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VCE Specialist Mathematics ½  
Complex Numbers I [8.1]  
**Test Solutions**

29.5 Marks. 1 Minute Reading. 24 Minutes Writing.

Results:

Test Questions	_____ / 23.5
Extended Test Questions	_____ / 6



## Section A: Test Questions (23.5 Marks)

### Question 1 (4.5 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. $-9$ is not a complex number.		<input checked="" type="checkbox"/>
b. The imaginary part of $z = -5 + 6i$ is $6i$ .		<input checked="" type="checkbox"/>
c. $\text{Arg}(2 - 3i) = -\tan^{-1}\left(\frac{3}{2}\right)$ .	<input checked="" type="checkbox"/>	
d. To add/subtract two complex numbers from each other, we simply have to add/subtract their real and imaginary parts.	<input checked="" type="checkbox"/>	
e. To divide two complex numbers, we simply have to divide their real and imaginary parts.		<input checked="" type="checkbox"/>
f. For any complex number to be a real number, its argument must be a multiple of $\pi$ .	<input checked="" type="checkbox"/>	
g. If you multiply any complex number by $i$ 10 times, it becomes the negative number of itself.	<input checked="" type="checkbox"/>	
h. $z + \bar{z} = 2\text{Re}(z)$ .	<input checked="" type="checkbox"/>	
i. $z - \bar{z} = \text{Re}(z)$ .		<input checked="" type="checkbox"/>

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**Question 2** (2 marks)

Express each of the following in their simplest term.

a.  $\sqrt{-175}$ . (1 mark)

$$5i\sqrt{7}$$

b.  $\sqrt{2}(\sqrt{-50} + \sqrt{5})$ . (1 mark)

$$\sqrt{10} + 10i$$

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**Question 3** (2 marks)

If  $z = 2 - 3i$ , then find  $z^2$ .

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$$\begin{aligned}
 &\text{Given,} \\
 &z = 2 - 3i \\
 &z^2 = z \cdot z \\
 &= (2 - 3i)(2 - 3i) \\
 &= 2(2) - 2(3i) - 3i(2) + (3i)(3i) \\
 &= 4 - 6i - 6i + 9i^2 \quad \{\text{since } i^2 = -1\} \\
 &= 4 - 12i + 9(-1) \\
 &= 4 - 12i - 9 \\
 &= -5 - 12i \\
 &\text{Therefore, } z^2 = -5 - 12i
 \end{aligned}$$

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**Question 4** (3 marks)

Simplify:  $\frac{1-2i}{3+4i} - \frac{2+i}{5i}$ .

$$\frac{1-2i}{3+4i} - \frac{2+i}{5i}$$

This can be written as:

$$\begin{aligned} &= \frac{1-2i}{3+4i} \cdot \frac{3-4i}{3-4i} - \frac{2+i}{5i} \cdot \frac{-i}{-i} \\ &= \frac{(3-4i-6i+8i^2)}{9-16i^2} - \frac{(-2i-i^2)}{-5i^2} \\ &= \frac{(3-10i-8)}{(9+16)} - \frac{(-2i+1)}{5} \\ &= \frac{(-5-10i)}{25} + \frac{2}{5}i - \frac{1}{5} \\ &= -\frac{1}{5} - \frac{2}{5}i + \frac{2}{5}i - \frac{1}{5} = -\frac{2}{5} \end{aligned}$$

Therefore,  $\frac{1-2i}{3+4i} - \frac{2+i}{5i} = -\frac{2}{5}$

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**Question 5** (2 marks)

Find the conjugate of  $z_1 - z_2$  if  $z_1 = 2 + 3i$  and  $z_2 = 5 + 2i$ .

Given,

$$z_1 = 2 + 3i$$

$$z_2 = 5 + 2i$$

$$\begin{aligned} z_1 - z_2 &= (2 + 3i) - (5 + 2i) \\ &= (2 - 5) + i(3 - 2) \\ &= -3 + i \end{aligned}$$

As we know conjugate of  $z = x + iy = x - iy$ .

$$\text{Conjugate of } z_1 - z_2 = -3 - i$$

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**Question 6** (1 mark)

Simplify:  $i^{59}$ .

$-i$

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**Question 7** (3 marks)

Suppose  $z = 1 + \sqrt{3}i$  and  $\frac{w}{z} = 2 + 2i$ .

Find the exact value of the modulus of  $w$  and the exact value of the argument of  $w$ .

Method B

$$w = (2 + 2i)(1 + \sqrt{3}i)$$

$$\Rightarrow |w| = |(2 + 2i)(1 + \sqrt{3}i)|$$

$$\Rightarrow |w| = |2 + 2i||1 + \sqrt{3}i|$$

$$\Rightarrow |w| = \sqrt{2^2 + 2^2} \times \sqrt{1^2 + (\sqrt{3})^2}$$

$$\Rightarrow |w| = \sqrt{8} \times \sqrt{4}$$

$$\Rightarrow |w| = 2\sqrt{2} \times 2$$

$$\Rightarrow |w| = 4\sqrt{2}$$

$$\arg(w) = \arg[(2 + 2i)(1 + \sqrt{3}i)]$$

$$\Rightarrow \arg(w) = \arg(2 + 2i) + \arg(1 + \sqrt{3}i)$$

$$\Rightarrow \arg(w) = \arctan\left(\frac{2}{2}\right) + \arctan\left(\frac{\sqrt{3}}{1}\right)$$

$$\Rightarrow \arg(w) = \arctan 1 + \arctan(\sqrt{3})$$

$$\Rightarrow \arg(w) = \frac{\pi}{4} + \frac{\pi}{3}$$

$$\Rightarrow \arg(w) = \frac{7\pi}{12}$$

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**Question 8** (3 marks)

Let  $z = 1 - i$ .

Find the value of  $z^6$ .

$$z^6 = (1 - i)^6$$

$$= 8i$$

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**Question 9** (3 marks)

Consider the complex number  $z = 4 + 4i$ .

Find the value(s) of  $k$  such that  $z^k$  is a negative imaginary number.

**Solve** $\left[\frac{\pi}{4} \mathbf{k} == -\pi / 2 + 2 \pi * \mathbf{n}, \mathbf{k}\right]$  // **Expand**  
 [풀이 함수] [확장]

$\{\{\mathbf{k} \rightarrow -2 + 8 \mathbf{n}\}\}$

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## Section B: Extended Test Questions (6 Marks)

### Question 10 (6 marks)

- a. Show that  $\frac{4\sqrt{2}+4\sqrt{2}i}{\sqrt{3}+i} = \sqrt{6} + \sqrt{2} + (\sqrt{6} - \sqrt{2})i$ . Do NOT use polar form. (2 marks)

$$\begin{aligned} & \frac{4\sqrt{2} + 4\sqrt{2}i}{\sqrt{3} + i} \cdot \frac{\sqrt{3} - i}{\sqrt{3} - i} \\ &= \frac{4\sqrt{6} - 4\sqrt{2}i + 4\sqrt{6}i + 4\sqrt{2}}{3 + 1} \quad // \text{ Expand} \\ &= \frac{4\sqrt{6} + 4\sqrt{2} + 4(\sqrt{6} - \sqrt{2})i}{4} \\ &= \sqrt{6} + \sqrt{2} + (\sqrt{6} - \sqrt{2})i \end{aligned}$$

- b. By converting the numerator and denominator to polar form, express  $\frac{4\sqrt{2}+4\sqrt{2}i}{\sqrt{3}+i}$  in polar form. (2 marks)

$$\begin{aligned} & \frac{4\sqrt{2}(1+i)}{\sqrt{3}+i} = \frac{4\sqrt{2}(\sqrt{2} \operatorname{cis}(\frac{\pi}{4}))}{2 \operatorname{cis}(\frac{\pi}{6})} \\ &= \frac{8 \operatorname{cis}(\frac{\pi}{4})}{2 \operatorname{cis}(\frac{\pi}{6})} = 4 \operatorname{cis}\left(\frac{\pi}{12}\right) \end{aligned}$$

- c. Hence, find the values of  $\sin\left(\frac{\pi}{12}\right)$  and  $\cos\left(\frac{\pi}{12}\right)$ . (2 marks)

$$4 \operatorname{cis}\left(\frac{\pi}{12}\right) = \sqrt{6} + \sqrt{2} + (\sqrt{6} - \sqrt{2})i$$

$$4 \cos\left(\frac{\pi}{12}\right) = \sqrt{6} + \sqrt{2}, \quad 4 \sin\left(\frac{\pi}{12}\right) = \sqrt{6} - \sqrt{2}$$

$$\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}, \quad \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$$

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## VCE Specialist Mathematics ½

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