²=32094
Sx=9.030811456
σx=8.567379996
↓n=10
```

The standard deviation is 9.0308.. and to one decimal place it becomes 9.0. Response C is more accurate however accuracy was only wanted to one decimal place as in response B 9.0.

Question 3 B

The upper and lower medians are

$$(x_U, y_U) = (6, 20)$$

$$(x_L, y_L) = (2, 10)$$

The gradient of the 3-median regression line is:

$$\begin{aligned} \frac{y_U - y_L}{x_U - x_L} &= \frac{20 - 10}{6 - 2} \\ &= \frac{10}{4} \\ &= 2.5 \end{aligned}$$

Question 4 C

95% limit is

mean plus or minus two standard deviations

or

$$\begin{aligned} 95\% \text{ limit} &= \bar{x} \pm 2s \\ &= 12 \pm 2 \times 0.1 \\ &= 12 - 0.2 \text{ and } 12 + 0.2 \\ &= 11.8 \text{ to } 12.2 \end{aligned}$$

Question 5 E

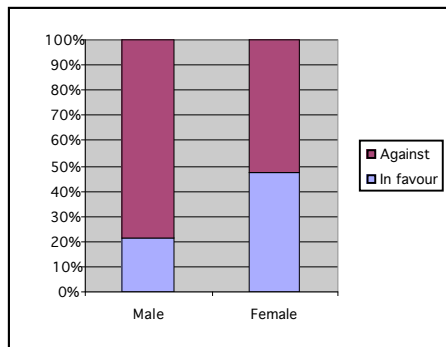
Back-to-back stem plot are for Categorical vs Numerical Bivariate analysis.

Response E *best bench press weight and age of gymnast* are both numerical variables.

Question 6 D

From this table, the independent variables are gender; male and female. Percentaging by columns, we obtain

	Male	Female
In favour of change	$= \frac{123}{575} \times \frac{100}{1}$ $= 21.4\%$	$= \frac{230}{485} \times \frac{100}{1}$ $= 47.4\%$
Against change	$= \frac{452}{575} \times \frac{100}{1}$ $= 78.6\%$	$= 100\% - 47.4\%$ $= 52.6\%$
Total	100%	100%

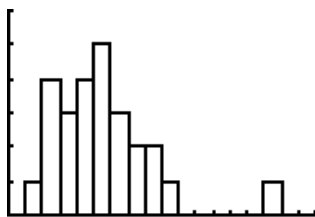


Question 7 D

Median and mean are affected by outliers is a false statement as only mean is affected by an outlier.

Question 8 A

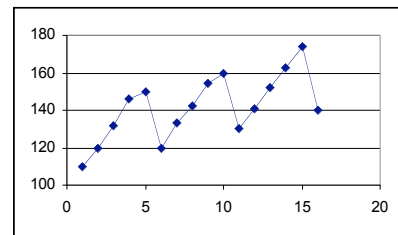
The data set contains one outlier with a value of 15 and thus histogram A is correct as it shows the outlier. Option E is not to be considered as the minimum number of groupings is usually five.



Question 9 B

Sum of all seasonal index = number of seasons
 Spring + Autumn + 0.78 + 0.92 = 4
 Spring + Autumn = 4 - (0.78 + 0.92)
 Spring + Autumn = 2.3
 Spring = 1.15 as spring and autumn seasonal indices are the same.

Question 10 E



It is **seasonal** as it repeats low (and high) values every 5th period namely 1st, 6th, 11th and 16th time period and **increasing** as these values show an upward trend.

Question 11 B

The median of the three values (37.1, 37.0, 37.5) centred about $t = 10$ is 37.1

t	8	10	12
Temperature	37.1	37.0	37.5

Question 12 C

If June 2005 has a timecode of 1 then June 2006 has a timecode of 13.

$$\begin{aligned} \text{Deseasonalised Monthly sales} &= 1200 \times \text{timecode} + 10000 \\ &= 1200 \times 13 + 10000 \\ &= 25600 \end{aligned}$$

Actual expected sales where June seasonal index is 0.8,

$$\begin{aligned} \text{Actual figure} &= \text{deseasonalised} \times \text{seasonal index} \\ &= 25600 \times 0.8 \\ &= 20480 \\ &= \$20\,480 \end{aligned}$$

Question 13 B

The above graph can be either compressed in the x-axis or stretched in the y-axis
 y axis transformation is y^2

x axis transformation can be either $\log x$ or $\frac{1}{x}$

The only option with y^2 and $\log x$ or $\frac{1}{x}$ is B

Section B (Modules)**Module 1: Number patterns and applications****Question 1** **C**

The common ratio, $r = \frac{t_2}{t_1} = \frac{1.5}{1} = 1.5$

The 4th term $= t_3 \times 1.5$
 $= 2.25 \times 1.5$
 $= 3.375$

Question 2 **B**

An arithmetic sequence with first term $a = 5$ and common difference, $d = 5$

The sum of ten terms:

$$\begin{aligned} S_{10} &= \frac{10}{2} [2 \times 5 + (10 - 1) \times 5] \\ &= 5 [10 + 9 \times 5] \\ &= 5 [55] \\ &= 275 \end{aligned}$$

Question 3 **E**

The sequence is arithmetic with $a = 41$ and $d = -3$

Solving $t_n < 0$

$$a + (n-1)d < 0$$

$$41 + (n-1) \times -3 < 0$$

$$44 - 3n < 0$$

$$44 < 3n$$

$$14.667 < n$$

n is greater than 14.667,

so $n = 15$

Question 4 **D**

$$t_4 = 9 : a + 3d = 9 \quad \dots (1)$$

$$t_8 = -11 : a + 7d = -11 \quad \dots (2)$$

Subtract equation (1) from equation (2)

$$4d = -11 - 9$$

$$= -20$$

$$d = -5 \text{ substituting in (1) gives}$$

$$a + 3 \times -5 = 9$$

$$a = 24$$

Question 5 **C**

Substituting in the difference equation:

$$t_2 = 2 \times t_1 - 1 = 2 \times 3 - 1 = 5$$

$$t_3 = 2 \times 5 - 1 = 9$$

$$t_4 = 2 \times 9 - 1 = 17$$

Question 6 **D**

This difference equation generates a sequence that is neither arithmetic nor geometric. You could 'solve' the difference equation by substituting ' a ' = 2 and ' b ' = -1 and $t_1 = 3$ in :

$$t_n = a^{n-1}t_1 + \frac{b(a^{n-1} - 1)}{a - 1}$$

$$t_n = 2^{n-1} \times 3 + \frac{-1(2^{n-1} - 1)}{2 - 1} = 3 \times 2^{n-1} - (2^{n-1} - 1) =$$

$$2 \times 2^{n-1} - 1 = 2^n - 1$$

Alternatively test the solutions to give the sequence 3, 5, 9, 17,

A gives 3, 5, 7, ...

B gives 3, 6, ...

C gives 3, 5, 7, ...

D gives 3, 5, 9, 17 correct

E gives 3, 9, ...

Question 7 **D**

The original number in the class must be divisible by $2 + 3 = 5$. (Only whole numbers are possible for 'people')

Add 2 to the original number and the answer must be divisible by $3 + 4 = 7$

The only answer that this is possible for is 40;

$$\frac{40}{5} = 8 \text{ and } \frac{40 + 2}{7} = 6$$

Alternatively you could solve the equations:

$$m : f = 2 : 5 \Rightarrow \frac{m}{f} = \frac{2}{5}$$

$$\text{and } m + 2 : f = 3 : 4 \Rightarrow \frac{m + 2}{f} = \frac{3}{4}$$

Question 8 **A**

$$t_1 = a = 4.5 \times (0.4)^1 = 1.8$$

$$t_2 = 4.5 \times (0.4)^2 = 0.72$$

$$r = \frac{0.72}{1.8} = 0.4$$

$$S_{\infty} = \frac{a}{1 - r} = \frac{1.8}{1 - 0.4} = \frac{1.8}{0.6} = 3$$

Question 9 **E**

Working through the answers:

A: Arithmetic 5, 7, 9, ...

B: Geometric but decreasing 100, 90, 81, ...

C: Increasing but not geometric $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \dots$

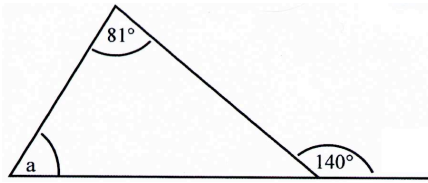
D: Geometric but alternating positive, negative.

E: $\frac{3}{2}, \frac{9}{4}, \frac{27}{8}, \dots$ geometric, positive, increasing

Module 2: Geometry & trigonometry

Question 1

E



Exterior angle = sum of the two interior angles

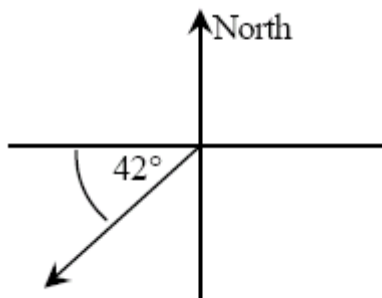
$$140^\circ = a + 81^\circ$$

$$a = 140^\circ - 81^\circ$$

$$a = 59^\circ$$

Question 2

C



The angle from south is the complimentary angle of 42° which is 48° .

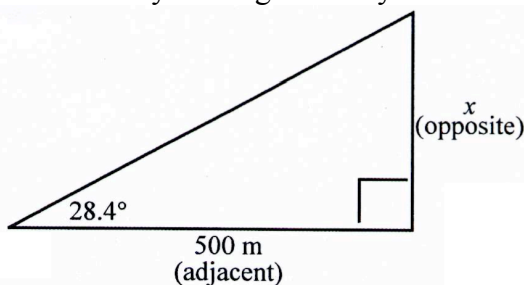
$$\text{true bearing} = 180 + 48 = 228^\circ \text{T} \quad \text{and}$$

$$\text{compass bearing} = \text{S } 48^\circ \text{ W}$$

Question 3

A

For right-angled triangle, given an angle and one side only use trigonometry.



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 28.4^\circ = \frac{x}{500}$$

$$x = 500 \times \tan 28.4^\circ$$

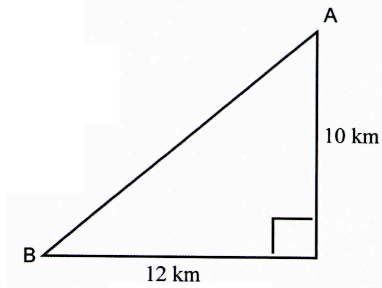
$$x = 270.35 \text{ m}$$

$$\text{A } 270 \text{ m}$$

Question 4

D

Given the two lengths of a right-angled triangle, to find the third length use Pythagoras' Theorem.



$$c^2 = a^2 + b^2$$

$$c^2 = 10^2 + 12^2$$

$$c^2 = 100 + 144$$

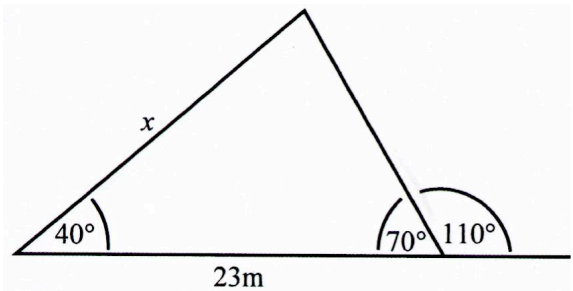
$$c = \sqrt{244}$$

$$c = 15.62 \text{ km}$$

The distance from A to B is closest to D (15.6 km)

Question 5

C



From the figure above, the value of the angle opposite side x can be found using the supplementary rule.

$$\text{Angle } X = 180^\circ - 110^\circ = 70^\circ$$

The angle opposite the side of length 23 metres can be found using sums of angles in a triangle rule.

$$180^\circ = 40^\circ + 70^\circ + \text{third angle}$$

$$\text{third angle} = 70^\circ$$

To find the value of x use the Sine Rule

$$\frac{a}{\sin A^\circ} = \frac{b}{\sin B^\circ}$$

$$\frac{x}{\sin(70^\circ)} = \frac{23}{\sin(70^\circ)}$$

Question 6
E

Volume of a pyramid is

$$V_{\text{pyramid}} = \frac{1}{3}AH \text{ where area of a square base is}$$

$$A_{\text{square}} = l^2.$$

Therefore given $V_{\text{pyramid}} = 4500 \text{ cm}^3$
and $H = 10 \text{ cm}$

$$V_{\text{pyramid}} = \frac{1}{3}AH = \frac{1}{3} \times l^2 \times H$$

$$4500 = \frac{1}{3} \times l^2 \times 10$$

$$l^2 = 1350$$

$$l = 36.7423$$

Perimeter of a square is

$$\begin{aligned} \text{Perimeter} &= 4l \\ &= 4 \times 36.7423 \\ &= 146.969 \\ &= 147 \end{aligned}$$

Then the perimeter of the base
is closest to E (147 m)

Question 7
A

Need one other side before the area can be found.

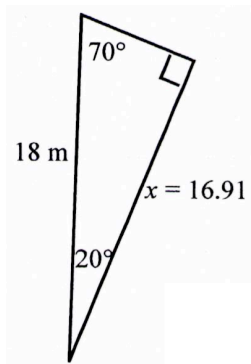
For side x , using trigonometry

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 20^\circ = \frac{x}{18}$$

$$x = 18 \times \cos 20^\circ$$

$$x = 16.9145 \text{ m}$$



The area of the triangle at
right is

$$\begin{aligned} A &= \frac{1}{2}ab \sin C \\ &= \frac{1}{2} \times 18 \times 16.9145 \times \sin 20^\circ \\ &= 52.065 \\ &= 52.1 \text{ cm}^2 \end{aligned}$$

A 52.1 cm²

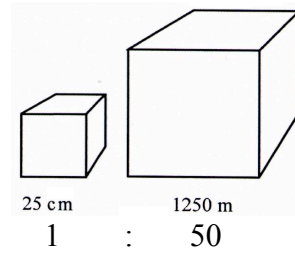
Question 8
D

Map scale = Linear scale factor, $k = 1 : 50$ or $\frac{1}{50}$

$$k = \frac{1}{50} = \frac{\text{Length of image}}{\text{Length of original}}$$

$$\frac{1}{50} = \frac{25 \text{ mm}}{\text{Length of original}}$$

$$\text{Length of original} = 50 \times 25 \text{ mm} = 1250 \text{ mm}$$



$$1250 \text{ mm} = 1.25 \text{ m}$$

$$V_{\text{prism}} = AH$$

$$= l^3$$

$$= (1.25 \text{ m})^3$$

$$= 1.953 \text{ m}^3$$

Conversion is

$$1 \text{ m}^3 = 1000 \text{ litres}$$

$$1.953 \text{ m}^3 = 1953 \text{ litres}$$

The volume of the real refrigerator
is closest to D (1950 litres)

Question 9
D

$$1:250\,000$$

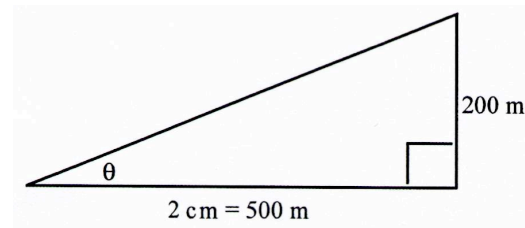
$$1 \text{ cm} = 250000 \text{ cm}$$

$$1 \text{ cm} = 2500 \text{ m}$$

Now from the contour map distance from A to B
is 2 cm

$$2 \text{ cm} = 2 \times 2500 \text{ m}$$

$$\text{From A to B is } 5000 \text{ m}$$



$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{altitude}}{\text{horizontal distance}}$$

$$= \frac{200 \text{ m}}{5000 \text{ m}}$$

$$= \frac{1}{25} \quad \text{D (0.04)}$$

Module 3 : Graphs and relations.

Question 1 C

The line goes through (0, 2) and (6, 10)

$$\text{Gradient } m = \frac{10 - 2}{6 - 0} = \frac{8}{6} = \frac{4}{3}$$

y-intercept : $c = 2$

$$\text{Equation : } y = \frac{4}{3}x + 2$$

$$3y = 4x + 6$$

$$3y - 4x = 6$$

Question 2 E

Substitute $3y = x + 2$ into equation

$3y - 5x = 6$ to give

$$(x + 2) - 5x = 6$$

$$x + 2 - 5x = 6$$

$$-4x = 4$$

$$x = -1$$

Substitute $x = -1$ into $3y = x + 2$

$$3y = -1 + 2$$

$$3y = 1$$

$$y = \frac{1}{3}$$

Question 3 C

The pulse rate is between 100 and 140 beats/minute from time 7 minutes to 10 minutes (3 minutes) and from time 32 minutes to 40 minutes (8 minutes).

A total of 11 minutes.

Question 4 D

The average rate of change between two points = the gradient of the straight line joining the two points.

= the gradient of the line joining (10, 140) and (40, 100)

$$= \frac{100 - 140}{40 - 10}$$

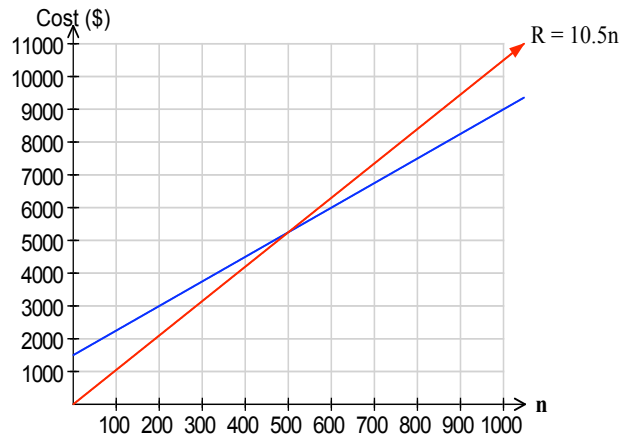
$$= \frac{-40}{30}$$

$$= \frac{-4}{3}$$

$$\approx -1.33$$

Question 5 C

Graphing $R = 10.5n$ on the set of axes: the line goes through the origin (0, 0) and the point (1000, 10500)



The break-even point is the point where the graphs intersect ie. at $n = 500$

Question 6 E

Photocopying costs:

A lot of 50 costs $50 \times 20^c = \$10$

A lot of 250 costs $250 \times 10^c = \$25$

A lot of 500 costs $500 \times 10^c = \$50$

Total cost \$85

One lot of 800 costs $800 \times 5 = \$40$

$\$85 - \$40 =$ a saving of \$45

Question 7 E

Considering the four boundary lines of the feasible region:

The lines $x = 0$ and $x = 8$ mean x is between 0 and 8 giving the constraint $0 \leq x \leq 8$

A line goes through the points (0, 20) and (20, 0).

This line has the equation $x + y = 20$ and since the point (0, 0) is in the region the constraint is $x + y \leq 20$

The line going through the points (0, 0) and (10, 10) has the equation $y = x$ and the feasible region contains the points where the y values are greater than the x values so the constraint is $y \geq x$

Question 8

The maximum value will occur at one of the **extreme points** (vertices of the feasible region)

Extreme point	$V = 3x + 4y$
(0, 0)	$3 \times 0 + 4 \times 0 = 0$
(0, 20)	$3 \times 0 + 4 \times 20 = 80$
(8, 12)	$3 \times 8 + 4 \times 12 = 72$
(8, 8)	$3 \times 8 + 4 \times 8 = 56$

The maximum value for $V = 3x + 4y$ is 80.

Question 9**A**

From the shape the graph is either $y = kx^2$ or

$y = kx^3$ (Eliminates answers D and E)

Substituting the point (4, 20) into these equations gives:

$$y = kx^2: 20 = k \times 4^2$$

$$k = \frac{20}{16} = \frac{5}{4}$$

$$\text{Equation } y = \frac{5}{4}x^2$$

$$y = kx^3: 20 = k \times 4^3$$

$$k = \frac{20}{64} = \frac{5}{16}$$

$$\text{Equation } y = \frac{5}{16}x^3$$

The equations from the answers:

$$\text{A: } y = \frac{5}{4}x^2$$

$$\text{B: } y = \frac{4}{5}x^3$$

$$\text{C: } y = \frac{4}{5}x^2$$

The only one that matches is answer A.

Module 4 : Business related Mathematics**Question 1****C**

If x is the amount that the retailer retains then:-

$$x + 10\% \text{ of } x = 59.95$$

$$x + \frac{10}{100}x = 59.95$$

$$x(1 + 0.1) = 59.95$$

$$1.1x = 59.95$$

$$x = \frac{59.95}{1.1}$$

Question 2**B**

Using the simple interest formula:

$$\begin{aligned} \text{Interest} &= \frac{\text{Pr}T}{100} \\ &= \frac{15000 \times 5.4 \times \frac{1}{4}}{100} \\ &= 202.5 \end{aligned}$$

Question 3**E**

Monthly interest rate

$$= \frac{6.72}{12} = 0.56\%$$

$$R = 1 + \frac{0.56}{100} = 1.0056$$

Using the compound interest formula :

Amount owing = PR^n where $n = 6$

$$\text{Amount owing} = 12500 \times 1.0056^6 = 12925.92$$

Question 4**D**

$$\text{The monthly interest rate} = \frac{4.8}{12} = 0.4\% \text{ so}$$

$$R = 1 + \frac{0.4}{1000} = 1.004$$

4 years = 48 months : $n = 48$

$P = 1000$; the original investment.

Regular payments of \$100 : $Q = 100$

Payments are being added to the initial amount in the account hence the '+' sign between the terms.

Question 5
A

Using the TVM solver on the calculator:-

```

N=74.46140774
I%=4.8
PV=-1000
PMT=-100
FV=10000
P/Y=12
C/Y=12
PMT: [ ] BEGIN
    
```

Question 6
A

The minimum balance for January and February is \$4280

$$\text{Interest} = \frac{4280 \times 1.5 \times \frac{2}{12}}{100} = 10.7$$

The minimum balance for March to June (4 months) is \$3420

$$\text{Interest} = \frac{3420 \times 1.5 \times \frac{4}{12}}{100} = 17.1$$

Total interest for the 6 months = \$27.80

Question 7
D

\$4400 is owed after the deposit is paid.

The repayments total

$$12 \times 402.60 = \$4831.20$$

The interest paid is $\$4831.20 - \$4400 = \$431.20$

$$\text{Interest rate} = \frac{431.20}{4400} \times 100 = 9.8\%$$

Question 8
B

Depreciation of 12% p.a. means a multiplying

$$\text{factor of } \left(1 - \frac{12}{100}\right) = 0.88$$

If the fixtures are valued at \$P initially then their value after n years will be $P \times (0.88)^n$

Solving for n when the value is 0.5P:-

$$0.5P = P \times (0.88)^n$$

$$0.5 = (0.88)^n$$

Using the calculator to solve:

```

Plot1 Plot2 Plot3
Y1=(0.88)^X
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
    
```

X	Y1	
1	.88	
2	.7744	
3	.68147	
4	.5987	
5	.52773	
6	.4644	
7	.40868	
X=7		

After 6 years the value will be less than half the original value. This will be at the beginning of the year 2005.

Question 9
E

If the interest rate rises then either they will need to pay more each month or the term of the loan will increase (or a bit of both). They will also pay more in interest over the term of the loan.

Using the TVM solver on the calculator will confirm answers A to D.

The TVM solver for answer E shows that they will be paying more than \$1739.61 if they increase the loan to 30 years.

```

N=360
I%=7
PV=270000
PMT=-1796.3167...
FV=0
P/Y=12
C/Y=12
PMT: [ ] BEGIN
    
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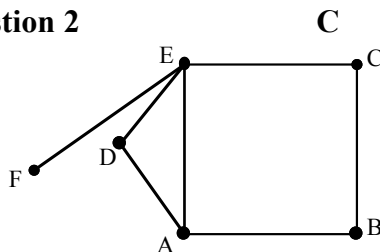

Module 5: Networks & decision mathematics

Question 1 A

The number of vertices is 5

The number of edges is 9, then respectively it is A (5 and 9).

Question 2



A Euler Path uses every edge only once and if there is a pair of vertices with odd degrees then the path must start at either; in this case F (degree of 1) or A (degree of 3)

Answer: C (A-E-C-B-A-D-E-F).

Question 3 B

The first row and first column of the matrix represents the number of edges originating from the vertex A, namely,

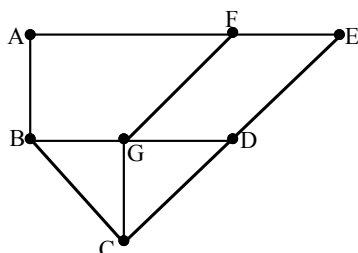
0 from A, 1 from B, 2 from C or 1 from D or

The matrix that represents the network shown above is B

$$B \begin{matrix} & A & B & C & D \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{bmatrix} \end{matrix}$$

Question 4 A

Hamiltonian path or circuit uses each vertex once only and for this example one possible path or circuit is B-C-G-D-E-F-A-B



No Euler path is possible.

The true statement is A:

The planar graph has a Hamiltonian path and circuit, but no Euler path or circuit.

Question 5 D

From A to E via B is 7 units

Then from E to H via D is 14 units

And from H to K either path is 2 units,

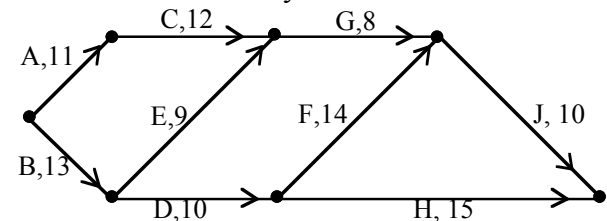
For a longest path of 23 units.

Question 6 C

Jade and Rhiannon have lived in four of the cities while Scott and Alisha have lived in three cities.

Question 7 C

The network diagram shown below gives the times for each activity in hours.



The activities that are immediately before predecessors of Activity G are C (C and E).

Question 8 D

Using Euler's formulae: $V = E - F + 2$

where V = number of vertices, E = number of edges, F = number of faces.

Then the number of faces or regions becomes

$$\begin{aligned} F &= E - V + 2 \\ &= 10 - 6 + 2 \\ &= 6 \end{aligned}$$

Question 9 B

Perform a row reduction by subtracting the smallest value from each row.

The smallest value in row A is 11

The smallest value in row B is 12

The smallest value in row C is 11

The smallest value in row D is 12

The reduction matrix becomes

	Cust 1	Cust 2	Cust 3	Cust 4
Driver A	4	2	0	3
Driver B	0	2	1	3
Driver C	2	1	3	0
Driver D	0	0	3	2

Answer is B where

Driver A gets Customer 3,

Driver B gets Customer 1,

Driver C gets Customer 4,

Driver D gets Customer 2.