

Year 2005

VCE

Further Mathematics

Trial Examination 2

Suggested Solutions

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Kilbaha Multimedia Publishing ABN 47 065 111 373
PO Box 2227
Kew Vic 3101
Australia
Tel: 03 9817 5374
Fax: 03 9817 4334
chemas@chemas.com
www.chemas.com

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Question 1
a. 15 months (1 mark)
b. It is negatively skewed (1 mark)
c. The median is the middle number = 81 (1 mark)
d. Interquartile range = upper quartile – lower quartile = $87 - 77 = 10$ (1 mark)
e. Use graphics calculator. Enter data in stat edit., then stat calc, 1-Var Stats, 2 nd L ₁ = 81 to the nearest whole number (1 mark)
f. M T Hooker has an outlier (1 mark) The outlier = 69 (1 mark)

Question 2
a. The independent variable is interest rate. (1 mark)
b. Use graphics calculator. Enter data in stat edit., in L ₁ and L ₂ then stat calc, linear regression L ₁ , L ₂ to get number of new houses = $31.1 + -1.3 \times \text{interest rate}$. (2 marks)
c. $8 = 31.1 - 1.3 \times I$ $1.3I = 23.1$ $I = 17.8$ $I = 18\%$ to the nearest percent. (1 mark)
d. Use graphics calculator as in b to get $r = -0.97$ (1 mark)
e. There is a very strong negative linear relationship. (1 mark)
f. As the interest rate increases by one percent then the number of new houses decreases by 2000 (2 marks)

Total = 15 marks

End of Core

Question 1

a. $55 + 15 = 70 \text{cals/hr.}$ (1 mark)	b. This is an arithmetic sequence with $a = 55 - 30 = 25$ at level 1 (1 mark) $t_n = a + (n - 1)d$ $370 = 25 + (n - 1)15$ $345 = (n - 1)15$ $n - 1 = 23$ $n = 24$ needs to set bike on level 24.(1 mark)
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Question 2

a. $C_1 = 4$ $C_2 = 1.5 \times 4 = 6$ $C_3 = 1.5 \times 6 = 9$ 4,6,9 (1 mark)	b. This is a geometric sequence with a common ratio of 1.5 (1 mark)
c. $S_n = \frac{a(r^n - 1)}{r - 1} = \frac{4(1.5^n - 1)}{0.5} = 8(1.5^n - 1) = 300$ $1.5^n = 38.5$ (1 mark) $n = \frac{\log_e(38.5)}{\log_e(1.5)}$ $n = 9 \text{ mins}$ (1 mark)	d. $C_n = 1.05^{n-1} \times 5 + \frac{1.325 \times (1.05^{n-1} - 1)}{1.05 - 1}$ $C_n = 1.05^{n-1} \times 5 + 26.5 \times (1.05^{n-1} - 1)$ (1 mark) $C_n = 31.5 \times 1.05^{n-1} - 26.5$ $C_n = 31.5 \times 1.05^n \times 1.05^{-1} - 26.5$ $C_n = \frac{31.5}{1.05} \times 1.05^n - 26.5$ $C_n = 30 \times 1.05^n - 26.5$ (1 mark)

Question 3 a. $A = 0.5^n$ (1 mark)	b. $A = 0.5^3 = 0.125$ litres (1 mark)
c. This means less than or equal to 5% sportade. $0.5^n \leq 0.05$ (1 mark) $n \leq \frac{\log_e(0.05)}{\log_e(0.5)}$ $n \geq 4.3$ $\therefore n = 5$ (1 mark)	d. $\frac{150}{1400} \times 100$ (1 mark) $= 10.7\%$ (1 mark)

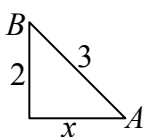
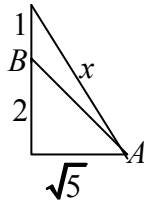
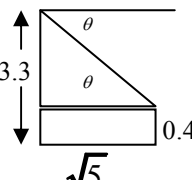
Total = 15 marks

End of Module 1

Question 1

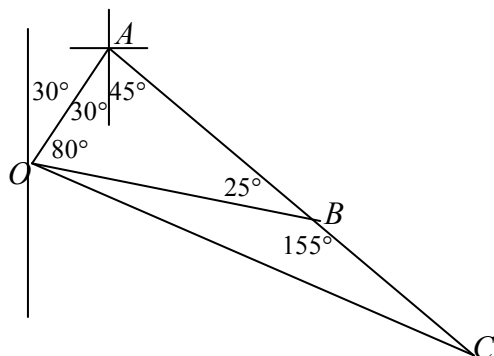
a. 1 : 25,000 $x : 5.2$ $x = \frac{5.2}{25,000} \text{ km}$ $x = \frac{5.2}{25,000} \times 100,000 \text{ cm}$ $x = 20.8 \text{ cm}$ (1 mark)	b. $180 - (35 + 42)$ $= 180 - 77$ $= 103^\circ$ (1 mark)
c. $BC^2 = 2.6^2 + 5.2^2 - 2 \times 2.6 \times 5.2 \cos 103^\circ$ $BC^2 = 39.8827$ $BC = 6.3 \text{ km}$ (1 mark)	d. $A = \frac{1}{2} bc \sin \theta$ $A = \frac{1}{2} \times 2.6 \times 5.2 \sin 103^\circ$ $A = 6.6 \text{ km}^2$ (1 mark)

Question 2

a.  $x^2 + 4 = 9$ $x^2 = 5$ $x = \sqrt{5}$ (1 mark) Average slope = $\frac{\text{rise}}{\text{run}} = \frac{2}{\sqrt{5}} = 0.89$ (1 mark)	b.  (1 mark) $x^2 = 5 + 9 = 14$ $x = \sqrt{14} = 3.74$ (1 mark)
c. $\tan \theta = \frac{\sqrt{5}}{3}$ $\theta = \tan^{-1} \left(\frac{\sqrt{5}}{3} \right) = 37^\circ$ (1 mark)	d.  $\tan \theta = \frac{2.9}{\sqrt{5}}$ $\theta = 52^\circ$

Question 3

a.



Using alternate angles,
angle $OAB = 30 + 45 = 75^\circ$

(1 mark)

Angle $AOB = 110 - 30 = 80$

Angle $ABO = 180 - (75 + 80)$
(angles in triangle sum to 180°)

Angle $ABO = 180 - 155 = 25^\circ$ (1 mark)

b.

$$\frac{OB}{\sin 75^\circ} = \frac{200}{\sin 25^\circ}$$

$$OB = \frac{200 \sin 75^\circ}{\sin 25^\circ}$$

$$OB = 457.1 \text{ m} \quad (1 \text{ mark})$$

c.

$$OC^2 = 240^2 + 457.1^2 - 2 \times 457.1 \times 240 \cos 155^\circ$$

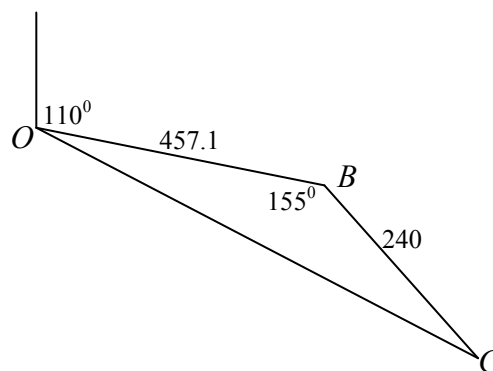
$$OC = 682.19615 \quad (1 \text{ mark})$$

$$\frac{682.19615}{\sin 155^\circ} = \frac{240}{\sin BOC}$$

$$\sin BOC = \frac{240 \sin 155^\circ}{682.19615}$$

$$BOC = 8.55^\circ \quad (1 \text{ mark})$$

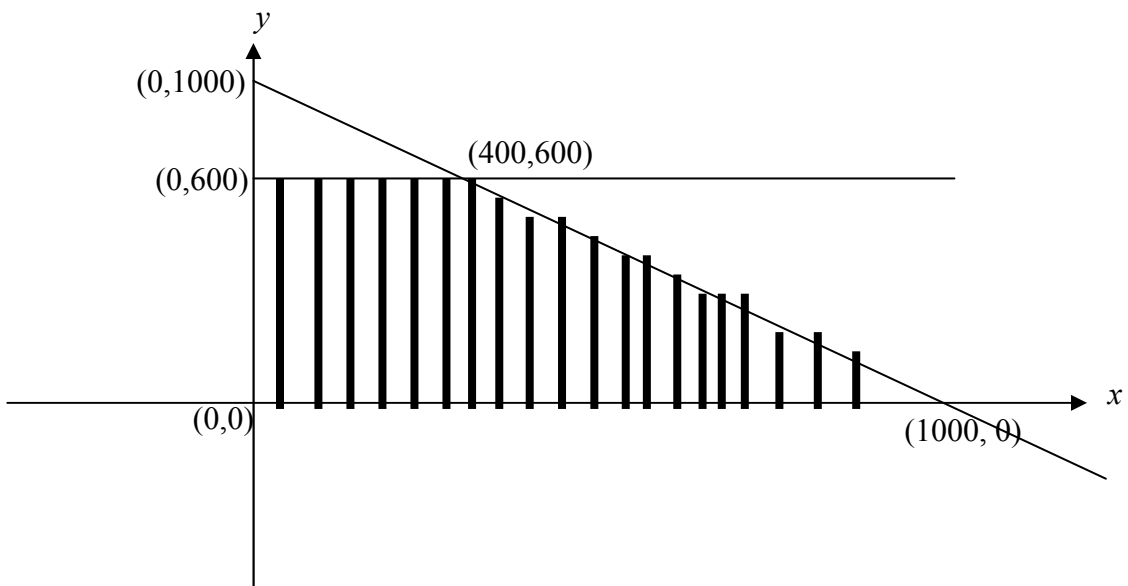
$$\therefore \text{bearing} = 30 + 80 + 9 = 119^\circ \quad (1 \text{ mark})$$



Total = 15 marks

End of Module 2

Question 1 a. $7 - 5 = 2$ mins. (1 mark)	b. 25 mins (1 mark)
c. $8,200 - 200 = 8000$ metres (1 mark)	d. From D to F = $8,200 - 7,200 = 1,000$ metres (1 mark)
e. 7,200 metres (1 mark)	f. $25 - 19 = 6$ mins. (1 mark)
g. $\frac{8000}{11} = 727.2727 \text{ m/min} \quad (1 \text{ mark})$ $= \frac{727.2727 \times 60}{1000} = 43.6 \text{ km/hr} \quad (1 \text{ mark})$	

Question 2 a. $x + y + z = 1000$ $z = 1000 - x - y$ (1 mark)	b. $y \leq 600$ $x \geq 0$ $y \geq 0$ (1 mark)
c.  Shaded region is the feasible region. • 1 mark for correct lines drawn and correct point of intersection • 1 mark for correct region	

Question 2 (continued) d. $P = 80x + 100y + 40z$ $= 80x + 100y + 40(1000 - x - y)$ $= 80x + 100y + 40,000 - 40x - 40y$ $= 40x + 60y + 40,000 \quad (1 \text{ mark})$	e. Profit at (0,0) $= \\$40,000$ Profit at (0,600) $= 0 + 36,000 + 40,000 = \\$76,000$ Profit at (400,600) $= 16,000 + 36,000 + 40,000 = \\$92,000$ Profit at (1000,0) $= 40,000 + 0 + 40,000 = \\$80,000$ Maximum profit = \$92,000 (1 mark)
f. Number of medals = z $z = 1000 - x - y$ Maximum profit when $x = 400$ and $y = 600$ and $\therefore z = 0$ (1 mark)	

Total = 15 marks

End of Module 3

Question 1 a. $0.5 \times 12 = 6\%$ (1 mark)	b. $I = \frac{PRT}{100} = \frac{15,000 \times 6 \times 8}{100}$ $I = \\$7,200$ (1 mark)
c. Total Amount = $15,000 + 7,200 + 5,000 = \\$27,200$ (1 mark)	d. Original value = 20,000 Residual value = 4,000 Total depreciation over 8 years = \$16,000 (1 mark) Depreciation each year = $16,000/8$ Depreciation each year = \$2,000 (1 mark)
Question 2 a. $222 \times 96 + 7,000 = \\$28,312$ (1 mark)	b. $222 \times 96 - 13,000 = \\$8,312$ (1 mark)
c. $I = \frac{PRT}{100}$ $8312 = \frac{13000 \times r \times 8}{100}$ $r = \frac{8312 \times 100}{13000 \times 8} = 8\%$ (1 mark)	d. Effective rate = $\frac{2n}{n+1} \times \text{flat rate}$ $= \frac{2 \times 96}{97} \times 8 = 15.8\%$ (1 mark)
Question 3 a. $P = 17,000$ $n = 96$ (1 mark) $R = 1 + \frac{6.5}{1200} = 1.005$ (1 mark)	b. $PR^n = \frac{Q(R^n - 1)}{R - 1}$ $17,000(1.005)^{96} = \frac{Q(1.005^{96} - 1)}{0.005}$ $Q = \\$223.40$ (1 mark)
c. $A = 17000(1.005)^{24} - \frac{223.4(1.005^{24} - 1)}{0.005} = 13480.21739$ $0 = 13480.21739(1.005)^n - \frac{446.8(1.005^n - 1)}{0.005}$ (1 mark) $13480.21739(1.005)^n = 89360(1.005^n - 1)$ $0.849147(1.005)^n = 1$ $(1.005)^n = 1.17765$ $n \log_{10} 1.005 = \log_{10} 1.17765$ $n = 32.8$ months (1 mark) He has paid 24 months $24 + 32.8 = 56.8$ instead of 96 months. $96 - 56.8 = 39.2$ months 3 years and 3 months (1 mark)	

Question 1 a. D and F (1 mark)	b. 9 weeks (1 mark)
c. (1 mark)	
d. A D E H • 1 mark for A D • 1 mark for A D E H	e. $12 + 31 + 25 + 5 = 73$ weeks (1 mark)
f. The earliest time E can commence is when A and D are completed $12 + 31 = 43$ At end of 43 weeks (1 mark)	g. $73 - 5 - 25 - 14 = 29$ Latest starting time is end of 29th week (1 mark)
h. Latest start time = 29 Earliest start time = 16 Float time = $29 - 16 = 13$ weeks (1 week)	

Question 2

a.

	Cut Out	Seams	Braid	Pockets
Wayne	16	17	10	5
Xavier	18	19	12	8
Yolande	18	20	12	12
Zeta	15	18	6	9
(1 mark)				
d. The length of the critical path = $12 + 2 + 4 + 3 = 21$ hours (1 mark)				

Question 2

a.

Row reduction

	C	S	B	P
W	11	12	5	0
X	10	11	4	0
Y	6	8	0	0
Z	-9	12	0	3

Column reduction

	C	S	B	P
W	5	5	5	0
X	4	3	4	0
Y	0	0	0	0
Z	-3	4	0	3

(1 mark)

	C	S	B	P
W	2	1	2	0
X	1	0	1	0
Y	0	0	0	1
Z	-3	4	0	4

(1 mark)

Allocate

Wayne	Pockets
Zeta	Braid
Xavier	Seams
Yolande	Cut

(1 mark)

c.

Pockets 5 mins

Braid 6 mins

Seams 19 mins

Cutting 18 mins

Total time = $5 + 6 + 19 + 18 = 48$ mins.

End of Module 5

Total = 15 marks

End of suggested solutions
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KILBAHA MULTIMEDIA PUBLISHING PO BOX 2227 KEW VIC 3101 AUSTRALIA	TEL: (03) 9817 5374 FAX: (03) 9817 4334 chemas@chemas.com www.chemas.com
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