2003 Further Mathematics
Written Examination 2 (Analysis task)
Suggested answers and solutions

Core: Data analysis

Question 1

a The male column must sum to 3458; so the number of men with 'normal' blood pressure is 2184. **[A1]**

b The two variables are *blood pressure* and *gender*. It is more likely that *blood pressure* depends on *gender.*

Gender is the independent variable. **[A1]**

c **i** $\frac{1672}{6504} \times 100 = 25.7\%$ **[A1]**

ii $\frac{930}{1672} \times 100 = 55.6\%$ **[A1]**

d The percentage of men with normal blood pressure is $\frac{2184}{3458} \times 100 = 63.2\%$

The percentage of men with high blood pressure is $\frac{930}{3458} \times 100 = 26.9\%$ **[M1]**

e Low blood pressure appears to be related to gender; 40.5%of the females had low blood pressure compared to only 9.9% of the males.

Evidence that high blood pressure is related to gender is not conclusive: 26.9% of the males compared to 24.4% of the females had high blood pressure; a difference of only 2.5%.

Blood pressure in the normal range seems to be related to gender; more men, 63.2%, had normal blood pressure compared to the 35.1% of females.

Either statement **[M1]**
Sensible figures quoted **[H1]**

Question 2

a There is moderate, positive correlation between the variables. **[A1]**

(This means that as weight increases, diastolic blood pressure also increases.)

b Diastolic blood pressure

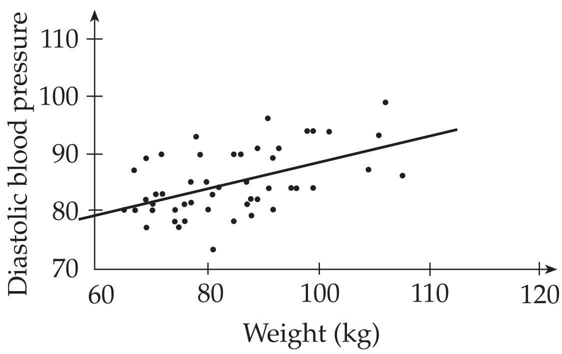
$= 66.4 + 0.22 \times 80$

$= 84$ (mm Hg) **[A1]**

c Another point (100, 88.4)

Plot both (80, 84) and (100, 88.4) then draw the line through these two points.

Any two points plotted **[M1]**
Correct line **[A1]**



[Line also goes through (60, 79.6) and (110, 90.6)]

d Slope = 0.22 **[A1]**

For each kilogram increase in weight the diastolic blood pressure increases by 0.22 (units). **[A1]**

e Coefficient of determination = r^2

$r^2 = 0.5114^2 = 0.2615$ **[A1]**

"26.2% of the variation in diastolic blood pressure can be explained by the variation in weight." **[H1]**

Total: 15 marks

Module 1: Number patterns and applications

Question 1

a 1 : 19 : 7 ratio gives a total number of shares of 1+19+7= 27

The swim of 800 metres has 1 share therefore the run leg with 7 shares is equivalent to 5600 metres

(7×800 m = 5600 m) **[A1]**

b The total distance of the race is 27×800m = 21 600 m = 21.6 km **[A1]**

c cycle leg = 12 km

run leg = 6 km (half the length of the cycle)

swim leg = 3 km (the balance of 21 km)

gives a ratio

swim : cycle : run

3 km : 12 km : 6 km **[A1]**

simplifies to

1 : 4 : 2 **[A1]**

Question 2

a Test for common ratio, $r = \frac{r_2}{r_1} = \frac{r_3}{r_2} = \frac{r_4}{r_3}$

Common ratio, $r\ \frac{r_2}{r_1} = \frac{6}{2} = 3$

and $\frac{r_3}{r_2} = \frac{18}{6} = 3$ **[M1]**

Hence it is a geometric sequence proven by the common ratio. **[A1]**

b If the common ratio = 3 **[A1]**

Then $r_4 = 3 \times r_3$

$= 3 \times 18 = 54$ **[A1]**

Checking $r_5 = 3 \times r_4$

$= 3 \times 54 = 162$

c $a = 2$ **[M1]**

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$S_8 = \frac{2\left(3^8 - 1\right)}{3 - 1}$$ **[M1]**

$S_8 = 6560$ **[A1]**

Question 3

a Net loss of fluids for each kilometre

$b = (100 - 120) = -20$

initial body fluid content after first kilometre

$t_1 = 7\,800$ **[A2]**

so $t_{n+1} = t_n - 20$, where $t_1 = 7800$

b The general form of a difference equation with $t_{n+1} = t_n - 20$; $t_1 = 7\,800$ is

$t_n = t_1 + (n-1)b$

and for $a = 1$ (arithmetic)

$b = -20$; $t_1 = 7\,800$ **[M1]**

$t_n = 7800 - 20(n-1)$

$t_{25} = 7800 - 20(25-1)$

$t_{25} = 7800 - 4800 = 3000$ ml **[A1]**

or ALTERNATIVELY an iteration technique using the difference equation to generate 25 terms e.g. 7800, 7780, 7760, 7740, 7720 ……

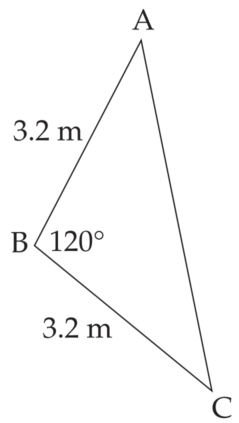
Therefore he should not continue the race in the hot conditions. **[A1]**

Total: 15 marks

Module 2: Geometry and trigonometry

Question 1

a



Using the cosine rule :

AC$^2 = 3.2^2 + 3.2^2 - 2 \times 3.2^2 \cos 120°$

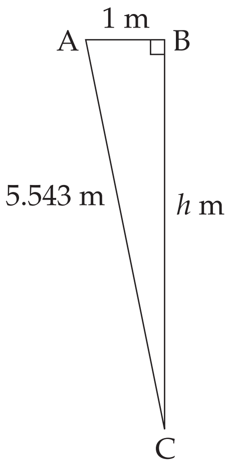
$= 30.75$ **[M1]**

AC = 5.543 m, correct to 3 decimal places.

b Area of Δ **ABC** $= \frac{1}{2} \times 3.2 \times 3.2 \times \sin 120°$ **[M1]**

$= 4.434$ m^2 **[A1]**

c



$h^2 = 5.543^2 - 1^2$ **[M1]**

$h = 5.45$; to 2 decimal places **[A1]**

d Area of a trapezium $= \frac{1}{2} \times h \times (\overrightarrow{AF} + \overrightarrow{CD})$

$= \frac{1}{2} \times 5.452 \times (7 + 5)$

[M1]

$= 32.7122$ **[A1]**

Total area of patio

$= 2 \times$ area of ΔABC + area of trapezium

$= 2 \times 4.434 + 32.712$

$= 41.6\ m^2$, to one decimal place. **[A1]**

Question 2

Ratio of lengths:

Scale drawing : Actual = 1 : 50

Ratio of areas:

Scale drawing : Actual $= 1^2 : 50^2$ **[M1]**

$= 1 : 2500$

Hence the area ratio equation:-

$25.2 : x = 1 : 2500$ **[A1]**

where x is the actual area of the sail.

$$\frac{x}{25.2} = \frac{2500}{1}$$

$$x = 2500 \times 25.2$$

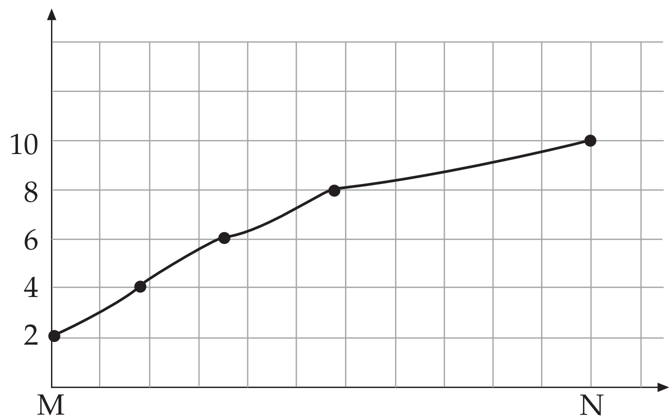
$$= 63\ 000\ cm^2$$

$63\ 000\ cm^2 = 6.3\ m^2$ **[A1]**

($1m^2 = 100cm \times 100cm = 10\ 000cm^2$)

Question 3

a



Scale **[A1]**

All correct **[A1]**

b Fencing$^2 = 42^2 + 8^2$ **[M1]**

Fencing = 42.755 metres

Need to allow 1 metre extra to account for the slope. **[A1]**

Total: 15 marks

Module 3: Graphs and relations

Question 1

a From the graph

For COST

200 runners at \$44000

400 runners at \$50000

$$gradient = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{50000 - 44000}{400 - 200} = \frac{6000}{200}$$

= \$30 per runner pair **[A1]**

For REVENUE

200 runners at \$20000

400 runners at \$50000

$$gradient = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{50000 - 20000}{400 - 200} = \frac{30000}{200}$$

= \$150 per runner pair **[A1]**

FUNCTION	Gradient	y-intercept
MANUFACTURING COST	\$ 30 per runner	\$38000
RETAIL REVENUE	\$150 per runner	–\$10000

[A1][A1]

b Using simultaneous equations show that the break even point is 400 runners per week.

Break even point is when

COST = REVENUE

30 × number of runners + 38000

= 150 × number of runners – 10000 **[M1]**

$$30x + 38000 = 150x - 10000$$

$$38000 + 10000 = 150x - 30x$$

$$48000 = 120x$$

$$x = 400$$

400 runners need to be manufactured and sold each week **[A1]**

c For 450 runners

COST = 30 × number of runners + 38000
= 30 × 450 + 38000
= 13500 +38000
= 51500

REVENUE = 150 × number of runners – 10000
= 150 × 450 – 10000
= 67500 – 10000
= 57500 **[M1]**

Profit = Revenue – Cost
= 57500 – 51500
= \$6000 **[A1]**

Question 2

a Since 2-person unit cost \$50 per night, the total cost for 2-person units would be $50x$

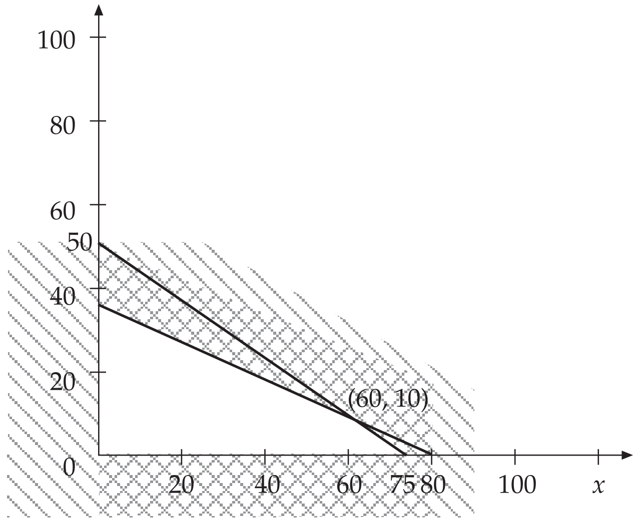
and for 3-person unit cabins the total cost is $100y$ with a total budget up to \$4000.

Therefore the budget constraint is

$50x + 100y \leq 4000$ **[A1]**

b $y \geq 0$ and $x \geq 0$ **[A1]**

c



for the $y \geq 0$ and $x \geq 0$ **[A1]**

for each of the two other constraints **[A2]**

d From the above graph, the three vertices are as follows

(0,40) NB: 0 units for 2 person and 40 units for 3 person (for 120 people and cost of \$4000)

(60,10) NB: 60 units for 2 person and 10 units for 3 person (for 150 people and cost of \$4000)

(75,0) NB: 75 units for 2 person and 0 units for 3 person (for 150 people and cost of \$3750)

for correct co-ordinates and **[A1]**

for correct costs. **[A1]**

Total: 15 marks

Module 4: Business related mathematics

Question 1

a \$50 000 + \$80 000 = \$130 000 **[A1]**

b 5% of \$80 000 = \$4 000 **[A1]**

c 10×4000 = \$40 000 interest

plus \$80 000 principal = \$120 000 **[A1]**

d Using the simple interest formula

$$I = \frac{Prt}{100} \Rightarrow 25000 = \frac{8000 \times r \times 5}{100}$$ **[M1]**

$r = 6.25\%$ **[A1]**

e \$120 000 – \$105 000 = \$15 000 **[A1]**

Question 2

a $n = 30 \times 12 = 360$ **[A1]**

$$R = 1 + \frac{\frac{6.5}{12}}{100} = 1.0054$$ **[A1]**

b Substituting correctly into the formula

$$0 = 130000 \times 1.0054^{360} - \frac{Q(1.0054^{360} - 1)}{1.0054 - 1}$$

[M1]

Solving to give $Q = \$819.98 \Rightarrow \$820$ to the nearest ten **[A1]**

Alternatively using the TVM solver

N=360

I%=6.5

PV=–130000

PMT (to be solved)

FV=0

P/Y=12

C/Y=12

PMT solves to give \$821.69 ⇒ \$820 to the nearest ten

(the answer differs slightly from the formula as that method used a rounded value of R)

c Using the TVM solver

N (to be solved)

I%=6.5

PV=–130000

PMT =900

FV =0

P/Y=12

C/Y=12 **[M1]**

N solves to give 282.32 monthly payments therefore he will make 283 payments in total. He will therefore save 77 payments. **[A1]**

Question 3

a $24 \times 125 - 2400 = \$600$ **[A1]**

b Using the formula

$$r_e = r_f \times \frac{2n}{n+1}$$

$$r_e = 12.5 \times \frac{2 \times 24}{24+1}$$ **[M1]**

$r_e = 24\%$ per annum **[A1]**

Total: 15 marks

Module 5: Networks and decision mathematics

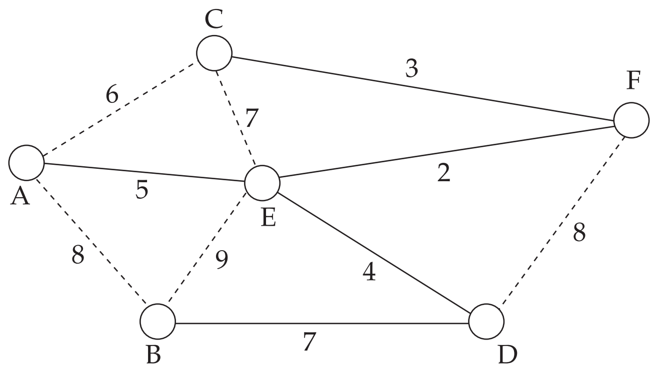
Question 1

a There are a number of possible routes. One typical answer being

$D \to B \to A \to C \to F \to E \to D$ **[A1]**

b A Hamiltonian circuit **[A1]**

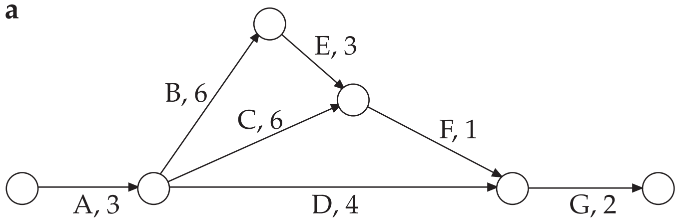
c **[A1]**



d 2+3+4+5+7 = 21 km **[A1]**

Question 2

a



E correct **[M1]**

F correct **[M1]**

(arrows must be included)

b Earliest start time for F is 12 **[A1]**

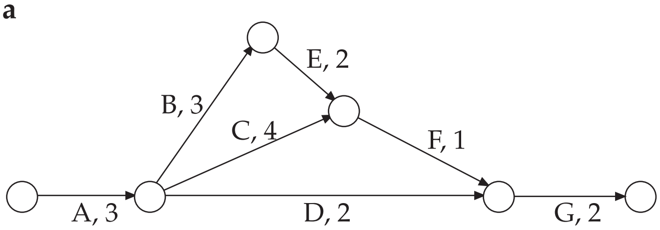
Latest start time for C is 6 **[A1]**

c Critical path $A \rightarrow B \rightarrow E \rightarrow F \rightarrow G$ **[A1]**

d 3+6+3+1+2 = 15 weeks **[A1]**

Question 3

a



Drawing up new network **[M1]**

3+3+2+1+2 = 11 weeks **[A1]**

b Cost of reducing by 3 weeks = \$12 000 **[A1]**

This is achieved by reducing E by 1 week and B by 2 weeks
i.e. $1 \times 2000 + 2 \times 5000$ **[M1]**

Activities to be reduced to save 4 weeks are E, B and C **[A1]**

B may be reduced by a further week to 3 but this would not be beneficial unless C is also reduced by 1 week otherwise C would take 1 more week than the combined totals of B and E.

Cost of reducing by 4 weeks = \$16 000

i.e. $1 \times 2000 + 2 \times 5000 + 1 \times 4000$

Total: 15 marks