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VCE Specialist Mathematics ½
Complex Numbers I [8.1]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2-Pg 23
Supplementary Questions	Pg 24-Pg 38

Section A: Compulsory Questions

Sub-Section [8.1.1]: Find Polar and Rectangular Forms of Complex Numbers

Question 1

a. Convert $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ into polar form.

$$\text{cis}\left(\frac{\pi}{4}\right)$$

b. Let $u = 3\text{cis}\left(\frac{\pi}{3}\right)$. Convert u into a rectangular form.

$$u = \frac{3}{2} + \frac{3\sqrt{3}}{2}i$$

c. Convert $-\sqrt{3} + i$ into polar form.

$$2\text{cis}\left(\frac{5\pi}{6}\right)$$

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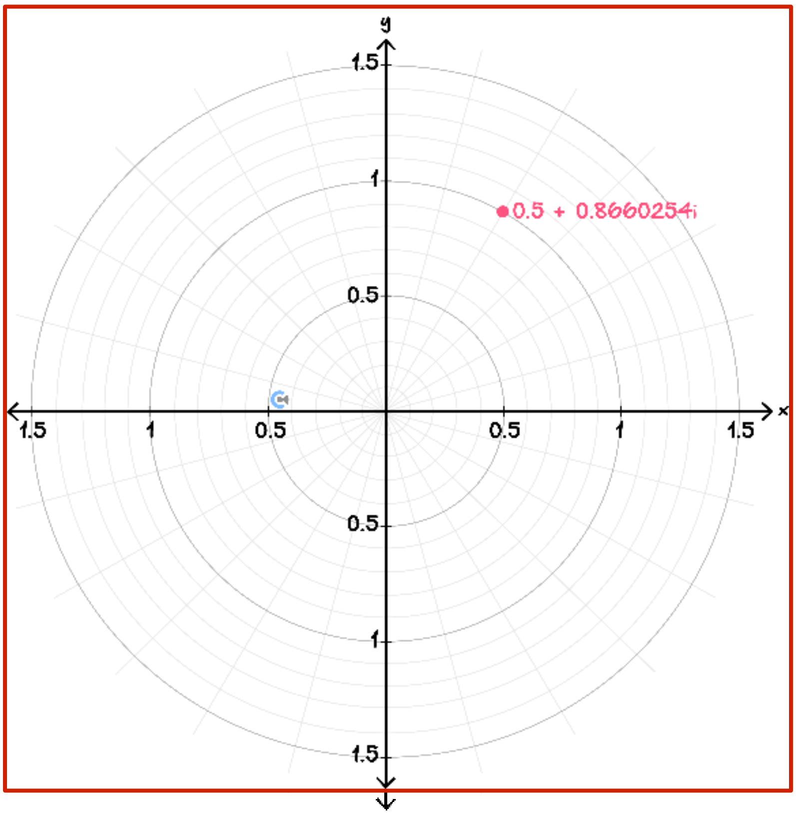
Question 2

a.

i. Convert $\text{cis}\left(\frac{7\pi}{3}\right)$ into a rectangular form.

$$\frac{1}{2}+\frac{\sqrt{3}}{2}i$$

ii. Plot your answer on the Argand diagram below:

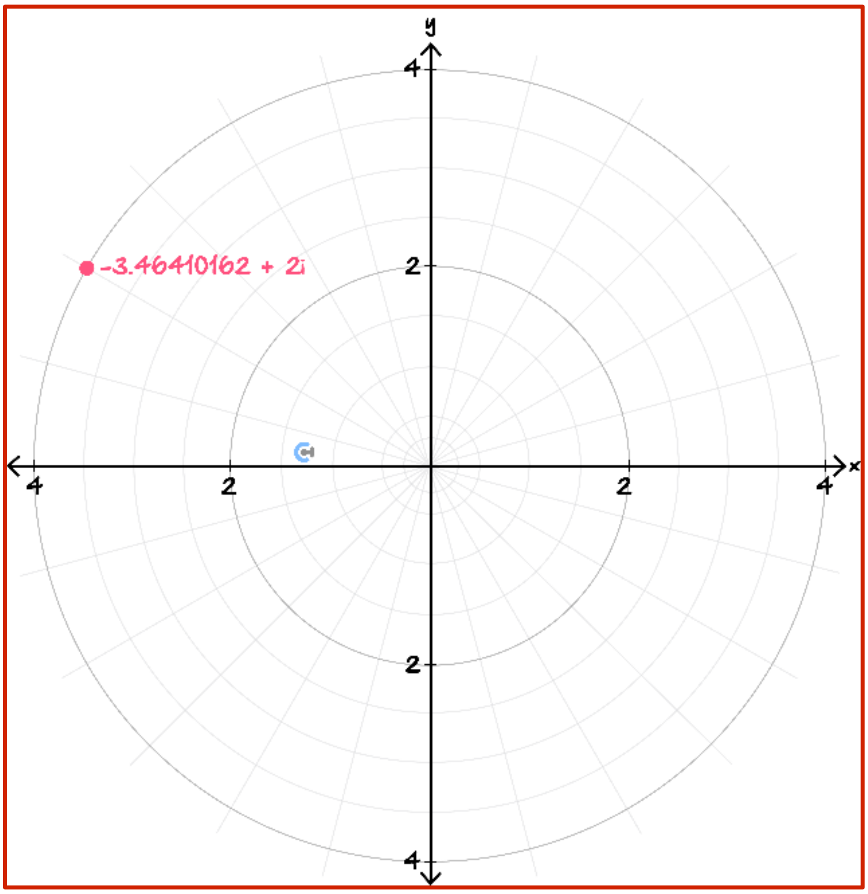


b.

i. Convert $-2\sqrt{3} + 2i$ into polar form.

$$4\text{cis}\left(\frac{5\pi}{6}\right)$$

ii. Plot your answer on the Argand diagram below:

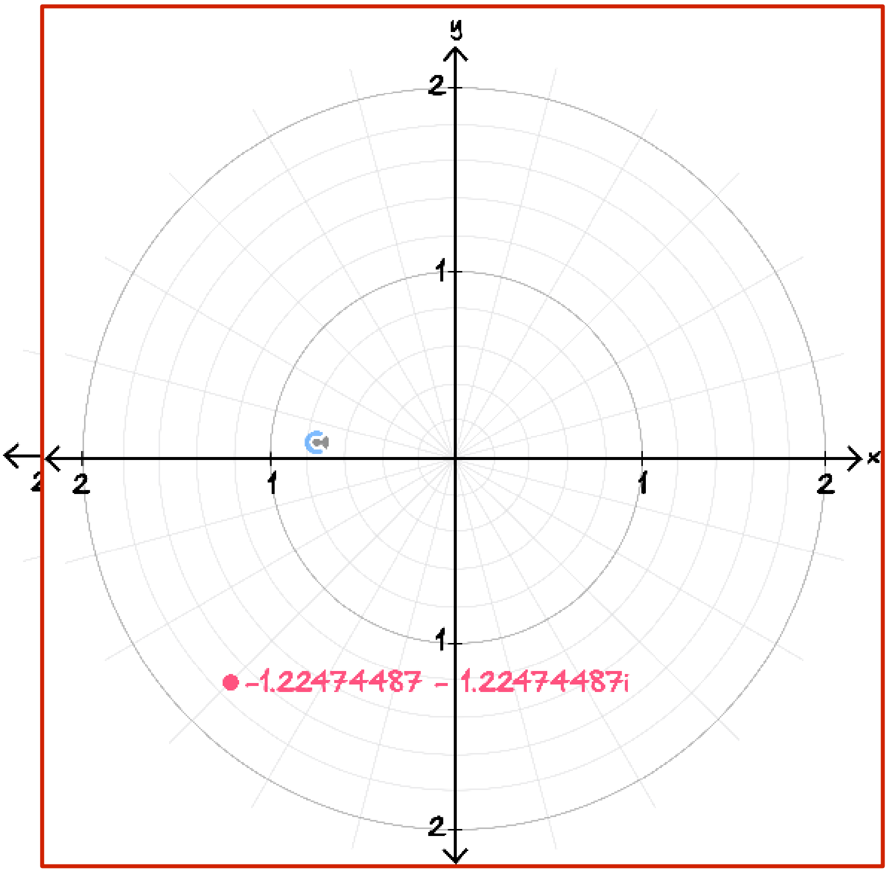


c.

i. Convert $-\frac{\sqrt{6}}{2}-\frac{\sqrt{6}}{2}i$ into polar form.

$$\sqrt{3}\text{cis}\left(-\frac{3\pi}{4}\right)$$

ii. Plot your answer on the Argand diagram below:



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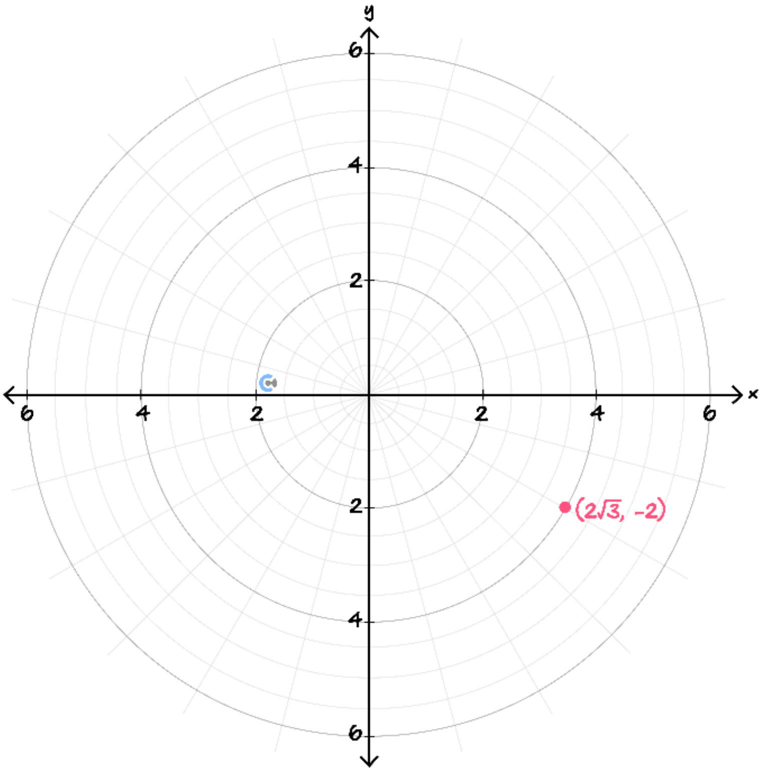
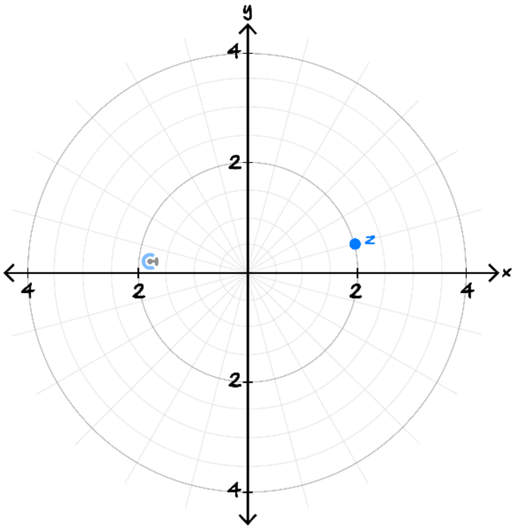
Question 3

a. Below is a complex number plotted on an Argand diagram. State the number in polar form.



$4\text{cis}\left(\frac{-\pi}{6}\right)$

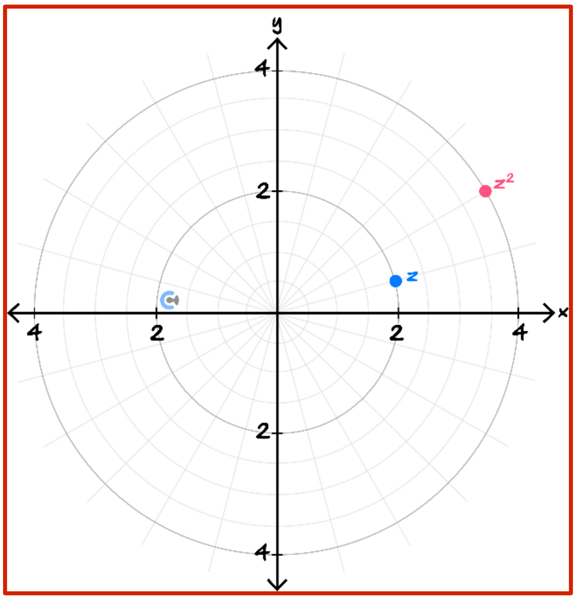
b. The complex number z is on the Argand diagram below:



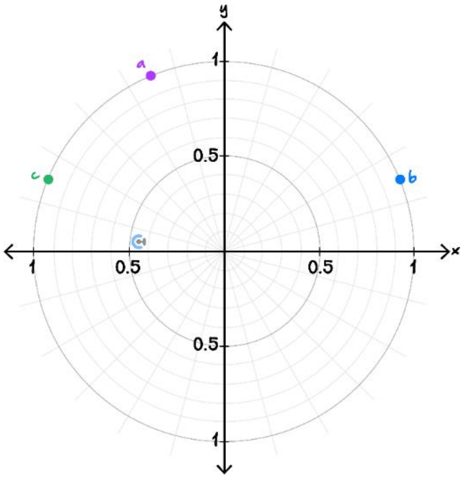
i. State z in polar form.

$$z = 2\text{cis}\left(\frac{\pi}{12}\right)$$

ii. Plot z^2 on the Argand diagram below:



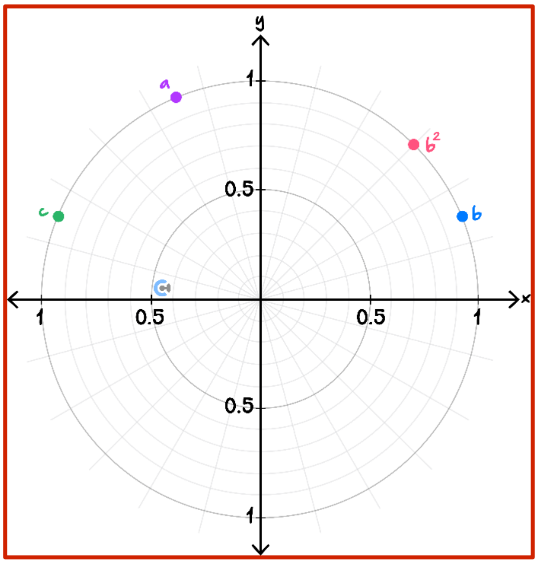
c. 3 complex numbers, a, b and c are plotted on the Argand diagram below:



i. State which complex number corresponds to $\text{cis}\left(\frac{23\pi}{8}\right)$.

c corresponds to $\text{cis}\left(\frac{23\pi}{8}\right)$.

ii. Plot b^2 on the Argand diagram below:



Question 4 Tech-Active.

a. Convert $\text{cis}\left(\frac{15\pi}{8}\right)$ into a rectangular form.

$$\frac{\sqrt{\sqrt{2}+2}}{2} - \frac{\sqrt{2-\sqrt{2}}}{2}i$$

b. Convert $\frac{\sqrt{6}-\sqrt{2}}{2} + \frac{\sqrt{6}+\sqrt{2}}{2}i$ into polar form.

$$2\text{cis}\left(\frac{5\pi}{12}\right)$$

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Sub-Section [8.1.2]: Evaluate Operations of Complex Numbers

Question 5

a. Let $u = 5\text{cis}\left(\frac{3\pi}{8}\right)$ and $v = 2\text{cis}\left(\frac{2\pi}{8}\right)$. Evaluate uv, leaving your answer in polar form.

$$10\text{cis}\left(\frac{5\pi}{8}\right)$$

b. Let $u = 3\text{cis}\left(\frac{5\pi}{6}\right)$ and $v = 2\text{cis}\left(\frac{5\pi}{12}\right)$. Evaluate $\frac{u}{v}$, leaving your answer in polar form.

$$\frac{3}{2}\text{cis}\left(\frac{5\pi}{12}\right)$$

c. Let $u = 4 + \frac{7}{2}i$ and $v = \frac{3}{5} + i$. Evaluate $u + v$.

$$\frac{23}{5} + \frac{9}{2}i$$

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Question 6

a. Let $u = 1 + 3i$ and $v = \sqrt{3} + \sqrt{3}i$. Evaluate uv, leaving your answer in rectangular form.

$$-2\sqrt{3} + 4\sqrt{3}i$$

b. Simplify $\frac{1+2i}{1-i}$.

$$-\frac{1}{2} + \frac{3}{2}i$$

c. Let $u = \text{cis}\left(\frac{3\pi}{4}\right)$ and $v = 2\text{cis}\left(\frac{2\pi}{3}\right)$. Evaluate $u + v$, leaving your answer in rectangular form.

$$-\frac{\sqrt{2}}{2} - 1 + \left(\sqrt{3} + \frac{\sqrt{2}}{2}\right)i$$

Question 7

a. Let $u = \frac{\sqrt{\sqrt{2}+2}}{2} + \frac{\sqrt{2-\sqrt{2}}}{2}i$. Evaluate u^2, leaving your answer in polar form.

$$\text{cis}\left(\frac{\pi}{4}\right)$$

b. Simplify $\frac{-1+\sqrt{3}i}{2-\sqrt{3}i}$.

$$-\frac{5}{7} + \frac{\sqrt{3}}{7}i$$

c. Let $u = \sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$ and $v = 1 + \left(1 - 2\sqrt{3}\right)i$. Evaluate $u + \bar{v}$, leaving your answer in polar form.

$$4\text{cis}\left(\frac{\pi}{3}\right)$$

Question 8 Tech-Active.

a. Evaluate $\text{cis}\left(\frac{5\pi}{7}\right) + \text{cis}\left(\frac{2\pi}{7}\right)$, leaving your answer in rectangular form to 2 decimal places.

$$1.56i$$

b. Let $u = 5\sqrt{3} - \sqrt{2}i$ and $v = \sqrt{7} - i$. Simplify $\frac{u}{v}$.

$$\frac{5\sqrt{21} + \sqrt{2}}{8} + \frac{5\sqrt{3} - 14}{8}i$$

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Sub-Section [8.1.3]: Apply De Moivre's Theorem

Question 9

a. Let $a = \sqrt{2}\text{cis}\left(\frac{\pi}{6}\right)$. Evaluate a^4, leaving your answer in rectangular form.

$-2 + 2\sqrt{3}i$

b. Let $v = \sqrt{3}\text{cis}\left(\frac{5\pi}{7}\right)$. Evaluate v^3, leaving your answer in polar form with the principal argument.

$3\sqrt{3}\text{cis}\left(\frac{\pi}{7}\right)$

c. Evaluate $(1+i)^5$, leaving your answer in rectangular form.

$-4-4i$

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Question 10

a. Let $u = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$. Evaluate u^3, leaving your answer in rectangular form.

$-i$

b. Let $v = 2^{\frac{1}{11}}\text{cis}\left(\frac{\pi}{6}\right)$. Evaluate v^{22}, leaving your answer in rectangular form.

$2 - 2\sqrt{3}i$

c. Evaluate $\frac{(1+\sqrt{3}i)^3}{(-\sqrt{2}+\sqrt{2}i)^4}$, leaving your answer in rectangular form.

$\frac{1}{2}$

Question 11

a. Find the value(s) of n such that $\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^n = \text{cis}\left(\frac{\pi}{3}\right)$.

$n = 2 + 12k, k \in Z$

b. Suppose $u = \text{cis}\left(\frac{2\pi}{7}\right)$. Find the value(s) of n such that u^n is a real number.

$n = \frac{7}{2}k$, where k is an integer.

c. Find the value(s) of n such that $(3+3i)^n = ai$, where $a \in R$ and $a < 0$.

$n = 6 + 8k, k \in Z$

Question 12 Tech-Active.

Evaluate $\left(\sqrt{2}\text{cis}\left(\frac{\pi}{12}\right)\right)^5$, leaving your answer in rectangular form.

$2\sqrt{3} - 2 + \left(2\sqrt{3} + 2\right)i$

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Sub-Section: Final Boss

Question 13

Consider the equation $z^6 - 1 = 0$, $z \in C$.

a. Use De Moivre's theorem to verify that $z = \text{cis}\left(\frac{\pi}{3}\right)$ is a solution to this equation.

$z^6 - 1 = \text{cis}(2\pi) - 1 = 1 - 1 = 0$

b. Find the other five solutions to the equation.

$z = 1, \text{cis}\left(\frac{2\pi}{3}\right), -1, \text{cis}\left(\frac{4\pi}{3}\right), \text{cis}\left(\frac{5\pi}{3}\right)$ using the fact that roots are evenly spaced.

c. Show that $\text{cis}(a) + \text{cis}(-a) = 2\cos(a)$.

$$\text{LHS} = \text{cis}(a) + \text{cis}(-a) = \cos(a) + i\sin(a) + \cos(a) - i\sin(a) = 2\cos(a)$$

d. Hence, show that the sum of the roots of $z^6 - 1$ is 0.

$$1 - 1 + \text{cis}\left(\frac{\pi}{3}\right) + \text{cis}\left(\frac{-\pi}{3}\right) + \text{cis}\left(\frac{4\pi}{3}\right) + \text{cis}\left(\frac{4\pi}{3}\right) = 2\cos\left(\frac{\pi}{3}\right) + 2\cos\left(\frac{4\pi}{3}\right)$$

$= 2\left(\cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{3}\right)\right) = 0$ from symmetry properties (supplementary angle).

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Question 14

De Moivre's theorem states that $(\cos(\theta) + i\sin(\theta))^n = \cos(n\theta) + i\sin(n\theta)$.

a. Show that De Moivre's theorem is true for $n = 2$.

$$(\cos(\theta) + i\sin(\theta))^2 = \cos^2(\theta) - \sin^2(\theta) + i\,2\sin(\theta)\cos(\theta)$$
$$= \cos(2\theta) + i\sin(2\theta)$$

b. Using induction, prove De Moivre's theorem for $n \in \mathbb{N}$.

Test for $n = 1$. Then assume true for $n = k$, prove for $n = k + 1$.

$$(\cos(\theta) + i\sin(\theta))^{k+1} = (\cos(\theta) + i\sin(\theta))(\cos(k\theta) + i\sin(k\theta)).$$

$$= \cos(\theta)\cos(k\theta) + i^2\sin(\theta)\sin(k\theta) + i\sin(\theta)\cos(k\theta) + i\sin(k\theta)\cos(\theta)$$

$$= \cos(\theta)\cos(k\theta) - \sin(\theta)\sin(k\theta) + i(\sin(\theta)\cos(k\theta) + \sin(k\theta)\cos(\theta))$$

$$= \cos((k+1)\theta) + i\sin((k+1)\theta)$$

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Section B: Supplementary Questions

Sub-Section [8.1.1]: Find Polar and Rectangular Forms of Complex Numbers

Question 15

a. Write $14i$ in polar form.

$$14\text{cis}\left(\frac{\pi}{2}\right)$$

b. Convert $-2-2\sqrt{3}i$ into polar form.

$$4\text{cis}\left(\frac{-2\pi}{3}\right)$$

c. Convert $\sqrt{2}\text{cis}\left(\frac{-\pi}{4}\right)$ into a rectangular form.

$$1 - i$$

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Question 16

a. Convert $-\frac{3\sqrt{3}}{2}+\frac{3}{2}i$ into polar form.

$3\text{cis}\left(\frac{5\pi}{6}\right)$

b. Convert $\text{cis}\left(\frac{420\pi}{4}\right)$ into Cartesian form.

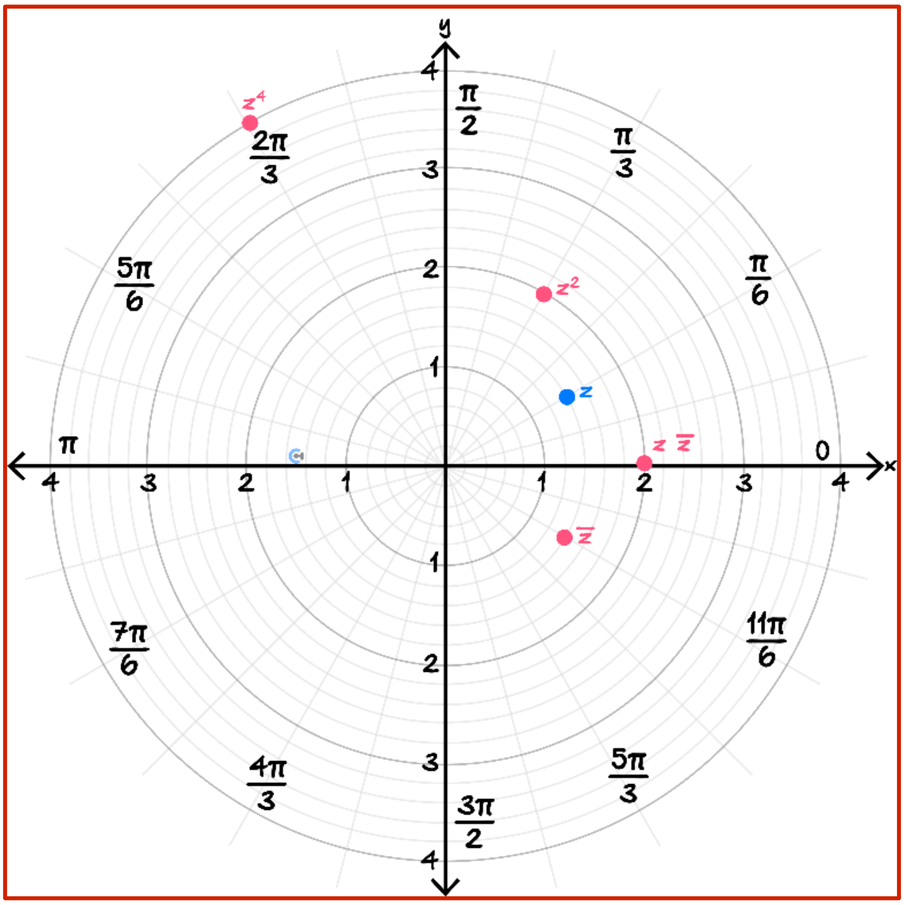
-1

c. Convert $-\frac{5\sqrt{2}}{2}-\frac{5\sqrt{2}}{2}i$ into polar form.

$5\text{cis}\left(\frac{-3\pi}{4}\right)$

Question 17

The complex number $z = \sqrt{2}\text{cis}\left(\frac{\pi}{6}\right)$ is shown in the Argand diagram below:



On the same set of axes, plot:

a. z^2

b. z^4

c. $\bar{z}$

d. $z\bar{z}$

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Question 18

a. Use a compound formula to show that $\sin\left(\frac{7\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$.

$$\text{LHS} = \sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}\frac{1}{2} + \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} = \text{RHS}$$

b. Hence, show that $\cos\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2}-\sqrt{6}}{4}$, using the fact that $\sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{6}-\sqrt{2}}{4}$.

$$\text{LHS} = -\sqrt{1-\sin^2\left(\frac{7\pi}{12}\right)} = -\sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2}-\sqrt{6}}{4}$$ since cos is negative in the second quadrant.

c. Hence, state $\text{cis}\left(\frac{17\pi}{12}\right)$.

$$\frac{\sqrt{2}-\sqrt{6}}{4} - \frac{\sqrt{6}+\sqrt{2}}{4}i$$

Sub-Section [8.1.2]: Evaluate Operations of Complex Numbers

Question 19

a. Let $u = 3\text{cis}\left(\frac{\pi}{5}\right)$ and $v = 2\text{cis}\left(\frac{-4\pi}{5}\right)$. Evaluate uv, leaving your answer in polar form.

$$6\text{cis}\left(\frac{-3\pi}{5}\right)$$

b. Let $u = \text{cis}\left(\frac{-3\pi}{4}\right)$ and $v = \text{cis}\left(\frac{\pi}{4}\right)$. Evaluate $\frac{u}{v}$.

$$-1$$

c. Let $u = \sqrt{3} + 2i$ and $v = \sqrt{48} - i$. Evaluate $u + v$.

$$5\sqrt{3} + i$$

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Question 20

a. Find $\frac{1-i}{2+3i}$.

$$-\frac{1}{13}-\frac{5}{13}i$$

b. Let $a = (3-\sqrt{2}i)$ and $b = (2+\sqrt{5}i)$. Evaluate ab, leaving your answer in rectangular form.

$$\sqrt{10}+6+(3\sqrt{5}-2\sqrt{2})i$$

c. Evaluate $2\text{cis}\left(\frac{3\pi}{4}\right)+\text{cis}\left(\frac{\pi}{3}\right)$, leaving your answer in an appropriate form.

$$\frac{1}{2}-\sqrt{2}+\left(\frac{\sqrt{3}}{2}+\sqrt{2}\right)i$$

Question 21

a. Show that $z\bar{z} = |z|^2$ for any complex number z.

LHS $= (a + bi)(a - bi) = a^2 + b^2 =$ RHS
Also works in polar form.

b. Evaluate $\frac{3+10i}{2-5i}$.

$$-\frac{44}{29} + \frac{35}{29}i$$

c. It is known that $z = 1 - 2i$. Find z^4 in rectangular form.

$$-7 - 24i$$

I would recommend finding z^2 then squaring it again.

Question 22

a. Show that for any complex number $z = a + bi$ where $a, b \in R$, $\text{Im}(z^3) = b(3a^2 - b^2)$.

Show by expansion.

b. Hence, prove or disprove the statement $z^3 \in R$ if and only if $z \in R$.

We can show this is false with counterexample, e.g., $z = 1 + \sqrt{3}i$.

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Sub-Section [8.1.3]: Apply De Moivre's Theorem

Question 23

Evaluate the following, leaving your answer in an appropriate form.

a. $\left(3\text{cis}\left(\frac{\pi}{6}\right)\right)^3$

$27i$

b. $(-2+2i)^3$

$16+16i$

c. $\left(2\text{cis}\left(\frac{-\pi}{10}\right)\right)^2$

$$4\text{cis}\left(\frac{-\pi}{5}\right)$$

Question 24

a. Evaluate $\left(1+\sqrt{3}i\right)^{48}$, leaving your answer in the form a^b where a and b are integers.

$$2^{48}$$

b.

i. Convert $2\sqrt{2} - 2\sqrt{2}i$ to polar form.

$$4\text{cis}\left(-\frac{\pi}{4}\right)$$

ii. Hence, evaluate $\frac{(2\sqrt{2}-2\sqrt{2}i)^7}{(-1+\sqrt{3}i)^{12}}$. State your answer in the form $a + bi$, where $a, b \in R$.

$$2\sqrt{2} + 2\sqrt{2}i$$

c. Evaluate $\left(\frac{\sqrt{2}+\sqrt{2}i}{\sqrt{3}+1}\right)^5$, leaving your answer in polar form.

$$\text{cis}\left(\frac{5\pi}{12}\right)$$

Question 25

a. Find the value of n such that $\left(\sqrt{2}+\sqrt{2}i\right)^n = 1024\,\text{cis}(\theta)$ where $\theta \in (-\pi, \pi]$, and state θ.

$n = 10, \theta = \pi/2$

b. Solve for the value(s) of n such that z is purely imaginary.

$$z = 9\text{cis}\left(\frac{\pi}{10}n\right)$$

$$\text{solve}\left(\frac{\pi}{10}\cdot n = \frac{\pi}{2} + \pi\cdot k, n\right) \qquad n = 5\cdot(2\cdot k + 1)$$

$k \in Z$

c. Let $z = \left(6\text{cis}\left(\frac{\pi}{7}\right)\right)^n$. Solve for the values of n such that z is a real number.

$n = 7k, k \in Z$

Question 26

Solve for the value(s) of n such that z is a real number.

$$z = 4\text{cis}\left(\frac{3(\pi + 2)}{8} n\right)$$

$$\text{solve}\left(\frac{3\cdot(\pi+2)}{8}\cdot n=\pi\cdot k,n\right) \qquad n=\frac{8\cdot k\cdot \pi}{3\cdot(\pi+2)}$$

$k \in Z$

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