

Further Mathematics Examination 2

Solutions

Section A

Core : Data analysis

Question 1

- a. categorical [A1]
- b.
$$\begin{array}{c|cccc} 0 & 1 & 7 & 8 & 9 \\ 1 & 2 & 3 & 6 & 8 & 9 & 9 \\ 2 & 0 & 1 & 3 & 7 \\ 3 & 3 \end{array}$$
 [A1]
- c. males are negatively skewed, females are symmetrical [A1]
- d. median = 18 [A1]
- e. Interquartile range = $22 - 6$
= 16 [A1]
- f. 1.5 IQR = 24 [M1]
Upper 'fence' = $Q_3 + 24$
= 46 $\Rightarrow$ not an outlier [A1]

Question 2

- a. Since gender may influence attitude to smoking, it is the independent variable. [A1]
- b. 22 [A1]
- c.
- | | Female | Male |
|-----------------|--------|------|
| For smoking | 65.3 | 53.7 |
| Against smoking | 34.7 | 46.3 |
| | 100% | 100% |
- [A1]
- d. Based on this sample there appears to be a relationship. [A1]
A higher percentage of females (65.3%) than males (53.7%) are in favour of smoking in casinos. [A1]

Question 3

- a. $M = 5.02 \times 2 - 2.65$ [A1]
 $M = 7.39$
 $M = \$7$ (to the nearest dollar)

- b. As the number of hours gambled increases by 1, money lost increases by \$5.02 (the gradient of the linear equation). [A1]
- c. The predicted loss before you start gambling is \$2.65. Therefore no practical significance can be applied to this result. [M1]

Total: 15 marks

Section B

Module 1 : Number patterns and applications

Question 1

- a. The sequence is arithmetic with $a = 1.8$ and $d = 0.6$
 4^{th} term = $3 + 0.6 = 3.6$
 5^{th} term = $3.6 + 0.6 = 4.2$ [A1]
- b. $a = 1.8, d = 0.6$ [A1]
 $L_n = a + (n - 1)d$
= $1.8 + (n - 1) \times 0.6$ [H1]
= $1.2 + 0.6n$
- c. 1 litre / minute = 60 litres / hour
We need to find n when $L_n = 60$
Substituting in L_n
 $60 = 1.2 + 0.6n$ [M1]
 $58.8 = 0.6n$
 $n = 58.8 \div 0.6$
= 98 [A1]
In the 98th hour
- d. Find the sum of the first 24 terms :
 $S_{24} = \frac{24}{2} [2 \times 1.8 + 23 \times 0.6]$ [M1]
= 208.8
208.8 litres in total will leak from the crack in the first 24 hours. [A1]

Question 2

- a. Increase = $7 - 5 = 2$ litres

$$\frac{2}{5} \times 100 = 40\% \text{ increase} \quad [\text{A1}]$$

- b. The difference equation

$T_n = aT_{n-1}$ can be rearranged to give

$$\frac{T_n}{T_{n-1}} = a$$

$$\text{Hence } a = \frac{T_2}{T_1} = \frac{7}{5} = 1.4 \quad [\text{A1}]$$

Alternatively, you can use the fact that a percentage increase of 40% gives a multiplying factor of 1.4

- c. 1 litre/minute = 60 litres/hour

We need to find n such that $T_n = 60$

Recognising that this is a geometric sequence we can substitute in and solve

$$T_n = a \times r^{n-1} \\ 60 = 5 \times 1.4^{n-1} \quad [\text{M1}]$$

$$12 = 1.4^{n-1}$$

Using trial and error on the calculator:

Hence $1.4^{n-1} = 12$ for a value of $(n-1)$ between 7 and 8

And so for a value of n between 8 and 9.

So the amount leaking will exceed 60 l/hour during the 9th hour. [A1]

Alternatively using **Sequence** mode on the calculator :

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Clearly the amount exceeds 60 l/hour in the 9th hour.

- d. Substituting $a = 5$ and $r = 1.4$ in the equation for S_n for a geometric sequence :

$$S_{10} = \frac{5(1.4^{10} - 1)}{1.4 - 1} \quad [\text{M1}]$$

$$= 349.06 \quad (349 \text{ l}) \quad [\text{A1}]$$

Or using the calculator:

sum(and **seq(** are accessed under LIST

```
sum(seq(u(n),n,1,10)
      349.0683187
```

Question 3

This is a geometric sequence with

$$a = 20 \text{ and } r = \frac{17}{20} = 0.85$$

Hence we can find a sum to infinity because $-1 < r < 1$ [M1]

$$S_{\infty} = \frac{a}{1-r} = \frac{20}{1-0.85} = 133.33... \quad [\text{A1}]$$

133 l will leak from the crack from the time Deidre starts monitoring.

Total: 15 marks

Module 2 : Geometry and trigonometry**Question 1**

- a The triangle is an isosceles right-angled triangle, therefore the angle of decline is 45° . Or using trigonometry

$$\theta = \tan^{-1}\left(\frac{4}{4}\right) = \tan^{-1} 1 = 45^\circ \quad [\text{A1}]$$

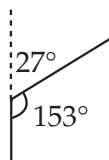
- b Lengths $\overline{AB}$ and $\overline{CD}$ are the same and using Pythagoras

Length $\overline{AB}$ or $\overline{CD}$	Total Length
$c^2 = a^2 + b^2$	$= 2 \times \sqrt{32} + 6$
$c^2 = 4^2 + 4^2 = 32$	$= 17.3137.. \quad [\text{M1}]$
$c = \sqrt{32}$	$\approx 17.3 \text{ metres} \quad [\text{A1}]$

Question 2

- a** The complementary angle to N27°E is 63° and added to 90° (from east to south) gives a total angle for $\angle OAB$ of 153° [A1]

- b** For Distance: Using Cosine Rule where
 $a = xm$ $b = 180m$ $c = 140m$
 and $\angle A = 153^\circ$,



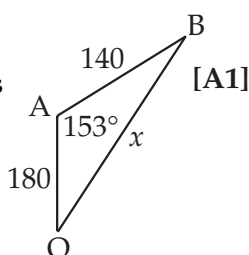
$$a^2 = b^2 + c^2 - 2bc \times \cos A$$

$$a^2 = 180^2 + 140^2 - 2 \times 180 \times 140 \times \cos 153^\circ \quad [\text{M1}]$$

$$a^2 = 96\,906.729$$

$$a = \sqrt{96906.729}$$

$$a = 311.298.. \approx 311 \text{ metres}$$



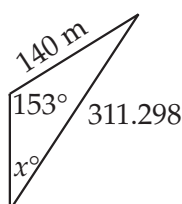
For Direction: Using the Sine Rule where

$$a = 311.298 \text{ m} \quad \angle A = 153^\circ$$

$$c = 140 \text{ m and } \angle C = x^\circ$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{311.298}{\sin 153^\circ} = \frac{140}{\sin x^\circ}$$



$$x^\circ = \sin^{-1}(0.20417..) = 11.781^\circ \approx 12^\circ$$

[M1]

where the bearing is N 12° E [A1]

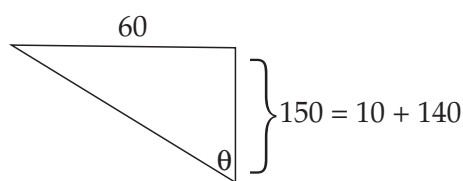
- c** G because the contour lines are closest together.

- d** Gradient = $\frac{\text{rise}}{\text{run}} = \frac{4}{15}$
or 1 in 3.75

Question 3

- a**
- | | | | | |
|--------|----|-----------|----|--------|
| | | Clubhouse | | |
| | | 0 | | |
| Hole 1 | 20 | 40 | | |
| | | 140 | 5 | Hole 5 |
| Hole 2 | 60 | 390 | | |
| | | 400 | 80 | Hole 4 |
| | | 540 | | |
| | | | | |
| | | Hole 3 | | |

- b** Using trigonometry where

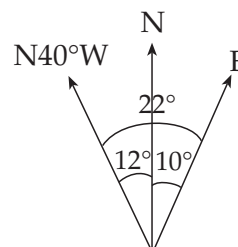


[M1]

$$\tan \theta = \frac{60}{150}$$

$$\theta = \tan^{-1}(0.4) = 21.8^\circ \approx 22^\circ \quad [\text{M1}]$$

and as a bearing 22° anticlockwise
from N 10° E becomes N 12° W



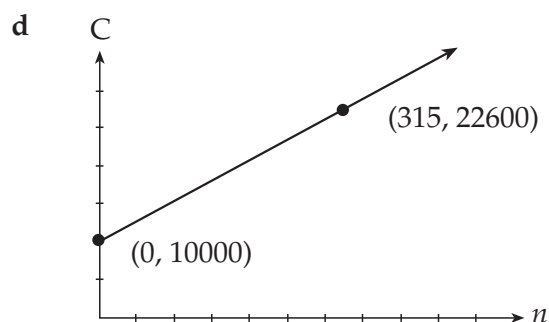
Total: 15 marks

Module 3 : Graphs and relations

- a** $a = 40, b = 10000$ [A1]

- $$\begin{aligned} \mathbf{b} \quad C &= 40 \times 140 + 10000 \\ &= \$15600 \end{aligned} \quad [\mathbf{A1}]$$

- $$\begin{aligned} \text{c} \quad 22600 &= 40n + 10000 \\ n &= 315 \end{aligned} \quad [\text{A1}]$$



2 points correctly used
graph drawn correctly

[M1]

[M1]

e $R = 140n$

[A1]

f Letting cost = revenue

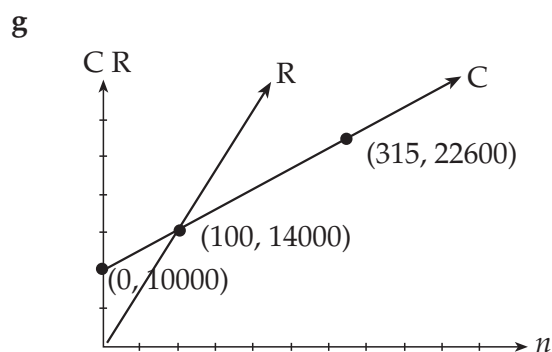
$$40n + 10000 = 140n$$

[M1]

$$100n = 10000$$

$n = 100$ i.e. 100 calculators must be
sold to break even

[A1]



Correct line with point labelled

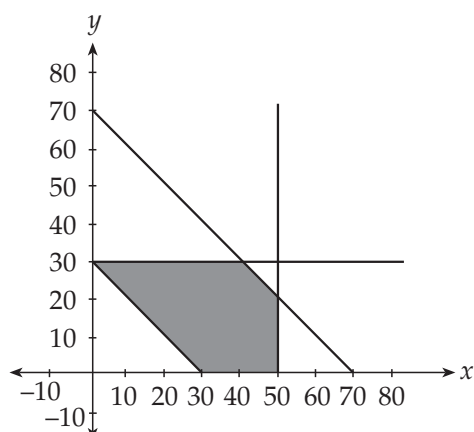
[M1]

Question 2

a $x + y \leq 70$

[A1]

b



Vertical and horizontal lines

[M1]

Diagonal lines

[M1]

Correct region shaded

[A1]

c From the graph, the maximum will
occur at the point (50, 20)

[M1]

$$\begin{aligned} \text{Maximum profit} &= (120 \times 50) + (80 \times 20) \\ &= \$7600 \end{aligned}$$

[A1]

Total: 15 marks

Module 4 : Business related mathematics

Question 1

a $\text{Interest} = \frac{PRT}{100} = \frac{10000 \times 8 \times 3}{100} = \2400

[M1] [A1]

b $\begin{aligned} \text{Total of Investment} &= \text{Principal} + \text{Interest} \\ &= \$10\,000 + \$2\,400 = \$12\,400 \end{aligned}$

[A1]

c

Time (years)	Balance at start of the year (\$)	Interest earned (\$)	Balance at the end of the year (\$)
1	10 000	800	10 800
2	10 800	$\frac{PRT}{100} = \frac{10800 \times 8 \times 1}{100} = 864$	11 664
3	11 664	$\frac{PRT}{100} = \frac{11664 \times 8 \times 1}{100} = 933.12$	$1164 + 933.12 = 12\,597.12$

[A1]

[A1]

d Compound interest bearing investment [A1]

Question 2

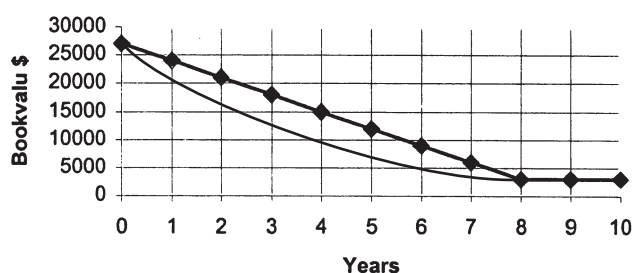
a A scrap value of \$3 000 [A1]

$$\begin{aligned}
 \text{b Rate of Depreciation} &= \frac{\text{Total Depreciation \$}}{\text{Years}} \\
 &= \frac{\$27000 - \$3000}{8 \text{ Years}} \\
 &= \$3000 \text{ per year} \quad [\text{M1}]
 \end{aligned}$$

$$\text{Rate of Depreciation \%} = \frac{\$3000}{\$27000} \times \frac{100}{1} = 11.1\% \quad [\text{A1}]$$

c

Computer Server Bookvalue \$



[A1]

Question 3

$$\text{a Loan Amount} = \text{Purchase} - \text{Trade In} = \$35\,000 - \$3\,000 = \$32\,000 \quad [\text{M1}]$$

Interest charged =

$$\frac{PRT}{100} = \frac{32000 \times 5 \times 5}{100} = \$8000$$

Monthly Repayments =

$$\frac{\text{Total repayments}}{\text{Number of Instalments}}$$

$$= \frac{\$32000 + \$8000}{5 \times 12} = \frac{\$40000}{60} = \$666.67 \quad [\text{A1}]$$

$$\begin{aligned}
 \text{b Effective interest rate} &= \frac{2 \times n}{n+1} \times \text{Flat Rat} \\
 &= \frac{2 \times 60}{60+1} \times 5\% \\
 &= 9.836\% = 9.8\% \quad [\text{A1}]
 \end{aligned}$$

c Loan Amount is the same as in a.

$$Q = \frac{PR^n(R-1)}{R^n-1}$$

$$\text{where } R = 1 + \frac{8.7/12}{100} = 1.00725 \quad [\text{M1}]$$

$$Q = \frac{32000 \times 1.00725^{60}(1.00725-1)}{1.00725^{60}-1}$$

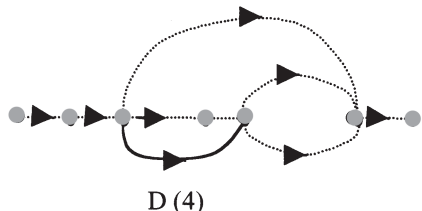
$$= \$659.62 \quad [\text{A1}]$$

Total: 15 marks

Module 5 : Networks and decision mathematics

Question 1

- a. Task D completes the network. [A1]



- b. Task C is not on the critical path so the latest it can start is 11 days before the end of the project. (Task C takes 7 days and task I takes 4 days)

Hence the **LST for C is 26.** [A1]

Task F is on the critical path so the **EST is day 23** (A takes 5 days, B takes 12 and E takes 5; a total of 22 days). [A1]

- c. The float (slack time) for task G is

$$\text{LST} - \text{EST} = 31 - 26 = 5 \text{ days} \quad [\text{A1}]$$

- d. The critical path is ABEFHI.

Any task on this path, if delayed will delay the completion time of the project. [A1]

- e. Completion time is 37 days [A1]

Question 2

- a. It is sensible to shorten the tasks that are on the critical path.

Hence tasks B, E, H and I should be shortened. (A and F cannot be shortened). [A1]

- b. B can be shortened 5 days, E can be shortened 1 day, H can be shortened 4 days and I can be shortened 2 days; a total of 12 days.

Hence the new completion time is $37 - 12 = 25$ days. [H1]

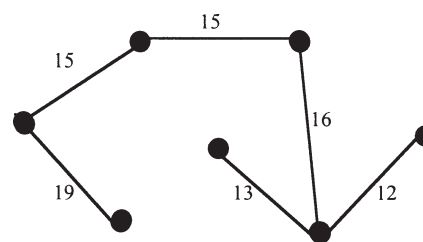
- c. The additional cost is

$$5 \times \$100 + 1 \times \$200 + 4 \times \$150 + 2 \times \$150 = \$1600$$

[H1]

Question 3

- a. The minimal-length spanning-tree is :



Tree [H1]

Correct [A1]

- b. The sum of the lengths on the **tree** above; 80 metres [H1]

Question 4

Using the Hungarian algorithm :

Subtract the minimum from each of the rows

	And.	Tom	Carl.	Eric
P	400	200	0	500
Q	160	250	0	400
R	1100	1500	0	1000
S	0	200	400	100

Subtract the minimum from each of the columns and cover the zeros with a minimum of lines : [M1]

	And.	Tom	Carl.	Eric
P	400	0	0	400
Q	160	50	0	300
R	1100	1300	0	900
S	0	0	400	0

There are only three lines (we need four for a solution)

Add the minimum uncovered number (160) to the elements at the intersections of the lines and subtract it from the elements not covered by the lines :

	And.	Tom	Carl.	Eric
P	240	0	0	240
Q	0	50	0	140
R	940	1300	0	740
S	0	160	560	0

We now have an additional line and an independent set of zeros

[M1] (circled)

Task P is allocated to Tom, Q to Andrew, R to Carlo and S to Eric [A1]

Total: 15 marks