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FURTHER MATHEMATICS

TRIAL EXAMINATION 2

(ANALYSIS TASK)

2003

Reading Time: 15 minutes
Writing time: 90 minutes

Instructions to students

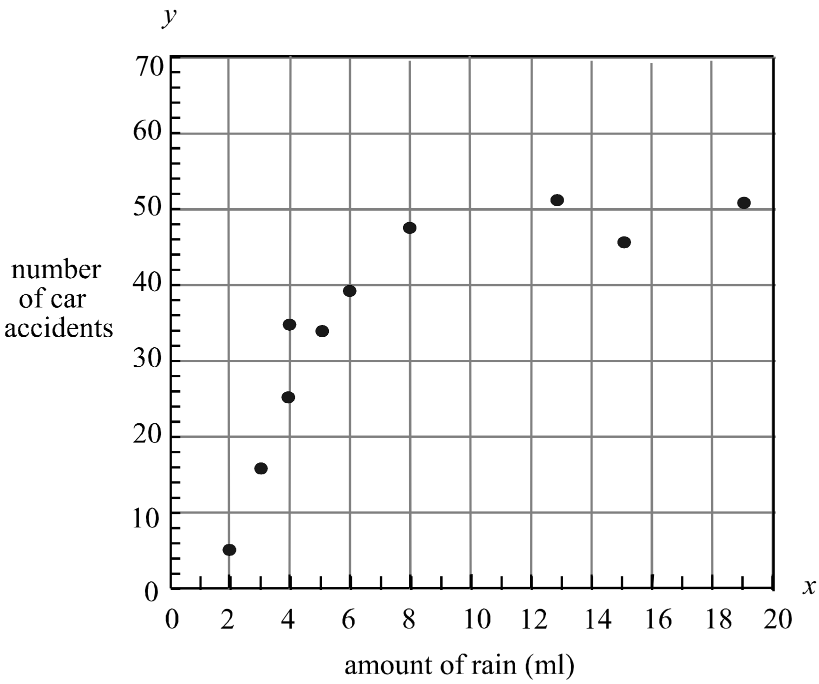
This exam consists of Section A and Section B.
Section A contains a set of extended answer questions from the core, "Data Analysis".
Section A is compulsory and is worth 15 marks.
Section B consists of 5 modules. You should choose 3 of these modules and answer every question in each of your chosen modules. Each of the modules is worth 15 marks.
There is a total of 60 marks available for this exam.
The marks allocated to each of the four questions are indicated throughout.
Students may bring up to two A4 pages of pre-written notes into the exam.

Section A - Core

This section is compulsory.

Question 1

The number of car accidents requiring a tow truck in metropolitan Melbourne on 10 weekdays is recorded against the amount of rain, which fell on each of the 10 respective days. The variable x represents the amount of rain, in ml, to have fallen in total on each day. The variable y represents the number of car accidents that occurred on the corresponding day.
A scatterplot of y against x is shown below



a. Which of the two variables is the independent variable. Explain your answer.

1 mark

b. Which one of the two variables could be described as discrete?

1 mark

In order to attempt to transform the data to linearity, a $\frac{1}{x}$ transformation was undertaken.

c. What other type of transformation would it have been appropriate to use in an attempt to transform the data to linearity?

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1 mark

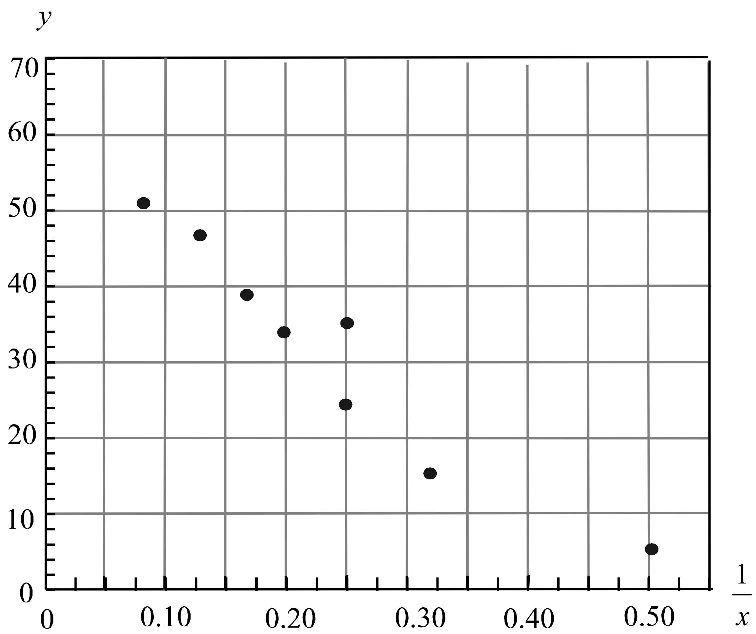
In the table below, the original data has been entered in the first row and the last row.

Amount of rain (x)	2	3	4	4	5	6	8	13	15	19
$\frac{1}{\text{Amount of rain}}$ $\left(\frac{1}{x}\right)$	0.50	0.33	0.25	0.25	0.20	0.17	0.13	0.08		
Number of accidents (y)	5	16	25	35	34	39	47	51	46	51

d. Complete the second row of the table above. Express your answers correct to 2 decimal places where appropriate.

1 mark

e. Plot the two points which involve the values you found in part **d.**, on the scatterplot below that shows y against $\frac{1}{x}$.



1 mark

The equation of the least squares regression line for this transformed data was calculated using data from the table and was found to be $y = -108.96 \times \frac{1}{x} + 57.02$.

f. According to this least squares regression line, write down how many accidents you would expect on a weekday in metropolitan Melbourne when there had been 12 ml of rain for the day.

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1 mark

g. A residual plot resulting from fitting the least squares regression line, given above, to the transformed data was undertaken. The residual plot is shown below with one point missing.

i. Find the residual value when $\frac{1}{x} = 0.17$. Express your answer correct to 2 decimal places.

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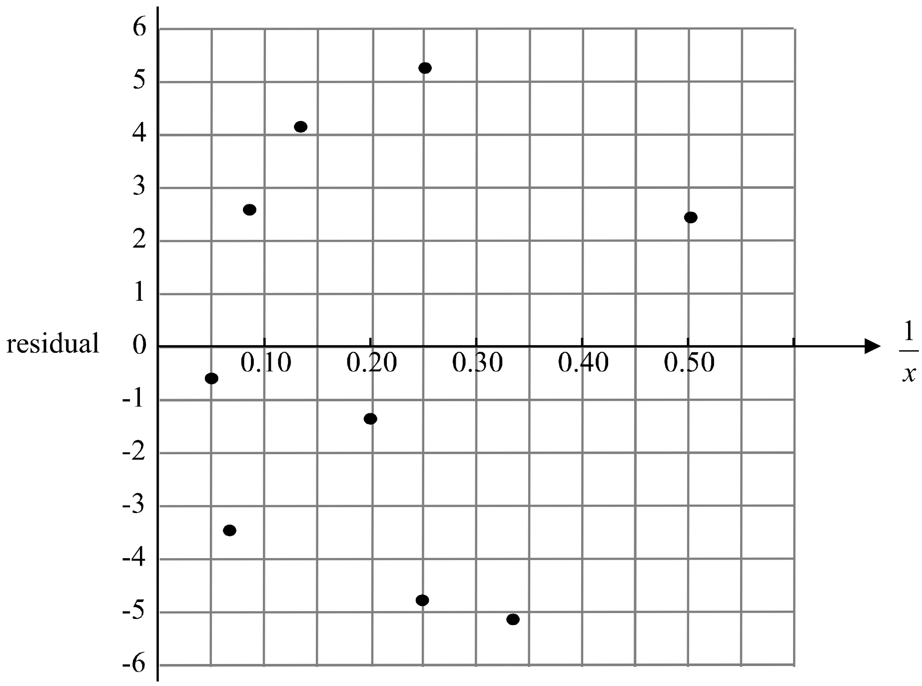
ii. Plot the missing point on the residual plot below.



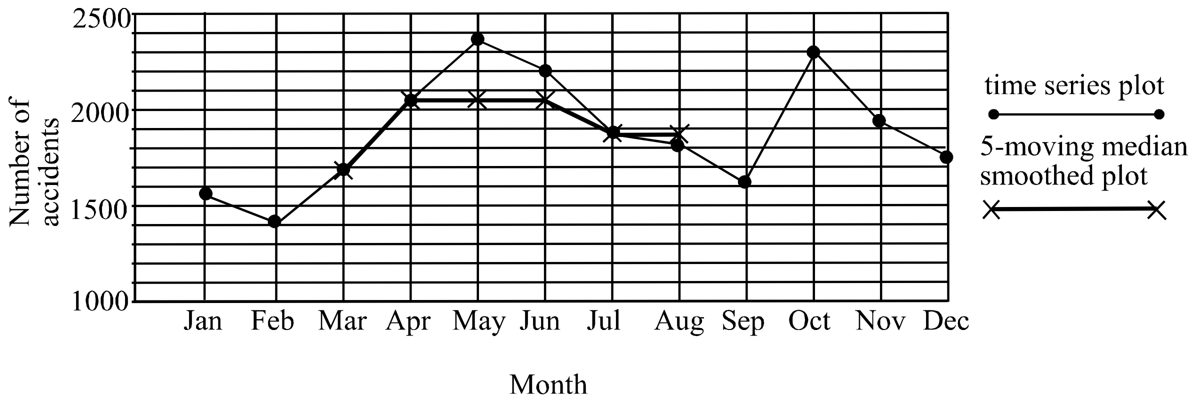
2 + 1 = 3 marks

h. Using the residual plot, state whether or not the $\frac{1}{x}$ transformation has been able to linearise the plot of the original data. Give a reason for your answer.

1 mark

Question 2

The number of car accidents per month that occur on a weekday and that require a tow truck in Metropolitan Melbourne is shown on the time series plot for a particular year.
In an attempt to smooth the data, a 5-moving median technique has been commenced for the data shown.



a. Complete the 5-moving median smoothed plot on the time series plot above.

2 marks

b. The table below shows the number of car accidents per quarter, over three consecutive years, that occurred on a weekday in metropolitan Melbourne and that required a tow truck.

Year	Number of Accidents				
	Summer	Autumn	Winter	Spring	Quarterly average
1	4607	5941	5541	6025	5529
2	4321	6282	5869	6722	
3	4742	6037	5416	6433	5657

i. Complete the table above by finding the quarterly average for the number of accidents in Year 2 to the nearest whole number.

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ii. Find the seasonal index for spring in Year 1 to 4 decimal places.

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iii. Hence find the overall seasonal index for spring for all three years to 4 decimal places.

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1 + 1 + 1 = 3 marks
Total 15 marks

Section B

Module 1: Number patterns and applications

If you choose this module then you must attempt **all** questions in it.

A farmhouse obtains its water from two sources. It has a household tank, which contains only rainwater, and a back-up tank, which contains water, pumped from a nearby river.

Question 1

During the drought, it was decided to ration the amount of water taken from the household tank each day to just 10 litres, which was to be used for drinking and cooking. At the start of day 1 of rationing the household tank contained 4500 litres of water.

a. Assuming no rain fell, how much water was left in the tank at the start of day 5 of rationing?

1 mark

b. Write down an equation relating A_n and n where A_n represents the amount of water, in litres, in the tank at the start of the n th day.

1 mark

c. Assuming no rain fell, at the start of which day of rationing would the amount of water in the household tank be reduced to 3000 litres?

1 mark

Question 2

When the household tank contained 4500 litres of water, the space inside the tank taken up by water compared to the space taken up by air was given by 2 : 11.
What is the capacity of the household tank?

2 marks

Question 3

Water in the back-up tank is used for household purposes other than drinking and cooking. Due to a ban on the pumping of water from the river because of the drought, the water in the back-up tank is also getting desperately low.
A decision is taken to ration the water taken from this tank. On the first day of rationing 150L of water can be used. If no rain falls, then the amount of water used from this tank on each subsequent day is to be just 95% of the previous days amount.

a. Assuming no rain falls, how much water can be used from the back-up tank on the tenth day of rationing. (Express your answer in litres correct to 1 decimal place).

1 mark

b. Assuming no rain falls, how much water would be used from this back-up tank after 20 days of rationing? (Express your answer to the nearest litre.)

2 marks

c. Assuming no rain falls and given that this back-up tank contained 3700 litres when the rationing began, explain why this back-up tank would never run out of water with this rationing in place.

3 marks

Question 4

After a further period of no rain, when the back-up tank contained just 1400 litres, a decision was made to buy water. The previous rationing described in **Question 3** was abandoned. Each week, water was trucked in and added to the back-up tank. A weekly ration of 980 litres only was to be used from this tank. Let A_n represent the amount of water in litres, in the back-up tank at the start of the *n th* week of this rationing program. The difference equation relating A_n and A_{n+1} is given by

$$A_{n+1} = 2A_n - 980 \qquad A_1 = 1400$$

a. Explain, in terms of A_n, how much water is added to the tank each week.

1 mark

b. **i.** For the difference equation $A_{n+1} = aA_n + b$, A_n may be expressed in terms of n according to the rule

$$A_n = A_1 a^{n-1} + b\frac{\left(a^{n-1} - 1\right)}{a - 1}.$$

Use this to show that $A_n = 420 \times 2^{n-1} + 980$.

2 marks

ii. Verify that $A_n = 420 \times 2^{n-1} + 980$ for $n = 1$ to $n = 4$.

1 mark

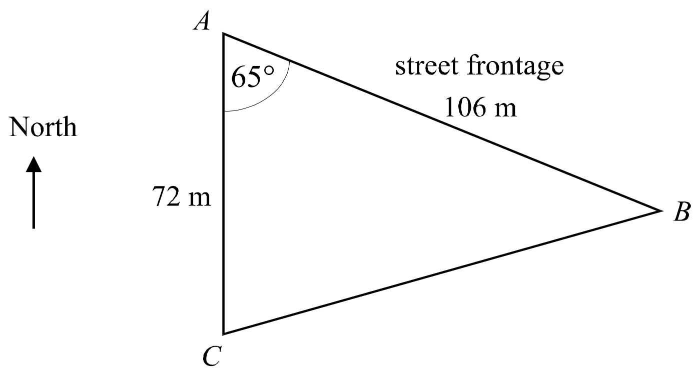
Total 15 marks

Module 2: Geometry and trigonometry

If you choose this module then you must attempt **all** questions in it.

Question 1

In 1958 Emilio bought a triangular shaped block of land shown on the diagram below. The boundary *AC* runs in a north-south line and is 72 metres long. The street frontage, indicated by *AB* is 106 metres long and angle *BAC* is 65°.



a. Find the length of the boundary *BC* correct to the nearest metre.

2 marks

b. Hence find the angle *ABC*. Express your answer to the nearest minute.

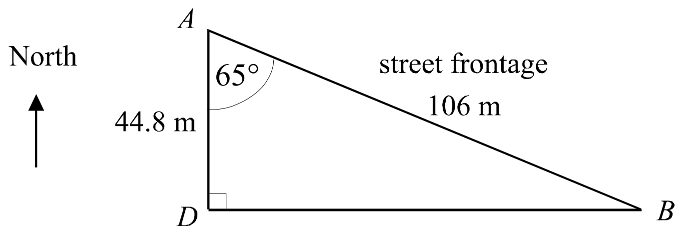
2 marks

c. Find the area of Emilio's block of land to the nearest square metre.

2 marks

Question 2

In 1974, Emilio subdivided his land and sold the back section of it to his neighbour. The boundary running in a north-south line was reduced in length to 44.8 metres and the new boundary *BD* ran in an east-west line. Emilio's remaining block of land is shown in the diagram below.



a. Find the length of the new boundary *BD* to the nearest metre.

1 mark

b. Find the bearing of *A* from *B*, correct to the nearest degree.

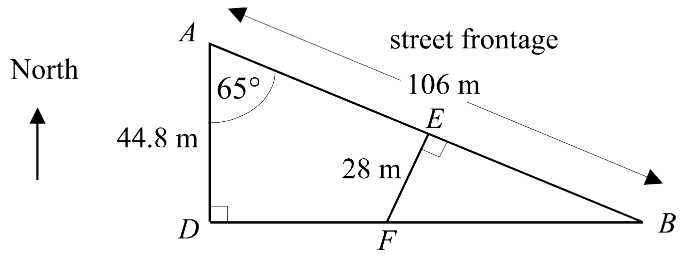
2 marks

c. On a scale diagram of Emilio's newly subdivided block of land, the area of the land measured $5376\ \text{cm}^2$. What was the ratio of the area of the land on the diagram compared to the actual area of the land?

2 marks

Question 3

In 2001 Emilio decided to further subdivide his remaining land so that his daughter Teresa could build a unit there.
The line of subdivision, indicated on the graph below as *EF*, was 28 metres long and ran at right angles to the street frontage *AB*.



a. Using your answer to **Question 2** part **a.** or otherwise, find the length of Emilio's new street frontage, that is *AE*. Justify your answer.

3 marks

b. Without evaluating any areas, find the ratio of the area of Teresa's block of land (indicated by ΔBEF), to the area of Emilio's block of land before it was subdivided (indicated by ΔABD).

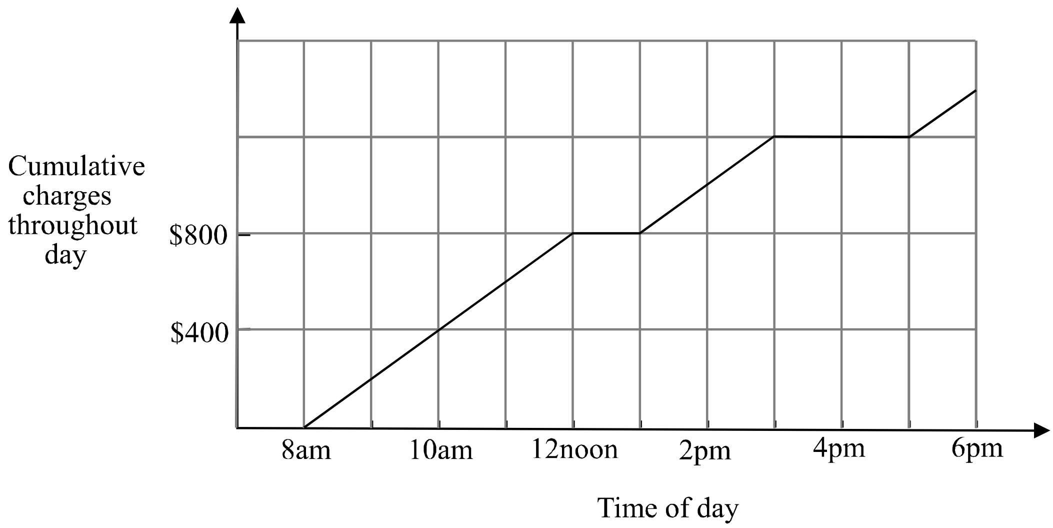
1 mark
Total 15 marks

Module 3: Graphs and relations

If you choose this module then you must attempt **all** questions in it.

Question 1

A junior solicitor in a large firm charges his time out at $200 per hour. A graph showing the amount he has charged out during the course of a day is shown below.



The junior solicitor had a lunch break and attended a seminar during the day. For the rest of the day he was charging out his time.

a. Write down the likely times during which he attended the seminar.

1 mark

b. How much did the junior solicitor charge out in total for his day's work?

1 mark

c. If the junior solicitor had not had a lunch break, had not attended the seminar but had worked continuously and had started and finished at the same time, how much would he have charged out for the day?

1 mark

d. Explain why those straight line segments of the graph (other than the horizontal ones) have the same gradient.

1 mark

Question 2

In the same firm a small department is made up of a junior partner and his assistant. Together they spend a maximum of 100 hours and a minimum of 50 hours a week doing corporate work.
Let x represent the number of hours a week that the assistant spends on corporate work and let y represent the number of hours a week that the junior partner spends on corporate work.
The first two constraints can then be given by

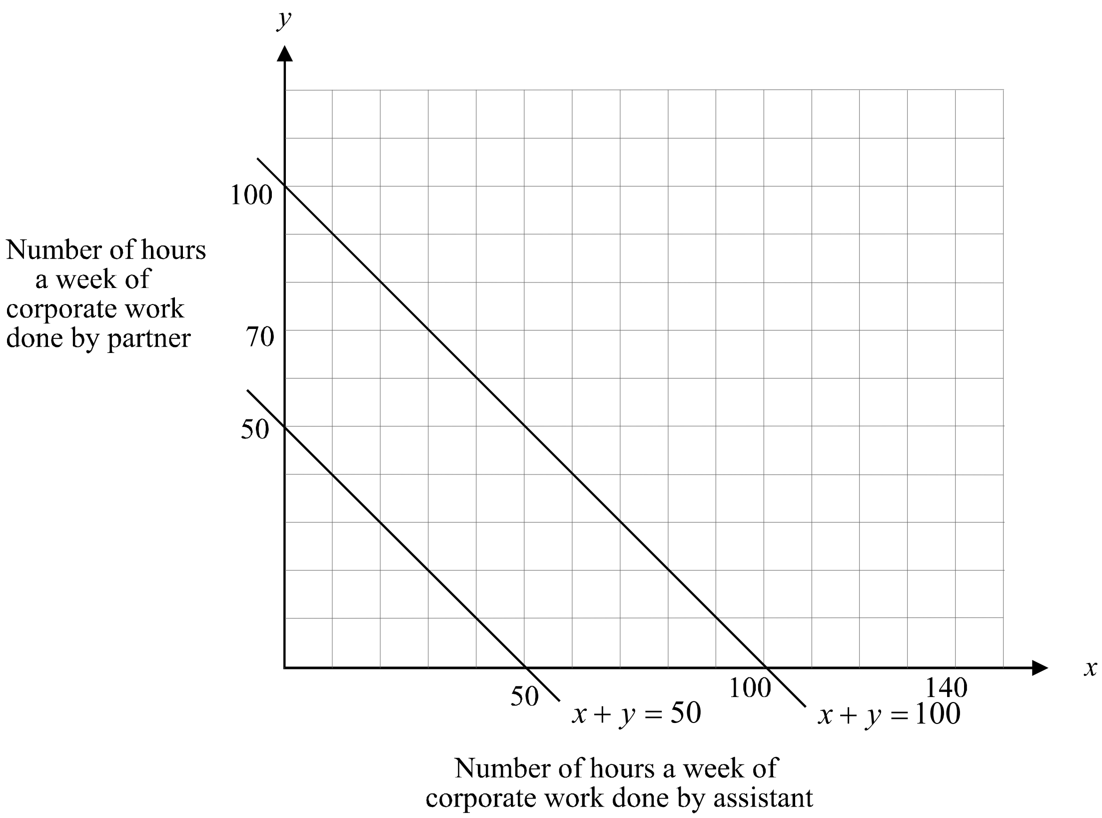
$$x + y \leq 100$$

$$x + y \geq 50$$

Resources available to this department, for example secretarial and computing, provide another constraint, which is given by

$$x + 2y \leq 140$$

The corresponding equations of the first two constraints are shown on the diagram below.



a. The two lines shown on the graph have a feature in common.

i. State what this feature is.

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ii. Write down the value of the feature that they have in common.

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1 + 1 = 2 marks

b. On the same diagram:

i. sketch the equation $x + 2y = 140$.

ii. shade the feasible region.

1 + 1 = 2 marks

c. Find the solution to the system of equations given by

$$x + y = 100$$
$$x + 2y = 140$$

1 mark

d. The junior partner charges $250 per hour and the assistant charges $200 per hour.

i. Write down an expression for *C*, the amount charged, in terms of *x* and *y*.

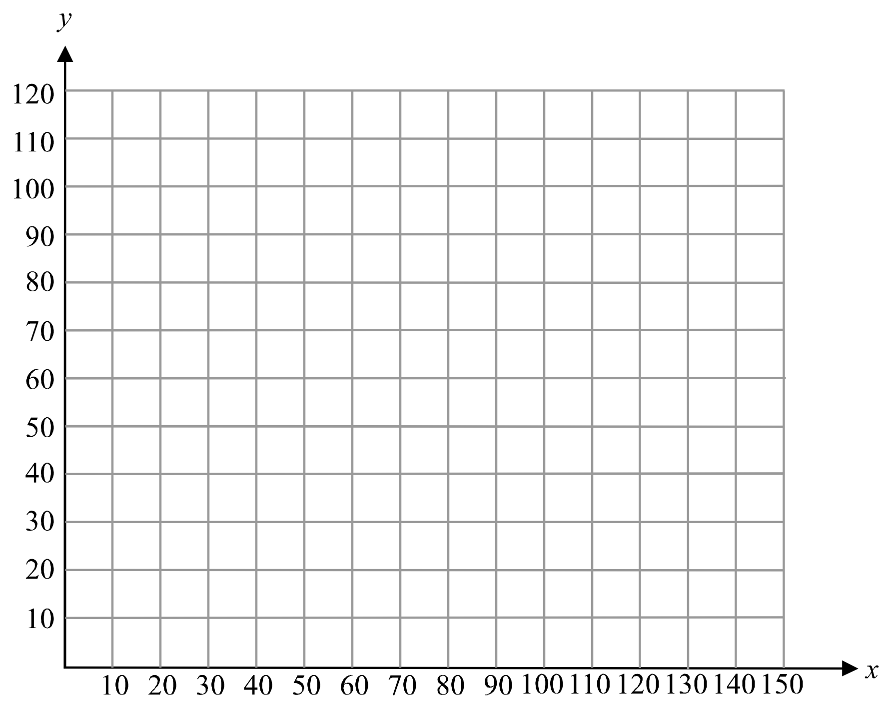
ii. Find the maximum amount that this department can charge for the corporate work that it does in a week.

1 + 2 = 3 marks

The junior partner and his assistant each renegotiate their contracts with the firm. The key features of the new contracts are as follows:

- The junior partner and his assistant will together spend a maximum of 100 hours a week on corporate work.
- The junior partner will spend a maximum of 70 hours a week on corporate work.
- The assistant will spend a maximum amount of 70 hours a week on corporate work.
- The constraint involving resources such as secretarial and computing will be removed.
- The constraint involving the minimum number of hours worked will be removed.
- The expression for *C* is now $C = 210x + 280y$.

e. Use the set of axes below to assist in explaining whether or not the maximum amount that the department can charge will be increased or decreased as a result of these changes.



3 marks
Total 15 marks

Module 4: Business-related mathematics

If you choose this module then you must attempt **all** questions in it.

Question 1

When Peter was born, his grandparents invested $1000 for him in a fixed interest bank account offering compound interest of 6% compounding annually. The account had no further deposits or withdrawals made.

a. What was the amount in the account when Peter turned 18 years of age?

1 mark

b. If Peter's grandparents had invested the $1000 in an account which earned 6% per annum simple interest, what would the investment have been worth after 18 years?

1 mark

c. If Peter's grandparents had invested the $1000 in an account, which earned 6% per annum compound interest, compounding monthly, what would the investment have been worth after 18 years?

2 marks

d. Peter's sister also had $1000 invested on her behalf by her grandparents when she was born. The account earned compound interest fixed at 6% per annum compounding quarterly and had no further deposits or withdrawals made to it. After how many years was the amount in this account equal to $2168.87?

2 marks

Question 2

When Peter finished studying he began work in his father's business and was assigned a company car, which was bought for $32 000.

a. This company car is depreciated according to the flat rate method. Each year the car depreciated by 16% of the amount it was purchased for. After how many years does the car have a book value of $0?

1 mark

b. Consider the case where the company car was to be depreciated according to the unit cost method, at the rate of 21 cents per kilometre driven and assume that Peter drove, on average, 20 000 kilometres a year.

i. How many years would it take for the car to have a book value of $0? Express your answer correct to 2 decimal places.

1 mark

ii. What annual percentage flat rate of depreciation does this unit cost method represent?

1 mark

Question 3

Peter takes out a reducing balance loan of $50 000 on 1st March 2003. He makes monthly repayments of $400. The interest charged on the loan is 7.8% per annum and is calculated at the end of each month.
The table below shows the statement for Peter's loan account for the first two months of the loan.

Date	Deposit	Interest Charged	Amount owing
1-3-03			50 000
10-3-03	400		49 600
31-3-03			
10-4-03	400		
30-4-03			

a. Complete the 4 missing entries, indicated by the shaded boxes, in the table above.

2 marks

b. The annuities formula is given by

$$A = PR^n - \frac{Q(R^n - 1)}{R - 1}$$

If this formula were applied to Peter's loan then

i. write down the value of Q.

ii. write down the value of R.

1 + 1 = 2 marks

c. After how many years will Peter have repaid the loan? Express your answer correct to 1 decimal place.

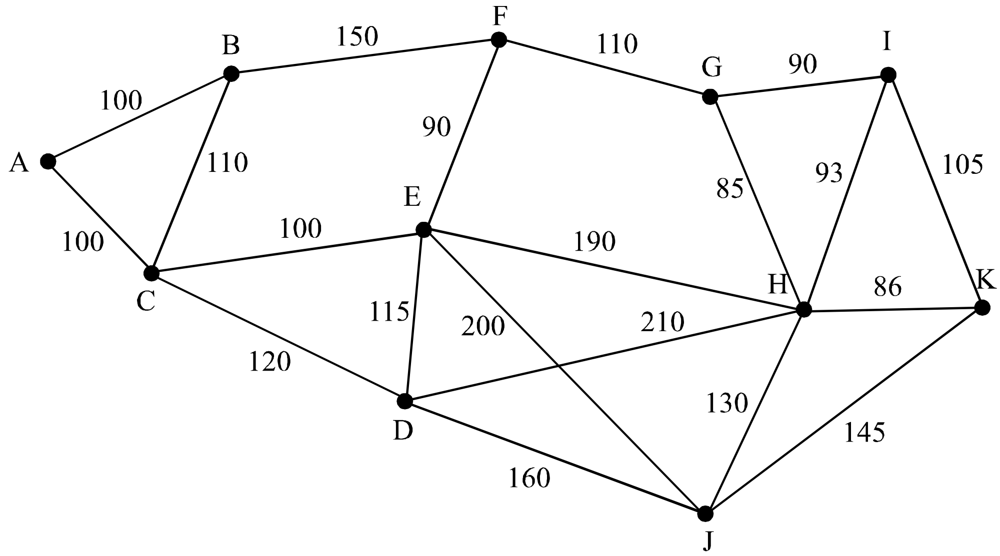
2 marks
Total 15 marks

Module 5: Networks and decision mathematics

If you choose this module then you must attempt **all** questions in it.

Question 1

An historical society has a walking tour of historical sites around town. The map below shows the 11 sites, $A - K$ and the walking tracks between them. The distances along the walking tracks are indicated in metres.



a. Explain whether or not the map above could be described as a planar graph.

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1 mark

b. Find the length of the shortest path from A to H.

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1 mark

c. Explain whether or not a person can start at site A and visit each site just once before returning to site A.

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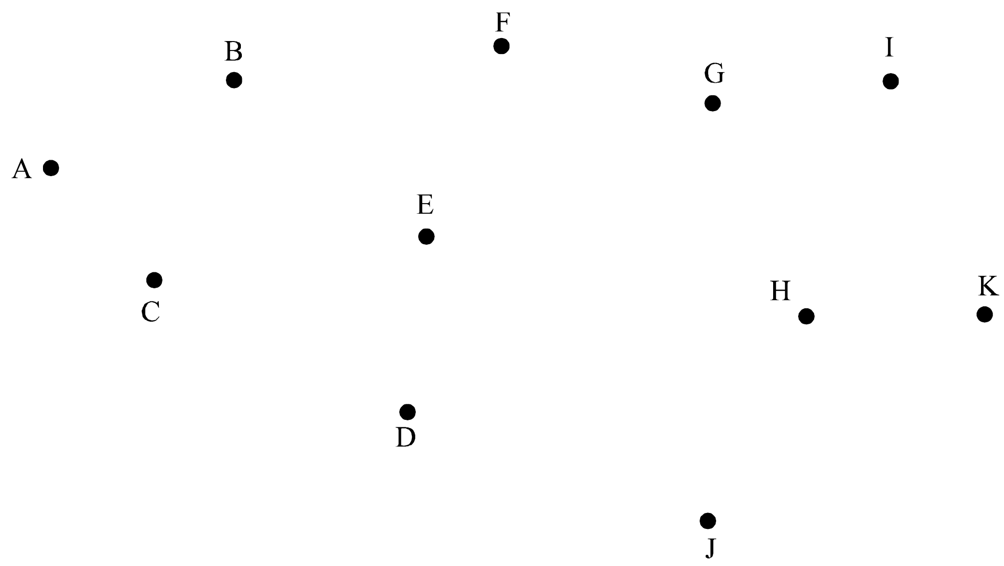
1 mark

d. Explain whether or not a person can start at site A and walk along each of the walking tracks exactly once before returning to *A*.

1 mark

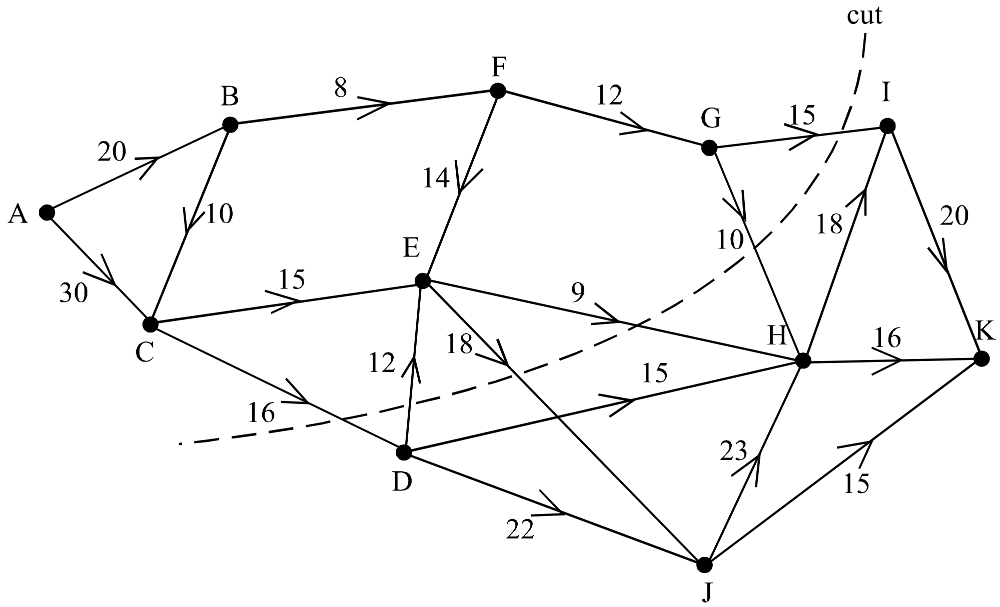
e. To minimise the upkeep of the walking tracks it has been proposed that some tracks be eliminated.
Use the diagram below to draw a minimum spanning tree and hence find the minimum length of walking tracks needed so that each site can still be accessed.



2 marks

Question 2

The historical society offers a special tour package to schools. This package has all students starting at site A and finishing at site K but not visiting every site. Different students visit different sites.
For health and safety reasons the maximum number of students and staff on each of the walking tracks is indicated on the network below. A cut is shown on this network.



a. Write down the capacity of the cut shown on this network.

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1 mark

b. What is the maximum number of students and staff who can do this special tour package offered by the historical society?

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2 marks

Question 3

The historical society has been offered another property of historical significance. In order to purchase the property, a series of steps involving local and state government, banks, insurance companies, solicitors and so on needs to be completed. The list of steps labeled A – K together with the expected time for their completion and the order in which they should proceed are given on the network below.

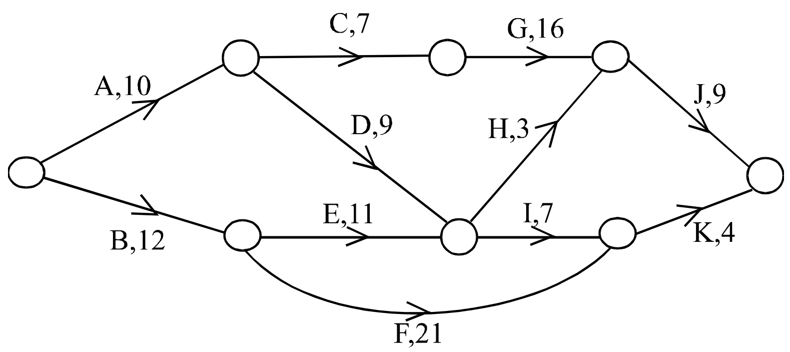

Table 1 below shows the list of steps to be taken together with the expected time taken for each and the immediate predecessor required for each step.

Activity	Expected time taken (days)	Immediate predecessor
A	10	-
B	12	-
C	7	A
D	9	A
E	11	
F	21	B
G	16	C
H	3	
I	7	D,E
J	9	G,H
K	4	F,I

Table 1

a. Complete the shaded cells in Table 1.

2 marks

b. Use the information shown in Table 1 to complete Table 2 below which shows the earliest and latest start times for each step which must be taken.

Activity	Earliest start time	Latest start time
A	0	0
B	0	5
C	10	10
D	10	
E	12	19
F	12	17
G	17	17
H	23	30
I	23	31
J	33	33
K		38

Table 2

2 marks

c. What is the shortest period of time that all the steps can be completed?

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1 mark

d. Name a step, which, if it were to be reduced in time taken, would shorten the time taken for all the steps to be completed. Explain your answer.

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1 mark
Total 15 marks

