

MAV Further Mathematics Examination 2

Answers & Solutions

Solutions

Core : Data analysis

Question 1

- a. 5-number summary:

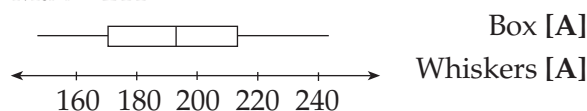
Min = 153

Lower quartile = 174

Median = 196

Upper quartile = 217

Max = 244



- b. i Shape : Symmetrical [A]

ii Centre : Attendance figures are centred around the median, 196. [H]

iii Spread : Attendance figures are spread over 91 values (range) [H]

OR The middle 50% of attendances are spread over 43 values (IQR)

Question 2

- a. i Positive correlation. [A]

ii As **maximum daily temperature** increases the **attendance** also increases. [A]

- b. **Attendance** = $79.3 + 4.01 \times \text{max. daily temp.}$
Variables correct [A]
Figures correct [A]

- c. "increases by 4 " [A]

- d. 20°:
Attendance = $79.3 + 4.01 \times 20$
 ≈ 160 [H]

35°:
Attendance = $79.3 + 4.01 \times 35$
 ≈ 220 [A]

Correctly plotted line.
(20,160); (35, 220) or sensible points [H]

- e. i. $r^2 = (0.884)^2 \approx 0.781$ [A]

ii "78.1% of the variation in **attendance** can be explained by the variation in **maximum daily temperature**." (sensible) [H]
Total 15 marks

Module 1 : Number patterns and applications

Question 1

- a. 13,15 or a total of 28 seats [A]

The pattern goes up by +2 each time, that is 7, 9, 11, 13, 15

- b. Common difference,
 $d = t_2 - t_1 = t_3 - t_2 = t_4 - t_3$
Common difference
 $= t_2 - t_1 = 9 - 7 = +2$
 $= t_3 - t_2 = 11 - 9 = +2$
 $= t_4 - t_3 = 13 - 11 = +2$ and so on [A]

- c. Arithmetic sequence; substitute first term, $a = 7$, common difference, $d = 2$ and $n = 25$ into

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{25}{2}(2 \times 7 + (25-1)2)$$
 [M]

$$S_{25} = 12.5(14 + 48) = 775$$

Total of middle section is 775 seats.
Hence total seating capacity for the dress circle is

$$3 \times 775 = 2\,325 \text{ dress circle seats} \quad [A]$$

- d. From the ratio 25 shares = 2 325 seats
therefore
1 share = $2\,325 \div 25 = 93$ seats [M]
total seats = $(10 + 25) \times 93 = 35$ shares [M]
= $35 \times 93 = 3255$ seats [A]
- Or alternatively
Upper circle = 10 shares
= $10 \times 93 = 930$ seats
Total seats = lower + upper
= $2325 + 930 = 3\,255$ seats in total

Question 2

- a. Common ratio, $r = \frac{r_2}{r_1} = \frac{r_3}{r_2} = \frac{r_4}{r_3}$ if geometric
- $$\frac{r_2}{r_1} = \frac{27000}{30000} = 0.9$$
- $$\frac{r_3}{r_2} = \frac{24300}{27000} = 0.9$$
- $$\frac{r_4}{r_3} = \frac{21870}{24300} = 0.9$$
- and so on $\Rightarrow r = 0.9$ [A]
- b. Common ratio $r = 0.9$ is 90% of previous weekly sales or $(100\% - 90\%)$ a 10% weekly decrease [A]
- c. Geometric sequence $a = 30\,000$, $r = 0.9$ and $t_n = 7\,000$
- $$t_n = ar^{n-1}$$
- $$7000 = 30000 \times 0.9^{n-1}$$
- [M]
- $$\frac{7000}{30000} = 0.9^{n-1}$$
- $$(n-1) \log 0.9 = \log \frac{7}{30}$$
- $$n-1 = \frac{\log_e(7 \div 30)}{\log_e 0.9}$$
- $$n-1 = 13.812..$$
- $$n = 14.812..$$
- Therefore it will take 15 weeks before the total weekly sales drops below 7000. [A]

Or use Trial & Error Technique

$$t_n = ar^{n-1}$$

$$7000 = 30000 \times 0.9^{n-1} \quad \text{Or [M]}$$

$$\frac{7000}{30000} = 0.9^{n-1}$$

$$0.9^{n-1} = 0.23333...$$

try $n = 10$
 $0.9^{10-1} = 0.9^9 = 0.387..$
 which is greater than 0.23333
 $n = 16$
 $0.9^{16-1} = 0.9^{15} = 0.205..$ which is less than 0.23333
 $n = 14$
 $0.9^{14-1} = 0.9^{13} = 0.254..$ which is greater than 0.23333
 $n = 15$
 $0.9^{15-1} = 0.9^{14} = 0.228..$ which is just less than 0.23333

Question 3

- a. $a = 100\% - 5\% = 95\%$ or 0.95
 $b = +50$ [A]
- b. The general form of a difference equation of the form $t_{n+1} = at_n + b$; $t_1 = 30\,000$ is
- $$t_n = a^{n-1}t_1 + \frac{b(1-a^{n-1})}{1-a} \text{ and for}$$
- $$a = 0.95$$
- $$b = +50; \quad t_1 = 30\,000$$
- $$t_n = 0.95^{n-1} \times 30000 + \frac{50(1-0.95^{n-1})}{1-0.95} \quad \text{[M]}$$
- $$t_{30} = 0.95^{30-1} \times 30000 + \frac{50(1-0.95^{30-1})}{1-0.95}$$
- $$t_{30} = 7552.13068.. \text{ which is greater than } 7000 \quad \text{[M]}$$

or ALTERNATIVELY an iteration technique using the difference equation to generate 30 terms e.g. 30000, 28550, 27172.5, 25863.875, 24620.68125

Therefore the show will still be above the 7000 limit [A]

Total 15 marks

Module 2 : Geometry and trigonometry

Question 1

a. $DB^2 = 33^2 + 30^2$ [M]

$$DB = \sqrt{1989}$$

$$\approx 44.60\text{m} \quad [A]$$

b. i. $\tan(\angle ABD) = \frac{33}{30}$ [M]
 $\angle ABD = 47.73^\circ$ [A]

(Correct to two decimal places)

ii. $\angle DBC = 125^\circ - \angle ABD$ [A]
 $= 125^\circ - 47.73^\circ$
 $= 77.27^\circ$

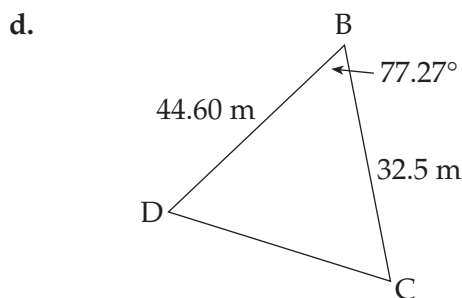
(Correct to two decimal places)

c. Right-angled triangle :

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 30 \times 33 \quad [M]$$

$$= 495 \text{ m}^2 \quad [A]$$



$$\text{Area} = \frac{1}{2} dc \sin B$$

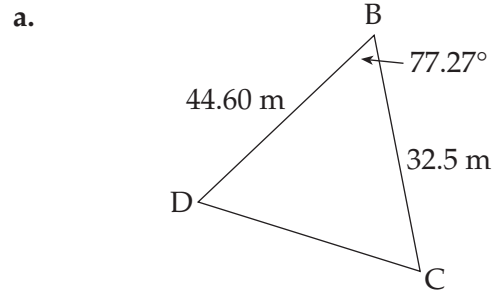
$$= \frac{1}{2} \times 32.5 \times 44.6 \sin 77.27^\circ \quad [M]$$

$$= 706.94 \text{ m}^2 \quad [A]$$

(Correct to two decimal places)

e. $706.94 + 495 = 1201.94\text{m}^2$
The estate agent has underestimated the area of the block by 1.94m^2
(2m^2 acceptable) [H]

Question 2

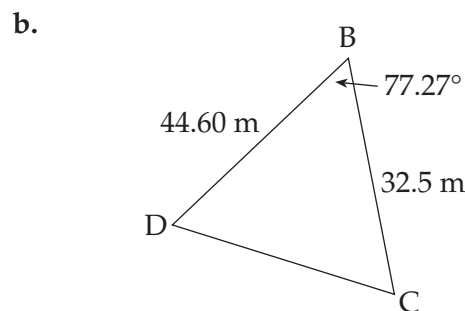


Using cosine rule :

$$DC^2 = 44.6^2 + 32.5^2 - 2 \times 44.6 \times 32.5 \cos 77.27^\circ \quad [M]$$

$$DC = 49.06\text{m} \quad [A]$$

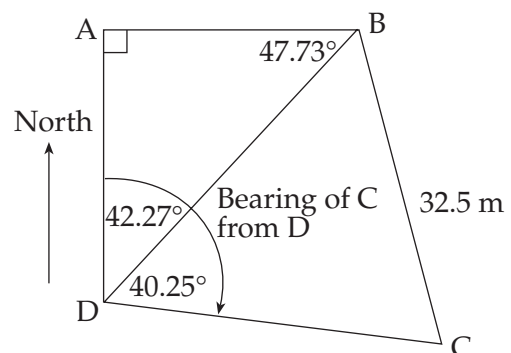
(Correct to two decimal places)



$$\frac{32.5}{\sin \angle BDC} = \frac{49.06}{\sin 77.27^\circ}$$

$$\sin \angle BDC = \frac{32.5 \times \sin 77.27^\circ}{49.06} \quad [M]$$

$$\angle BDC = 40.25^\circ \approx 40^\circ \quad [A]$$



$$\text{Bearing} = (42.27 + 40.25)^\circ \text{T}$$

$$= 82.52^\circ \text{T}$$

$$\approx 083^\circ \text{T} \quad [A]$$

(082°T acceptable)

Total 15 marks

Module 3 : Graphs and relations**Question 1**

- a. X-axis label – Distance from Speakers (metres) [A]

b.

Distance from Speakers, x (metres)	5	10	20	40
$\frac{1}{x^2}$	0.04	0.01	0.0025	0.000625
Sound Intensity, I	0.1	0.025	0.00625	0.001563

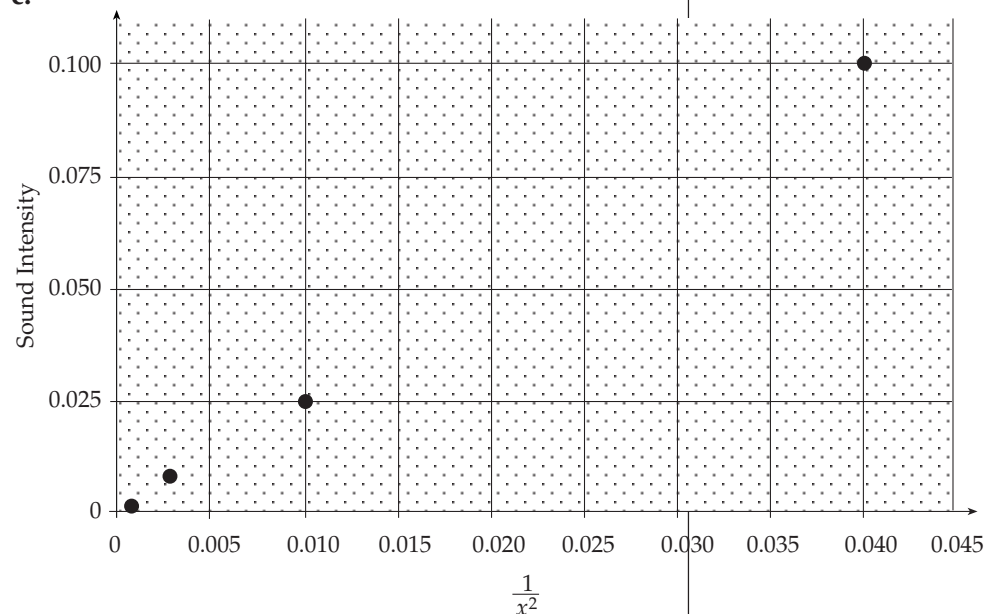
[A]

- d. Yes as it forms a straight line [A]
- e. Gradient of line using the two points (0.01,0.025) and (0.04,0.1)

$$k = \frac{y_2 - y_1}{x_2 - x_1}$$

$$k = \frac{0.1 - 0.025}{0.04 - 0.01} = \frac{0.075}{0.03} = 2.5 \quad [\text{M}] [\text{A}]$$

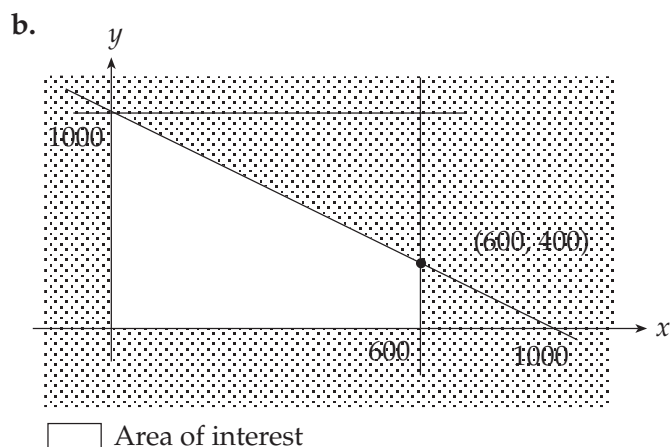
c.



[A][A]

Question 2

- a. Constraint 4 $x \leq 600$ [A]
 Constraint 5 $x + y \leq 1000$ [A]



- [M]
 Correctly drawing
 $x = 600$ and $y = 1000$ [A]
 Correctly drawing $x + y = 1000$ [A]
 c. $Z = 10x + 5y$ [A]
 d. Maximum value of objective function occurs at (600,400) [A]
 Maximum ticket sales = \$8000 [A]
 They must sell 600 front section tickets and 400 rear section tickets in order to achieve this maximum ticket sales revenue. [A]

Total 15 marks**Module 4 : Business related mathematics****Question 1**

- a. $\$30\,000 - \$20\,000 = \$10\,000$ [A]
 b. Investment A : \$750 (since simple interest) [A]
 Investment B : \$749 (7% of \$10700) [A]
 c. Points plotted at (2, 11500) and (2, 11449) [M]
 d. Investment A drawn as a straight line through the points plotted [M]
 Investment B drawn as an upward curve through the points plotted [M]
 e. Year 3 [A]
 f. Using a trial method or by setting up 2 equations on the calculator [M]
 e.g. on the TI – 83
 $y_1 = 10000 + 750x$
 $y_2 = 10000 \times 1.07^x$
 After year 7 both investments have reached \$15000 [A]
 (this can be found from the table of values on the calculator)

Question 2

- a. $n = 25 \times 12 = 300$ [A]
 $R = 1 + \frac{6}{\frac{12}{100}} = 1.005$ [A]
 b. Substituting all the correct values into the formula
 $0 = 180000 \times 1.005^{300} - \frac{Q(1.005^{300} - 1)}{1.005 - 1}$ [M]
 Solving to give $Q = \$1159.74$ [A]
 This could be done using the TVM solver on the TI-83 as follows
 $N = 300$
 $I\% = 6$
 $PV = -180000$
 PMT (to be solved)
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$

- c. $300 \times 1159.74 - 180000 = \167922 [A]
 d. The extra interest paid because of the higher principal would mean that Bob would pay more to the bank overall. [A]

Total 15 marks

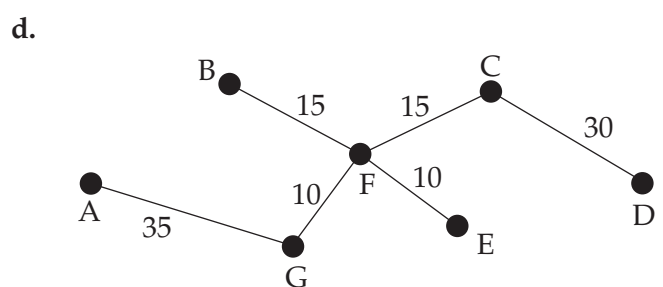
Module 5 : Networks and decision mathematics

Question 1

- a. It is a connected graph with all vertices having an even degree [A]

- b. [A]
 $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow C \rightarrow F \rightarrow E \rightarrow G \rightarrow F \rightarrow B \rightarrow G \rightarrow A$
 other answers possible

- c. $A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F \rightarrow G \rightarrow A$ [A]



[M]

- e. sum of weights = 115 [M]
 Award this mark provided the sum is correct for their graph (must be a tree)

Question 2

- a. E drawn from end of activity A.
 F drawn from end of activities C and D [M]

G drawn from the meeting of the ends of activities E and F [M]

Note : arrows must be included

- b. Earliest start time for F is 7 [A]
 Latest start time for D is 3 [A]

- c. Critical path $A \rightarrow C \rightarrow F \rightarrow G$ [A]

- d. Project completion time is 20 mins [A]

- e. Float (slack) time for this activity is 2 minutes therefore they will go 1 minute overtime resulting in a \$10 fine [A]

Question 3

Ken to Oval 3 , Mark to Oval 2,
 Richard to Oval 1, Tony to Oval 4

Use of Hungarian algorithm or other appropriate method [M]

Two coaches assigned to the correct ovals [A]

All four coaches correctly assigned [A]

Working for Hungarian algorithm (generally accepted method)

1. Subtract the minimum entry in each row from each element in the row

	Oval 1	Oval 2	Oval 3	Oval 4
Ken	5	0	11	3
Mark	5	0	13	3
Richard	0	2	18	0
Tony	3	4	9	0

2. Repeat for any column without a zero entry

	Oval 1	Oval 2	Oval 3	Oval 4
Ken	5	0	2	3
Mark	5	0	4	3
Richard	0	2	9	0
Tony	3	4	0	0

3. Minimum number of lines to cover the zero elements is 3 (through column 2 and rows 3 and 4). Since this is less than the number of rows proceed to 4.

4. The lowest uncovered element is 2. Add this to the covered rows and columns

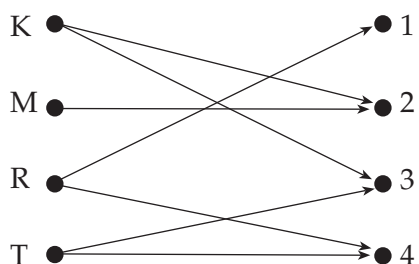
	Oval 1	Oval 2	Oval 3	Oval 4
Ken	5	2	2	3
Mark	5	2	4	3
Richard	2	6	11	2
Tony	5	8	2	2

5. This lowest uncovered element of 2 is now subtracted from all entries

	Oval 1	Oval 2	Oval 3	Oval 4
Ken	3	0	0	1
Mark	3	0	2	1
Richard	0	4	9	0
Tony	3	6	0	0

6. The minimum number of lines needed to cover the zeros is now 4 so a bipartite graph can be drawn with the edges chosen from the zero elements.

From this graph Mark must go to Oval 2 and Richard to Oval 1. This leaves Ken with Oval 3 as his only option and Tony with Oval 4 as his only option.



Total 15 marks