



The Australian Education Academy 2005

FURTHER MATHEMATICS - UNIT 3 & 4

TRIAL EXAMINATION 2 - Solutions

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Section A –

Core – Data Analysis Solutions

Question 1

- a. Write the data in ascending order.

Then the data set x_i : = 12 14 25 28 32 36 36 43 50

$$\text{Mean} = \frac{\sum_{i=1}^8 x_i}{n}, n = 8 \text{ (number of data)}, \sum_{i=1}^8 x_i = 240$$

$$\text{Hence mean } \bar{x} = 240 / 8 = 30$$

1 mark

- b. Here there are even number of data. Hence median is $(n+1) / 2$

Then $(8+1) / 2 = 4.5$ (position)

$$\text{Hence median} = (28 + 32) / 2 = 30$$

1 mark

- c. The variance is $\frac{\sum_{i=1}^8 (x_i - \bar{x})^2}{n - 1} = 174$

1 marks

- d. The standard deviation is the root of variance (s) .

$$s = 13.19 \approx 13$$

Now calculate $\bar{x} - s, \bar{x} + s = 27$ to 43

Hence percentage is 65 %

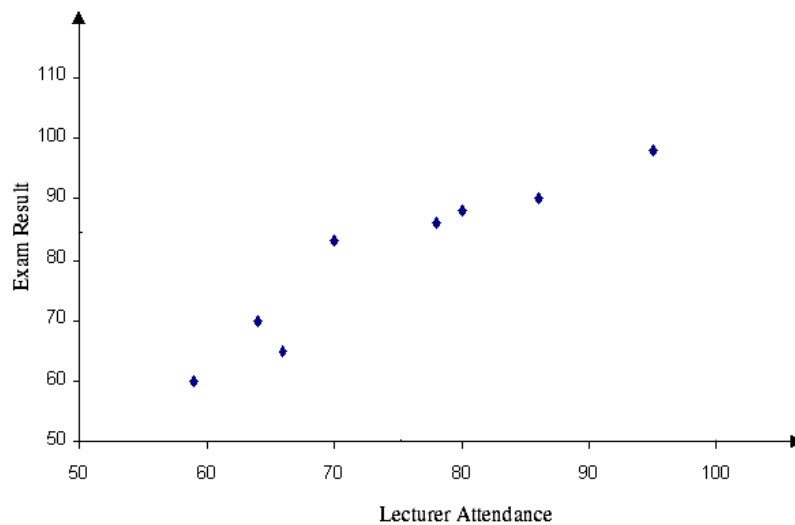
2 marks

Question 2

- a. independent variable := Lecture attendance
dependent variable := exam result

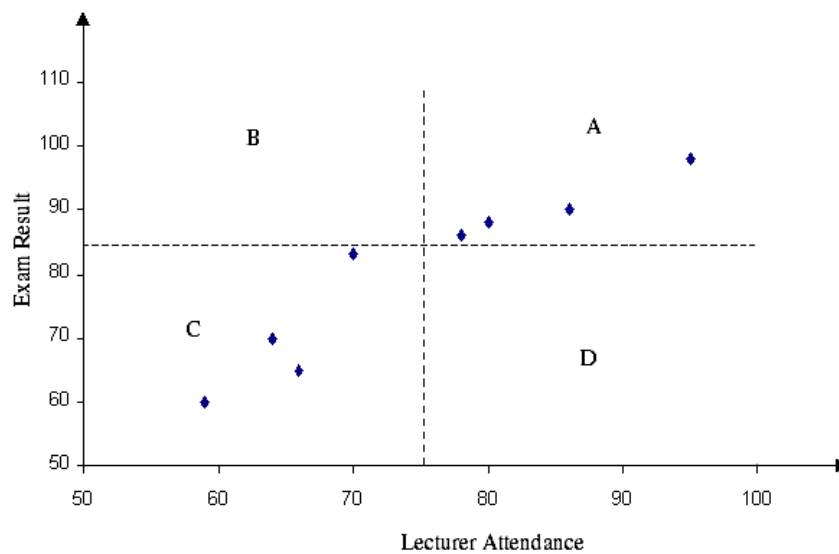
1 mark

b.



2 marks

c.



After drawing the scatter plot, locate the medians of x values and y values, then draw the vertical horizontal lines respectively the medians of x and y values.

Median of x = 74

Median of y = 85.5

Then calculate the q-correlation coefficient using the above graph and the following formula.

$$q = \frac{(a+c)-(d+d)}{a+b+c+d} = \frac{(4+4)-(0+0)}{4+4} = 1$$

1 mark

- d. Moderate positive linear association.

1 mark

Question 3

- a. You have to divide the points in to three groups as 3, 4, 3. Then have to find three medians

$$\begin{array}{lll} x_l = 4 & x_m = 9 & x_u = 18 \\ y_l = 17 & y_m = 8.5 & y_u = 4 \end{array}$$

Then gradient is (m) = $(y_u - y_l) / (x_u - x_l) = -13/14 = -0.93$

$$\begin{aligned} \text{Intercept is (b)} &= 1/3 * [(y_l + y_m + y_u) - m * (x_l + x_m + x_u)] \\ &= 1/3 * (29.5 + 13/14 * 31) = 19.42 \end{aligned}$$

$$y = -0.93x + 19.42$$

2 marks

b.

y actual	18	17	16	10	8	9	6	5	4	2
y predict	17.56	15.7	14.77	13.84	11.98	10.12	6.4	5.47	2.68	0.82
residual	0.44	1.3	1.23	-3.84	-3.98	-1.12	-0.4	-0.47	1.32	1.18

y actual means the given y values. Substituting values given as y predict.
Residual values calculated as y actual values – y predict.

3 marks

Total 15 marks

Module 1: Number patterns and applications

Question 1

- a. First day = 3, second day = $3 \times 3 = 9$, third day = $9 \times 3 = 27$
Hence the geometric sequence is 3 9 27 81

2 marks

- b. Here $r = 3$ $t_1 = a = 3$ $n = 10$
We have to find the total number of people who know the rumour.
For geometry sequence, the sum of the sequence is $S_n = \frac{a(r^n - 1)}{r - 1}, (r > 1)$
Hence for a 10th day $S_{10} = 88572$

1 mark

- c. The nth term of the geometric sequence is $t_n = ar^{n-1}$
Hence 5th day means $n = 5$
Hence $t_5 = 3 \times 3^4 = 243$

2 marks

Question 2

- a. Here $t_1 = 6.4$, $t_2 - t_1 = 2.1$
Hence $d = 2.1$
We know $t_n = a + (n - 1) * d$
Hence $t_n = 6.4 + (n - 1) * 2.1$

2 marks

- b. Here $n = 5$
Hence $t_5 = 6.4 + (5 - 1) * 2.1 = 15.2$ tones

1 mark

- c. We know $t_n = a + (n - 1) * d$
Here $t_n = 40$
 $40 = 6.4 + (n - 1) * 2.1$
 $n = 17$

2 marks

Question 3

- a. Here $t_1 = 2$, $t_2 = 6$, $t_3 = 18$ and $a = 2$ (since $t_1 = 2$)
We know $t_n = a r^{n-1}$
Hence $t_n = 2 r^{n-1}$, $t_2/t_1 = 6 / 2 = 3$
Hence $r = 3$
Hence $t_n = 2 * 3^{n-1}$

2 marks

- b. The form of first order difference equation for the geometry series is
 $t_{n+1} = at_n$
We know $t_1 = 2$, $t_2 = 6$, $t_3 = 18$
When $n = 1$ $t_2 = at_1$ $6 = a2$,
Hence $a = 3$
Hence $t_{n+1} = 3 t_n$

1 mark

- c. From part (a) $t_n = 2 \cdot 3^{n-1} \longrightarrow (1)$
 From part (b) $t_{n+1} = 3 t_n \longrightarrow (2)$
 Substituting (1) for (2)
 $t_{n+1} = 3 \cdot 2 \cdot 3^{n-1}$
 Hence $t_{n+1} = 2 \cdot 3^n$

2 marks

Total 15 marks

Module 2: Geometry and Trigonometry

Question 1

- a. Volume of the cylinder = area of uniform cross section * height
 $= \pi r^2 h$
 $= (22 / 7) \cdot 25 \cdot 30 = 2357.14 \text{ cm}^3$

1 mark

- b. We know $1000 \text{ cm}^3 = 1 \text{ litre}$
 Hence 2.35714 litre

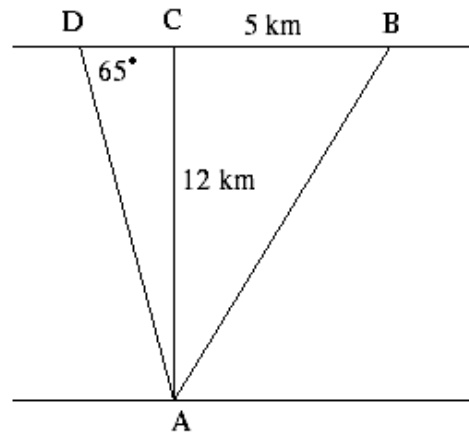
2 marks

- c. Volume of cylinder = volume of pyramid
 Hence $(22 / 7) \cdot 25 \cdot 30 = (1/3) \cdot a^2 \cdot 15$
 (a denotes the pyramids base length)
 $a = 21.71$

2 marks

Question 2

a.



Using Pythagoras Theorem

$$AC^2 + BC^2 = AB^2$$

$$5^2 + 12^2 = AB^2 \quad \text{Hence } AB = 13 \text{ km}$$

2 marks

- b. We can say sin value of the angle ABC = $12 / 13$
Using the sine rule to triangle ADB

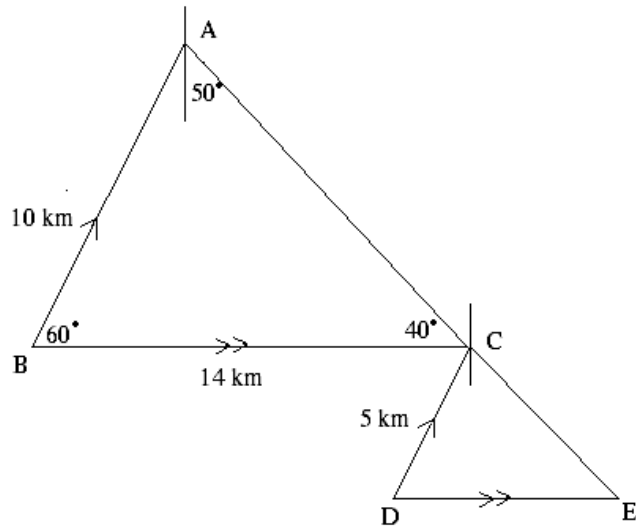
$$\frac{\sin 65}{13} = \frac{\sin(12/13)}{AD}$$

$$\text{Hence } AD = 13.33$$

3 marks

Question 3

a.



Using sine rule to ABD triangle

$$\frac{\sin 60^\circ}{AC} = \frac{\sin 40^\circ}{10}$$

$$AC = 13.47$$

2 marks

- b. Using similarity triangulation formula. Triangle ABC and CDE are similar triangles. (Since $AB \parallel CD$ and $BC \parallel DE$ hence $\angle ABC = \angle CDE$ and $\angle BAC = \angle DCE$)

$$\text{Hence } \frac{10}{DC} = \frac{AC}{CE}$$

$$\frac{10}{5} = \frac{13.47}{CE} \implies CE = 6.735$$

1 mark

- d. We can calculate $\angle BCD$ as 60° . (Since $AB \parallel CD$ and $BC \parallel DE$)
 Using the cosine rule
 $BD^2 = BC^2 + DC^2 - 2 \cdot BC \cdot DC \cdot \cos 30^\circ$
 $= 99.75$
 $BD = \sqrt{99.75} = 10$

2 marks

Total 15 marks

Module 3: Graphs and Relations

Question 1

- a. Commission is $= 25,000 \cdot (1.5 / 100) = 375$ \$
1 mark
- b. Commission is $= 25,000 \cdot (1.5 / 100) + 25,000 \cdot (1.2 / 100) + 20,000 \cdot (1 / 100)$
 $= 950$ \$
2 marks
- c. Commission $= 30,000 \cdot (1.5 / 100) + 25,000 \cdot (1.2 / 100) +$
 $(\text{sales price} - 55,000) \cdot (1 / 100)$
1 mark
- d. If he got \$ 1200
 The commission for first 30,000 $= 30,000 \cdot (1.5 / 100) = 450$ \$
 The commission for second 25,000 $= 25,000 \cdot (1.2 / 100) = 300$ \$
 Remainder of commission $= 1200 - (450 + 300) = 450$ \$
 This commission he gets in interest rate 1%
 Hence value $= 450 \cdot 100 = 45,000$
 Hence value of the car $= 30,000 + 25,000 + 45,000$
 $= 100,000$ \$

1 mark

Question 2

a. $\begin{array}{lcl} \text{Cost} & = & 230 + 27 * 15 = 635 \$ \\ \text{Revenue} & = & 300 * 15 - 250 = 4250 \$ \end{array}$

2 marks

b. $\begin{array}{lcl} 635 & = & 230 + 27n \\ n & = & 15 \\ \text{Hence number of computers} & & 15 \end{array}$

1 mark

c. $\begin{array}{lcl} \text{Profit} & = & \text{Revenue} - \text{Cost} \\ 13,170 & = & 300n - 250 - (230 + 27n) \\ 13,170 & = & 273n - 480 \\ n & = & 50 \end{array}$

2 marks

Question 3

a. $\begin{array}{l} \text{Let } x = \text{number of automobiles shipped to Victoria from Factory 1} \\ \text{Let } y = \text{number of automobiles shipped to New South Wales from} \\ \text{Factory 1} \\ \text{Hence } x + y \leq 130 \end{array}$

1 mark

b. $\begin{array}{l} \text{Now we can say } (150 - x) = \text{number of automobiles shipped to Victoria} \\ \text{from Factory 2} \\ (90 - y) = \text{number of automobiles shipped to New South Wales from} \\ \text{Factory 2} \\ \text{Hence } (150 - x) + (90 - y) \leq 175 \\ 240 - x - y \leq 175 \\ 65 \leq x + y \end{array}$

1 mark

- c. Factory 1 shipped x automobiles to Victoria at 120 \$ per car.
 Factory 1 shipped y automobiles to New South Wales at 350 \$ per car.
 Factory 2 shipped $(150-x)$ automobiles to Victoria at 400 \$ per car.
 Factory 2 shipped $(90-y)$ automobiles to New South Wales at 150 \$ per car.
 Hence Total cost $= x*150 + 350*y + (150-x)*400 + (90-y)*150$
 $= 73,500 - 280x + 200y$

3 marks

Total 15 marks

Module 4: Business Related Mathematics

Question 1

- a. Deposit value $= 700 * (10 / 100) = 70 \$$

1 mark

- b. Customer payment $= 950 \$ - 70 \$ = 880 \$$
 Hence quarters instalment $= 880 / 8 = 110 \$$

2 marks

- c. Flat rate interest $I = \frac{Pr t}{100}$
 $110 = \frac{880 * r * 8}{100}$
 $r = 1.5625$

1 mark

$$\begin{aligned}
 \text{d. Effective interest} &= \frac{2n}{n+1} * \text{flat rate} \\
 & \quad (n - \text{number of instalments}) \\
 &= \frac{2 * 8}{8+1} * 1.5625 \\
 &= 2.778
 \end{aligned}$$

1 mark

Question 2

$$\begin{aligned}
 \text{a. We know } A &= PR^T, \quad R = 1 + (r/100) \\
 A &= \text{current value} \\
 P &= \text{Principal value} \\
 R &= \text{Interest rate}
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence } 6047.92 &= 4500 * R^{10} \\
 R^{10} &= 1.3439 \\
 R &= 1.0299 = 1.03
 \end{aligned}$$

$$\begin{aligned}
 R &= 1 + (r/100) \\
 1.03 &= 1 + r / 100 \\
 3 &= r \\
 \text{Hence inflation rate } 3
 \end{aligned}$$

2 marks

$$\begin{aligned}
 \text{b. } A &= PR^T, \quad R = 1 + (r/100) \\
 13500 &= 4500 (1.03)^T \\
 3 &= (1.03)^T \\
 \ln 3 &= T \ln 1.03 \\
 T &= 37.16 \\
 T &= 37 \text{ years}
 \end{aligned}$$

2 marks

$$\begin{aligned}
 \text{c. } A &= PR^T, \quad R = 1 + (r/100) \\
 5216.74 &= 4500 (1.03)^T \\
 1.16 &= (1.03)^T \\
 \ln 1.16 &= T \ln 1.03 \\
 T &= 5 \text{ years} \\
 \text{Hence 10 years ago.}
 \end{aligned}$$

2 marks

Question 3

a. $\text{Rare of depreciation} = \frac{\text{total depreciation}}{\text{number of years}}$
But we want to calculate depreciation for a month.
Hence $\text{Rare of depreciation} = \frac{\text{total depreciation}}{\text{number of years} * 12}$
 $= 7000 / 8 * 12$
 $= 72.92$

2 marks

b. We know $BV = P - d T$
BV = Book value after T years
P = Cost price
T = Time in years
D = depreciation rate
Hence $BV = 25000 - 875 T$ (since we have to use depreciation rate for year. Hence use d as 875. (7000/8))

2 marks

c. Using part (b)
 $BV = P - d T$
 $33750 = 25000 - 875 T$
 $T = 10 \text{ years}$

1 mark

Total 15 marks

Module 5: Network and Decision Mathematics

Question 1

a. $AB \rightarrow BC \rightarrow CD \rightarrow DB \rightarrow BE \rightarrow ED \rightarrow DF \rightarrow FE \rightarrow EA \rightarrow AH \rightarrow HF \rightarrow$
 $FG \rightarrow HG \rightarrow HI \rightarrow IG \rightarrow GJ \rightarrow JI$
Here we have to use one edge at once as sub Euler path.

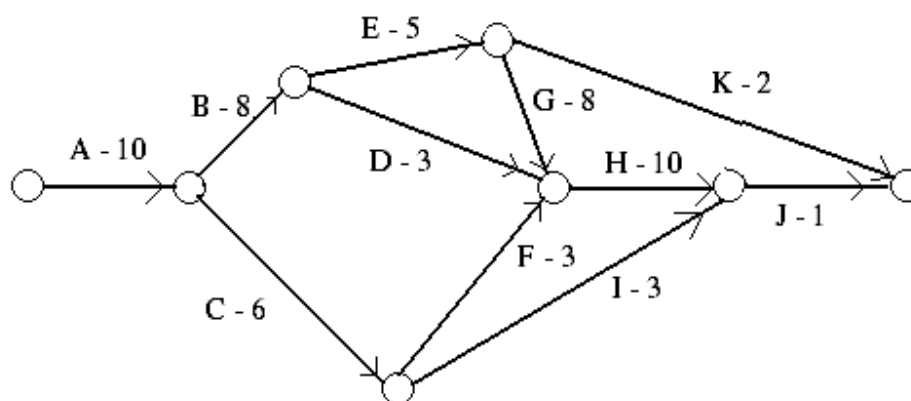
2 marks

- b. $AB \rightarrow BC \rightarrow CD \rightarrow DB \rightarrow BE \rightarrow ED \rightarrow DF \rightarrow FG \rightarrow GJ \rightarrow JI \rightarrow IG \rightarrow GH \rightarrow HI \rightarrow IA$
 Here we have to use one edge at once as sub Euler circuit with same start and ending point.

3 marks

Question 2

a.



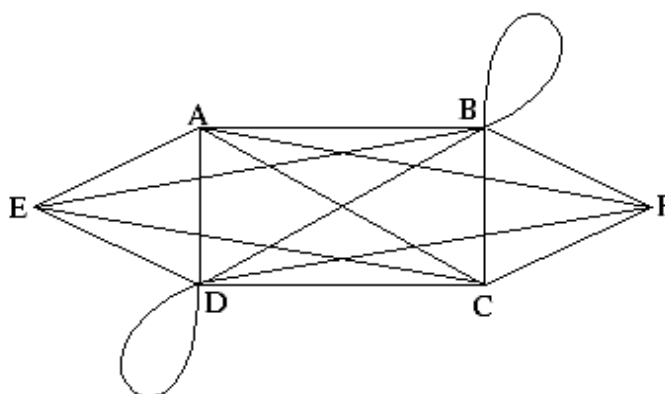
3 marks

- b. Earliest completion path is $A \rightarrow B \rightarrow E \rightarrow G \rightarrow H \rightarrow J$.
 Hence 42 days.

2 marks

Question 3

a.



3 marks

- b. $EA \rightarrow AB \rightarrow BF \rightarrow FC \rightarrow CD \rightarrow DE$.
Here we have to use one edge at once in Euler circuit with same start and ending point.

1 marks

- c. Here we have to find points which have loops.
Hence B and D.

1 mark

Total 15 marks