

2004 Further Mathematics Written Examination 2 (Analysis task) Suggested answers and solutions

Core: Data analysis

Question 1

12 26 27 28 39 40 41 42 44 50 55 56 56 63 64
70 70 71

Lowest score is 12

Highest score is 71

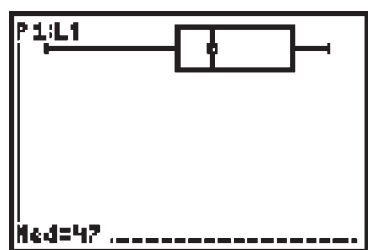
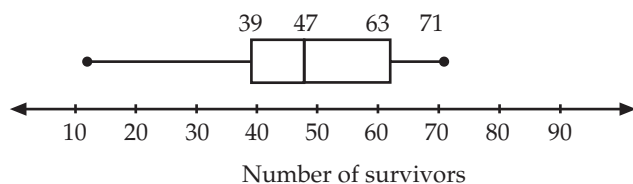
Median is between the 9th and 10th score

$$\text{Median} = \frac{44 + 50}{2} = 47 \quad [\text{A1}]$$

1st Quartile is the 5th score or 39

3rd Quartile is the 14th score or 63 [A1]

Number of survivors on 18 lifeboats of the Titanic



b Interquartile Range =

$$\text{3rd Quartile} - \text{1st Quartile} = 63 - 39 = 24 \quad [\text{consequential A1}]$$

Lower limit =

$$\text{1st Quartile} - 1.5 \times \text{IQR} = 39 - 1.5 \times 24 = 39 - 36 = 3$$

Upper Limit =

$$\text{3rd Quartile} + 1.5 \times \text{IQR} = 63 + 1.5 \times 24 = 63 + 36 = 99$$

All scores are within these limits and so there were no outliers

[A1 with workings]

c The distribution is negatively skewed. [A1]

It has a centre of 47 passengers and a range of 59 passengers. [A1]

Question 2

From the table:

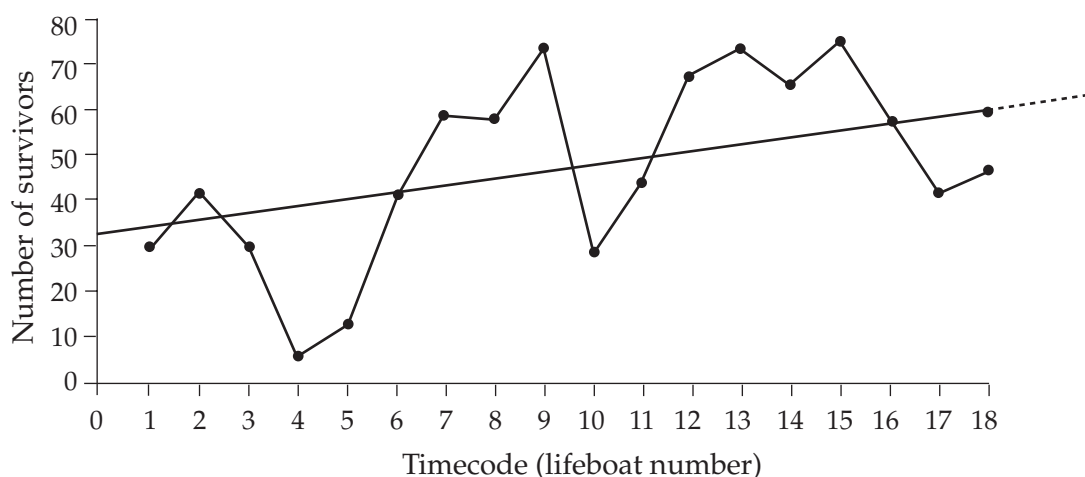
a The group that had the best survival rate was **female** (296 survivors and 106 deaths or 296 out of 402 survived $\frac{296}{402} \times \frac{100}{1} \approx 74\%$). [A1]

b For the men the class with the worst survival rate was **2nd class** (14 survivors and 154 deaths or 14 out of 168 survived $\frac{14}{168} \times \frac{100}{1} = 8.3\%$). [A1]

Question 3

a

Time series of number of survivors launched into 18 lifeboats



The above graph is **random**
(or **cyclical** is acceptable)

[A1]

with an overall **upward trend**

[A1]

b **Y intercept** above 30 at approximately 33

[A1]

Gradient of 1.5.

[A1]

c

Using

$$\text{Number of survivors} = 33 + 1.5 \times \text{Timecode}$$

For the 19th boat

$$\text{Number of survivors} = 33 + 1.5 \times 19$$

$$= 33 + 28.5$$

$$= 61.5 \approx 62 \text{ passengers}$$

[A1]

For the 20th boat

$$\text{Number of survivors} = 33 + 1.5 \times 20$$

$$= 33 + 30$$

$$= 63 \text{ passengers}$$

[A1]

This gives a total predicted number
of 125 extra survivors (62 + 63)

[A1]

Total Core = 15 marks

Module 1 : Number patterns and applications

Question 1

- a 4, 7, 10, 13 [A1]
- b Show, for example, that $t_2 - t_1 = t_3 - t_2$
i.e. $7 - 4 = 10 - 7 = 3$ [M1]
- c Use $t_n = a + (n - 1)d$ where
 $a = 4$ and $d = 3$ [M1]
 $t_n = 4 + (n - 1)3$
 $t_n = 4 + 3n - 3$
 $t_n = 3n + 1$ [A1]
- d Let $t_n = 100$ therefore $100 = 3n + 1$ [M1]
 $3n = 99$
 $n = 33 \Rightarrow$ on the 33rd Sunday [A1]
- e Use $S_n = \frac{n}{2}[2a + (n - 1)d]$ where
 $a = 4$, $d = 3$, $n = 52$ [M1]
 $S_{52} = 26[8 + 51 \times 3]$
 $S_{52} = 4186$ [A1]

Question 2

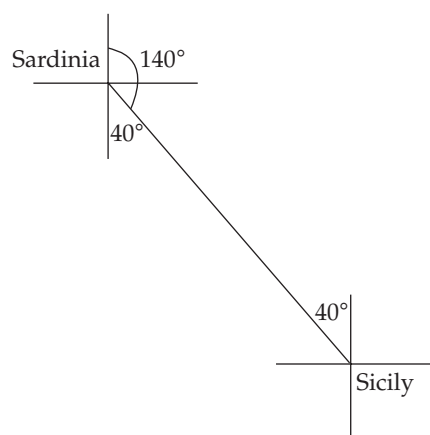
- a $a = 2$, $b = 1$, $c = 3$
Any two correct [A1]
All three correct [A1]
- b $t_1 = 3$, $t_2 = 7$, $t_3 = 15$, $t_4 = 31$ [A1]
 $7 - 3 \neq 15 - 7 \Rightarrow$ not arithmetic
 $\frac{7}{3} \neq \frac{15}{7} \Rightarrow$ not geometric [M1]
- c Use formula $t_n = a^{n-1}t_1 + b \frac{(a^{n-1} - 1)}{a - 1}$
where $a = 2$, $b = 1$ and $t_1 = 3$ [M1]
 $t_n = 2^{n-1} \times 3 + 1 \frac{(2^{n-1} - 1)}{1}$
 $t_n = 3 \times 2^{n-1} + 2^{n-1} - 1$
 $t_n = 4 \times 2^{n-1} - 1$ [M1]
- d $t_7 = 3 \times 2^6 + 2^6 - 1$
 $t_7 = 255$ [A1]

Total Module 1 = 15 marks

Module 2 : Geometry and trigonometry

Question 1

- a When 3 sides of a non right-angled triangle are given, the cosine rule may be used to determine any of the angles. [A1]
- b Using the cosine rule
$$\cos N = \frac{750^2 + 600^2 - 1000^2}{2 \times 750 \times 600}$$
 [M1]
 $\cos N = -0.0861$
 $N = 94.94^\circ$ [A1]
- c Angle $= 180^\circ - (94.94^\circ + 36.72^\circ)$
 $= 48.34^\circ$ [A1]
- d Choose any 2 sides and the included angle e.g.
Area $= \frac{1}{2} \times 750 \times 1000 \times \sin 36^\circ 43'$ [M1]
 $= 224200 \text{ km}^2$ to the nearest hundred square kilometres [A1]
- e There are 1000 metres in 1 kilometre therefore there are
 1000^2 metres^2 in 1 km^2 .
So the answer must be multiplied by
 $1000^2 = 1,000,000$ [A1]
- f The bearing of Sicily from Sardinia is
 $103.28^\circ + 36.72^\circ = 140^\circ T$ [A1]
- g Calculation of the angles shown in the diagram i.e. $180^\circ - 140^\circ = 40^\circ$ [M1]



Bearing of Sardinia from Sicily is
 $360^\circ - 40^\circ = 320^\circ T$ [A1]

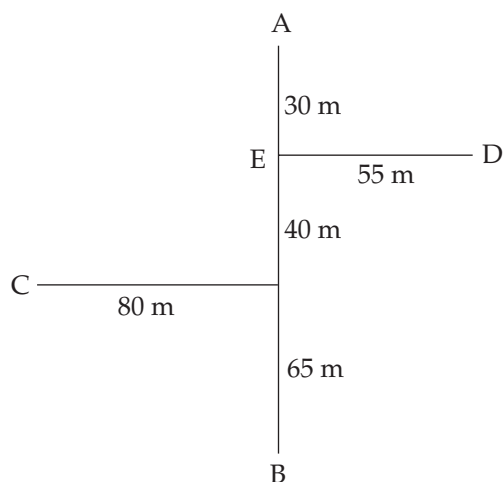
Question 2

- a Using Pythagoras' theorem

$$CD^2 = 135^2 + 40^2 \quad [\text{M1}]$$

$$CD = 140.8 \text{ m, correct to 1 decimal place.} \quad [\text{A1}]$$

- b Angle ADB = angle ADE + angle EDB



$$\text{angle ADE} = \tan^{-1} \frac{30}{55} = 28.61^\circ \quad [\text{M1}]$$

$$\text{angle EDB} = \tan^{-1} \frac{105}{55} = 62.35^\circ \quad [\text{M1}]$$

$$\begin{aligned} \text{Angle ADB} &= \text{angle ADE} + \text{angle EDB} \\ &= 28.61^\circ + 62.35^\circ \\ &= 91^\circ, \text{ to the nearest degree} \quad [\text{A1}] \end{aligned}$$

Total Module 2 = 15 marks

Module 3 : Graphs and relations**Question 1**

- a i 25 dogs require at least 200g of soft food.

$$25 \times 200 \text{ g} = 5000 \text{ g} = 5 \text{ kilograms} \quad [\text{A1}]$$

$$\text{ii } x \geq 5 \quad [\text{A1}]$$

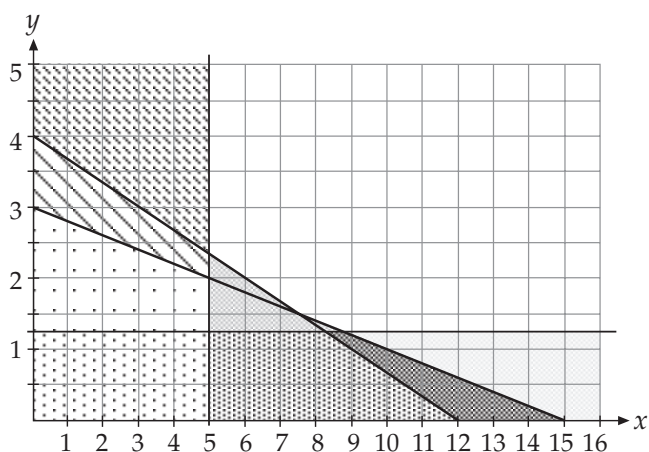
- b $50x + 150y \geq 24 \times 25$

$$\text{"}50x + 150y\text{"} \quad [\text{A1}]$$

$$\text{"}24 \times 25\text{"} \quad [\text{A1}]$$

Question 2

- a The feasible region is the unshaded region on the diagram below.



$$\text{Region } x + 5y \geq 15 \text{ correct} \quad [\text{A1}]$$

$$\text{Other regions correct} \quad [\text{H1}]$$

- b Points $(5, 2\frac{1}{3})$ and $(7.5, 1.5)$ [H1]

$$\text{Point } (8.75, 1.25) \quad [\text{A1}]$$

$$\text{Three points only} \quad [\text{A1}]$$

Question 3

$$\text{a } C = 1.5x + 5y \quad [\text{A1}]$$

$$\text{b Substitution of all points} \quad [\text{M1}]$$

$$(5, 7/3) \text{ gives } C = \$19.17$$

$$(7.5, 1.5) \text{ gives min.cost } \$18.75 \quad [\text{H1}]$$

$$(8.75, 1.25) \text{ gives } C = \$19.38$$

$$\text{c } \frac{\$18.75}{25} = \$0.75 = 75 \text{ cents} \quad [\text{A1}]$$

Question 4

At the break-even point:

$$\text{Costs} = 207 + 15 \times 0.8 \quad [\text{M1}]$$

Revenue = $15 \times d$ where d is the cost of boarding a dog for a day

$$\text{So } 207 + 15 \times 0.8 = 15d \quad [\text{A1}]$$

$$\Rightarrow d = \$14.60$$

Total Module 3 = 15 marks**Module 4 : Business-related mathematics****Question 1**

$$\text{a Percentage is } \frac{6149}{286000} \times 100 = 2.15\% \quad [\text{A1}]$$

$$\begin{aligned} \text{b } 47\% \text{ of } \$40\,000 \\ = \frac{47}{100} \times \$40\,000 = \$18\,800 \quad [\text{A1}] \end{aligned}$$

$$\begin{aligned} \text{c } 6\% \text{ of } (\$564\,000 - \$115\,000) \quad [\text{M1}] \\ = \frac{6}{100} \times \$449\,000 \\ = \$26\,940 \\ \text{plus } \$2560 \\ \text{Stamp duty} = \$29\,500 \quad [\text{A1}] \end{aligned}$$

Question 2a Monthly payments so interest rate, r , is

$$\frac{6.48}{12} = 0.54$$

$$R = I + \frac{r}{100} = 1.0054 \quad [\text{A1}]$$

$$n = 25 \times 12 = 300 \quad [\text{A1}]$$

b Using the TVM solver on the calculator:

```

N=300
I%=6.48
PV=-220000
PMT=1482.707524
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN

```

Payments are \$1482.71 per month [A1]

c Using the TVM solver on the calculator:

```

N=300
I%=7.48
PV=-220000
PMT=1622.919644
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN

```

Payments are \$1622.92 per month; [A1]an increase of \$140.21 [H1]

d Using the TVM solver on the calculator:

```

N=527.6097473
I%=6.48
PV=-220000
PMT=750
FV=0
P/Y=26
C/Y=26
PMT:END BEGIN

```

Term of the loan is 528 fortnights [M1]

$$\frac{528}{26} = 20.3 \text{ years} \quad [\text{A1}]$$

Question 3

$$\begin{aligned} \text{a } \$128 \times 52 \times 3 = \$19\,968 \\ \$19\,968 - \$16\,500 = \$3\,468 \quad [\text{A1}] \end{aligned}$$

$$\text{b } \frac{\$3\,468}{3} = \$1\,156 \text{ interest per year}$$

$$\frac{1156}{16500} \times 100 = 7.006\%$$

Flat rate of interest = 7.01% [H1]c Substituting $n = 156$ and*flat rate* = 7.01 in the formula for effective rate of interest

$$\text{effective rate} = \frac{2n}{n+1} \times \text{flat rate} \quad [\text{M1}]$$

gives

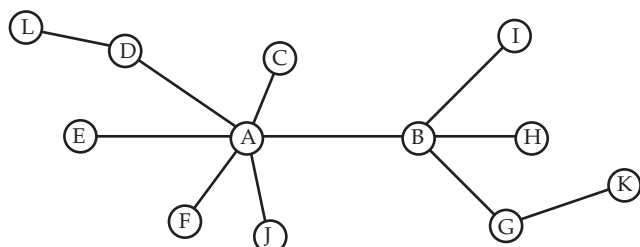
$$\begin{aligned} \text{effective rate} &= \frac{2 \times 156}{156 + 1} \times 7.01 \\ &= 13.93 \end{aligned}$$

Effective rate of interest is 13.9% [A1]**Total Module 4 = 15 marks**

Module 5 : Networks and decision mathematics

Question 1

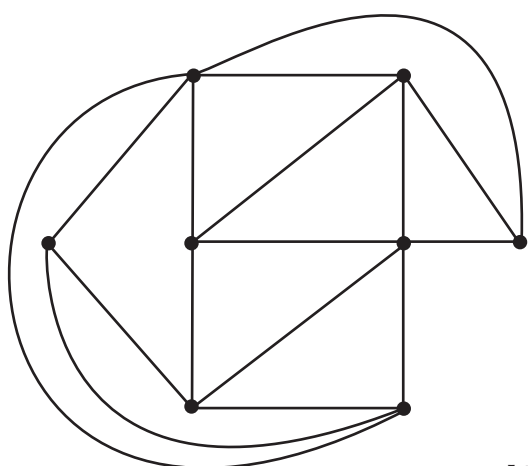
- a Set up servers A and B and add computers C-F, G-I.
Connect printers to A and G, scanner to D. [A2]



- b Output from computer H would travel via a pathway
H – B – G – K or H – B – A – J [A1]

Question 2

- a Move the connection between 1 and 6 outside 2.
Move the connection between 3 and 8 outside 7.
Move the connection between 1 and 8 to be outside.



- b This is a planar graph. [A1]

Question 3

- a Only vertices 3 or 7 are of odd degree. [A1]
- b If started with 3, end at 7, if started with 7 end at 3. [A1]
- c Starting at 3:
3 – 4 – 5 – 6 – 7 – 5 – 3 – 2 – 1 – 7
3 – 5 – 7 – 6 – 5 – 4 – 3 – 2 – 1 – 7
3 – 2 – 1 – 7 – 6 – 5 – 4 – 3 – 5 – 7
and other paths too
Starting at 7:
7 – 6 – 5 – 4 – 3 – 5 – 7 – 1 – 2 – 3
7 – 5 – 3 – 4 – 5 – 6 – 7 – 1 – 2 – 3
7 – 1 – 2 – 3 – 5 – 7 – 6 – 5 – 4 – 3
and other paths too [A1]

Question 4

a

Activity	Immediate Predecessor(s)
A	–
B	–
C	A
D	B
E	B
F	C,D
G	E,F
H	C,D
I	C,D
J	G,I

[A2]

- b Earliest is 14 weeks.
Enter latest completion time for each pathway.
Activity A: 2 weeks
Activity B: 3 weeks
Activity C: 6 weeks (via A)
Activity D: 7 weeks (via B).
Activity E: 5 weeks (via B)
Activity F: 8 weeks (via D)
Activity G: 11 weeks (via D and F)
Activity H: 11 weeks (via D)
Activity I: 8 weeks (via D)
Activity J: 14 weeks (via D, F and G) [A1]

c From above analysis the critical path is B – D – F – G – J [A1]

d From earliest completion time subtract the next critical path activity time and any activities leading from there.

Non Critical activities are A, C, E, H and I

$$\text{Float(H)} = 14 - 7 - 4 = 3 \text{ weeks}$$

$$\text{Float(I)} = 14 - 7 - 3 - 1 = 3 \text{ weeks}$$

$$\text{Float(E)} = 14 - 8 - 3 = 3 \text{ weeks}$$

$$\text{Float(C)} = 14 - 7 - 6 = 1 \text{ week}$$

$$\text{Float(A)} = 14 - 7 - 6 = 1 \text{ week} \quad [\text{A2}]$$

Total Module 5 = 15 marks