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VCE Specialist Mathematics ½
Complex Numbers I [8.1]
Test Solutions

29.5 Marks. 1 Minute Reading. 24 Minutes Writing.

Results:

Test Questions	________ / 23.5
Extended Test Questions	________ / 6

Section A: Test Questions (23.5 Marks)

Question 1 (4.5 marks)

Tick whether the following statements are **true** or **false**.

Statement	**True**	**False**
a. -9 is not a complex number.		✓
b. The imaginary part of $z = -5 + 6i$ is $6i$.		✓
c. $\text{Arg}(2 - 3i) = -\tan^{-1}\left(\frac{3}{2}\right)$.	✓	
d. To add/subtract two complex numbers from each other, we simply have to add/subtract their real and imaginary parts.	✓	
e. To divide two complex numbers, we simply have to divide their real and imaginary parts.		✓
f. For any complex number to be a real number, its argument must be a multiple of π.	✓	
g. If you multiply any complex number by i 10 times, it becomes the negative number of itself.	✓	
h. $z + \bar{z} = 2\text{Re}(z)$.	✓	
i. $z - \bar{z} = \text{Re}(z)$.		✓

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Question 2 (2 marks)

Express each of the following in their simplest term.

a. $\sqrt{-175}$. (1 mark)

$\boxed{5i\sqrt{7}}$

b. $\sqrt{2}\left(\sqrt{-50}+\sqrt{5}\right)$. (1 mark)

$\boxed{\sqrt{10}+10i}$

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Question 3 (2 marks)

If $z = 2 - 3i$, then find z^2.

Given,
$z = 2 - 3i$

$$
\begin{aligned}
z^2 &= z \cdot z \\
&= (2 - 3i)(2 - 3i) \\
&= 2(2) - 2(3i) - 3i(2) + (3i)(3i) \\
&= 4 - 6i - 6i + 9i^2 \quad \{\text{since } i^2 = -1\} \\
&= 4 - 12i + 9(-1) \\
&= 4 - 12i - 9 \\
&= -5 - 12i
\end{aligned}
$$

Therefore, $z^2 = -5 - 12i$

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Question 4 (3 marks)

Simplify: $\frac{1-2i}{3+4i}-\frac{2+i}{5i}$.

$$\frac{1-2i}{3+4i}-\frac{2+i}{5i}$$

This can be written as:

$$=\frac{1-2i}{3+4i}\cdot\frac{3-4i}{3-4i}-\frac{2+i}{5i}\cdot\frac{-i}{-i}$$

$$=\frac{(3-4i-6i+8i^2)}{9-16i^2}-\frac{(-2i-i^2)}{-5i^2}$$

$$=\frac{(3-10i-8)}{(9+16)}-\frac{(-2i+1)}{5}$$

$$=\frac{(-5-10i)}{25}+\frac{2}{5}i-\frac{1}{5}$$

$$=-\frac{1}{5}-\frac{2}{5}i+\frac{2}{5}i-\frac{1}{5}=-\frac{2}{5}$$

Therefore, $\frac{1-2i}{3+4i}-\frac{2+i}{5i}=-\frac{2}{5}$

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Question 5 (2 marks)

Find the conjugate of $z_1 - z_2$ if $z_1 = 2 + 3i$ and $z_2 = 5 + 2i$.

Given,
$z_1 = 2 + 3i$
$z_2 = 5 + 2i$

$$\begin{aligned} z_1 - z_2 &= (2 + 3i) - (5 + 2i) \\ &= (2 - 5) + i(3 - 2) \\ &= -3 + i \end{aligned}$$

As we know conjugate of $z = x + iy = x - iy$.
Conjugate of $z_1 - z_2 = -3 - i$

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Question 6 (1 mark)

Simplify: i^{59}.

$$\boxed{-i}$$

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Question 7 (3 marks)

Suppose $z = 1 + \sqrt{3}i$ and $\frac{w}{z} = 2 + 2i$.

Find the exact value of the modulus of w and the exact value of the argument of w.

Method B

$$w = (2 + 2i)(1 + \sqrt{3}i)$$

$$\Rightarrow |w| = \left|(2 + 2i)(1 + \sqrt{3}i)\right|$$

$$\Rightarrow |w| = |2 + 2i|\left|1 + \sqrt{3}i\right|$$

$$\Rightarrow |w| = \sqrt{2^2 + 2^2} \times \sqrt{1^2 + \left(\sqrt{3}\right)^2}$$

$$\Rightarrow |w| = \sqrt{8} \times \sqrt{4}$$

$$\Rightarrow |w| = 2\sqrt{2} \times 2$$

$$\Rightarrow |w| = 4\sqrt{2}$$

$$\arg(w) = \arg\left[(2 + 2i)(1 + \sqrt{3}i)\right]$$

$$\Rightarrow \arg(w) = \arg(2 + 2i) + \arg\left(1 + \sqrt{3}i\right)$$

$$\Rightarrow \arg(w) = \arctan\left(\frac{2}{2}\right) + \arctan\left(\frac{\sqrt{3}}{1}\right)$$

$$\Rightarrow \arg(w) = \arctan 1 + \arctan\left(\sqrt{3}\right)$$

$$\Rightarrow \arg(w) = \frac{\pi}{4} + \frac{\pi}{3}$$

$$\Rightarrow \arg(w) = \frac{7\pi}{12}$$

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Question 8 (3 marks)

Let $z = 1 - i$.

Find the value of z^6.

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:= (1 - I)^6
]= 8 i
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Question 9 (3 marks)

Consider the complex number $z = 4 + 4i$.

Find the value(s) of k such that z^k is a negative imaginary number.

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Solve[π/4 k == -π/2 + 2π*n, k] // Expand
풀이 함수                          확장

{{k → -2 + 8 n}}
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Section B: Extended Test Questions (6 Marks)

Question 10 (6 marks)

a. Show that $\frac{4\sqrt{2}+4\sqrt{2}i}{\sqrt{3}+i} = \sqrt{6}+\sqrt{2}+(\sqrt{6}-\sqrt{2})i$. Do NOT use polar form. (2 marks)

$$\frac{4\sqrt{2}+4\sqrt{2}\,i}{\sqrt{3}+i}\cdot\frac{\sqrt{3}-i}{\sqrt{3}-i}$$

$$= \frac{4\sqrt{6}-4\sqrt{2}\,i+4\sqrt{6}\,i+4\sqrt{2}}{3+1} \quad // \text{ Expand}$$

$$= \sqrt{2}+\sqrt{6}-\sqrt{2}\,i+\sqrt{6}\,i$$

$$= \sqrt{6}+\sqrt{2}+(\sqrt{6}-\sqrt{2})\,i$$

b. By converting the numerator and denominator to polar form, express $\frac{4\sqrt{2}+4\sqrt{2}i}{\sqrt{3}+i}$ in polar form. (2 marks)

$$\frac{4\sqrt{2}\,(1+i)}{\sqrt{3}+i} = \frac{4\sqrt{2}\left(\sqrt{2}\operatorname{cis}\left(\frac{\pi}{4}\right)\right)}{2\operatorname{cis}\left(\frac{\pi}{6}\right)}$$

$$= \frac{8\operatorname{cis}\left(\frac{\pi}{4}\right)}{2\operatorname{cis}\left(\frac{\pi}{6}\right)} = 4\operatorname{cis}\left(\frac{\pi}{12}\right)$$

c. Hence, find the values of $\sin\left(\frac{\pi}{12}\right)$ and $\cos\left(\frac{\pi}{12}\right)$. (2 marks)

$$4\,\text{cis}\left(\frac{\pi}{12}\right) = \sqrt{6}+\sqrt{2}+\left(\sqrt{6}-\sqrt{2}\right)i$$

$$4\cos\left(\frac{\pi}{12}\right) = \sqrt{6}+\sqrt{2},\quad 4\sin\left(\frac{\pi}{12}\right) = \sqrt{6}-\sqrt{2}$$

$$\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4},\quad \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$$

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