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VCE Specialist Mathematics ½
Vectors I [6.1]
Workbook

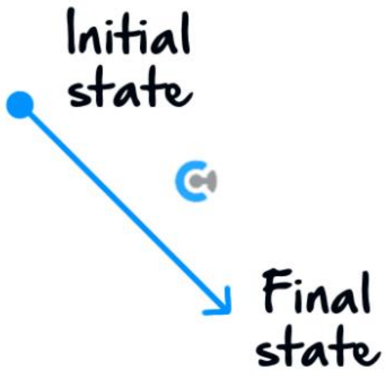
Outline:

Section A: Basics of Vectors

Sub-Section: Introduction to Vectors

What are vectors?

Vectors



- Vectors are mathematical objects with a ______________________________.
- They are used to represent an ____.
- Vectors are best visualised with an arrow between the initial state and the final state.

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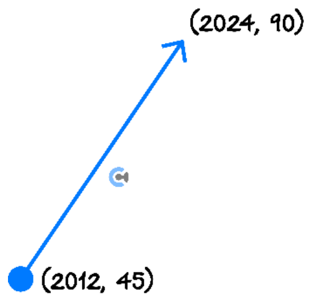
Question 1 Walkthrough.

A vector is drawn in a plane where the vertical component represents weight (in kg) and the horizontal component represents time (in years).

The following vector represents the change James underwent.



a. How much weight did James gain/lose, and over how many years?

b. Describe the vector using $\boldsymbol{y}$ and $\boldsymbol{w}$, given that $\boldsymbol{y}$ represents increase in years by 1 year and $\boldsymbol{w}$ represents increase in weight by 1 kg.

NOTE: When representing vectors, we use tildas!

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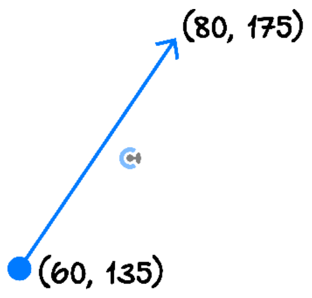
Question 2 Walkthrough.

A vector is drawn in a plane where the vertical component represents height (in cm) and the horizontal component represents weight (in kg).

The following vector represents the change Homan underwent.



a. Describe the change that Homan underwent.

b. Describe the vector using $\boldsymbol{h}$ and $\boldsymbol{w}$, given that $\boldsymbol{h}$ represents increase in height by 1 cm and $\boldsymbol{w}$ represents increase in weight by 1 kg.

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Sub-Section: Vectors vs Scalars

Vectors and Scalars

- Vectors are mathematical objects with a magnitude and size.
- Scalars however only have a _______________.

NOTE: For vectors, we represent them using tildas whereas for scalar we do not.

Question 3

Identify whether the following quantities are vectors or scalars:

a. Mass.

b. Change in mass.

c. Height.

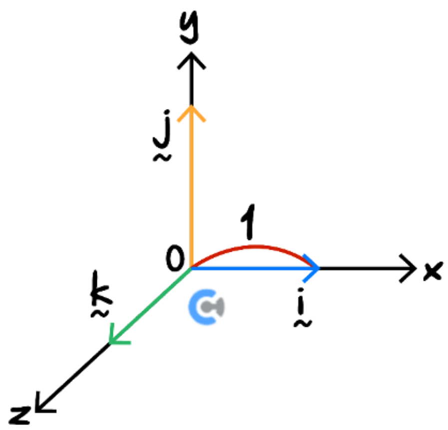
d. Change in height.

Section B: Vectors in Space

Sub-Section: Vectors in $3D$ space

How do we represent movements in 3D space?

Vectors in $3D$ Space



$$v = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$$

- $\boldsymbol{i}$ represents movement in the x-axis by 1 unit.
- $\boldsymbol{j}$ represents movement in the y-axis by 1 unit.
- $\boldsymbol{k}$ represents movement in the z-axis by 1 unit.

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Question 4

Write the vector required for the point $(1,1,1)$ to go to $(2,-3,3)$.

NOTE: $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ must be represented with tildas!

Discussion: Are $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ vectors on their own?

Is there another way to represent the same vector?

Column Vector Notation

$$\boldsymbol{v} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- Column vector can be used to represent vectors.

Question 5

Write the vector in the column vector form, required for the point $(-1,2,1)$ to go to $(3,-2,-1)$.

<u>Discussion:</u> Do vectors have to always start from the same point?

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Calculator Commands: Defining Vectors on Technology

Mathematica

$\boldsymbol{v} = \{x, y, z\}$

TI-Nspire

Either line works.

$[x \;\; y \;\; z] \to v$	$[x \;\; y \;\; z]$
Define $v = [x \;\; y \;\; z]$	*Done*

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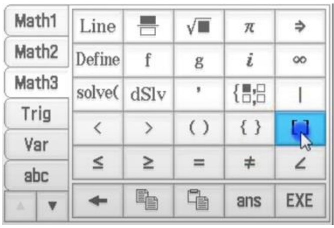
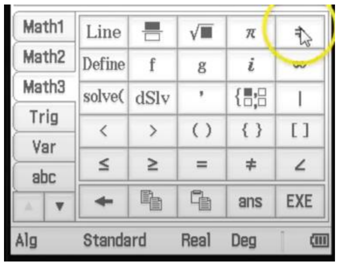
$[x, y, z] \Rightarrow v$





Question 6 Tech-Active.

Define a vector $v = \boldsymbol{i} - 2\boldsymbol{j} - 3\boldsymbol{k}$ on your technology.

NOTE: This will later be important to find other things about vectors!

Sub-Section: Position Vector

Discussion: How do we describe the position of a point using vectors?

Exploration: Position Vectors

- All positions are measured relative to the ________________________.

  Consider a point $P: (2,3)$.

  How do we go from the origin to the point $(2,3)$?

- What vector would describe the change necessary from the origin to the point?

$$\overrightarrow{OP} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Position Vector

$$\overrightarrow{OP} = \textbf{Position Vector of } P$$

- A position vector is a vector describing the position of a point.
- It starts from the ______________ to the _____________________.

Question 7

Find the position vector of $(1, -2, -1)$.

<ins>Discussion</ins>: Do position vectors have to always start from the same point?

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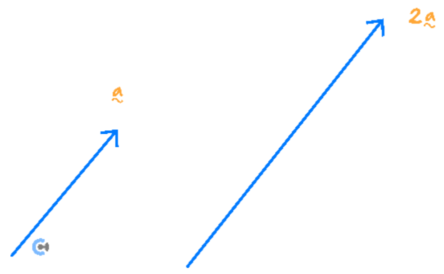
Section C: Operation of Vectors

Sub-Section: Scalar Multiplication

Scalar Multiplication



- Scalar multiplication does not change the ________________ of the vector.
  - It results in the ____ of the vector.

Question 8

Consider a vector $\boldsymbol{m}$ that visually looks like the diagram shown. Draw the vectors in the table.

$2\boldsymbol{m}$	$\frac{1}{2}\boldsymbol{m}$	$-\boldsymbol{m}$	$-2\boldsymbol{m}$

<u>Discussion</u>: Can scalar multiplication change the direction of the vector?

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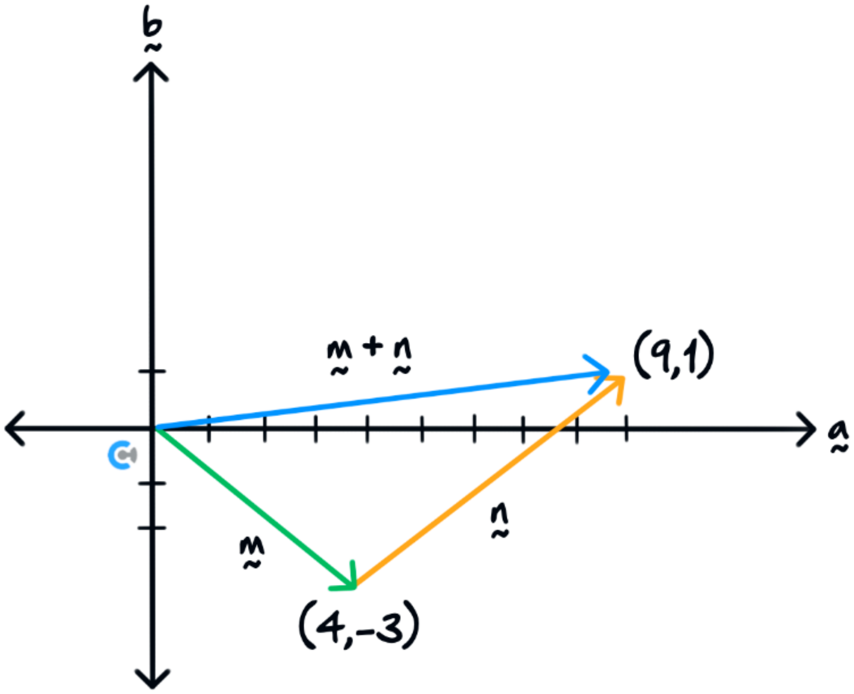
Sub-Section: Addition of Vectors

Discussion: Conceptually, given that vectors represent change, what does adding two vectors give us?

Addition of Vectors

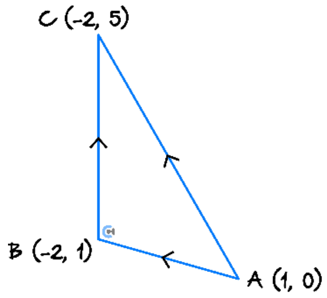


- When two vectors are added, the **sum vector** represents the overall (total) change.
- Visually, we line up the vectors and find the sum vector by joining the starting point to the final ____________________.

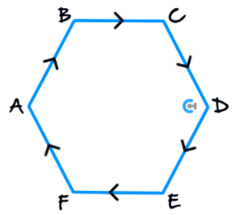
Question 9

Find the sum of the following vectors:

a. $\overrightarrow{AB} + \overrightarrow{BC}$



b. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} + \overrightarrow{FA}$



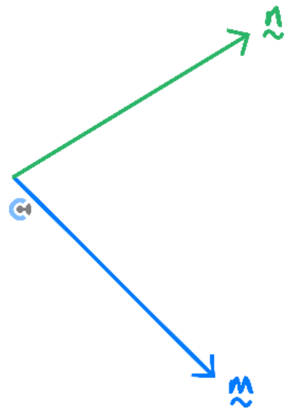
TIP: When adding two vectors with the same intersection:

- The intersection cancels out as the sum of vectors is only concerned with the overall change.
  - We need to get from the very start to the very end. We don't care **HOW** we got there.

$$A? + ?B = AB$$

Sub-Section: Subtraction of Vectors

Subtraction of Vectors



$$\boldsymbol{m} - \boldsymbol{n} = \boldsymbol{m} + (-\boldsymbol{n})$$

- Subtracting a vector is best thought of as adding a ________________________.
  - i.e., if we have to subtract $\boldsymbol{m} - \boldsymbol{n}$, we can instead **add the vectors $\boldsymbol{m}$ and $-\boldsymbol{n}$**.

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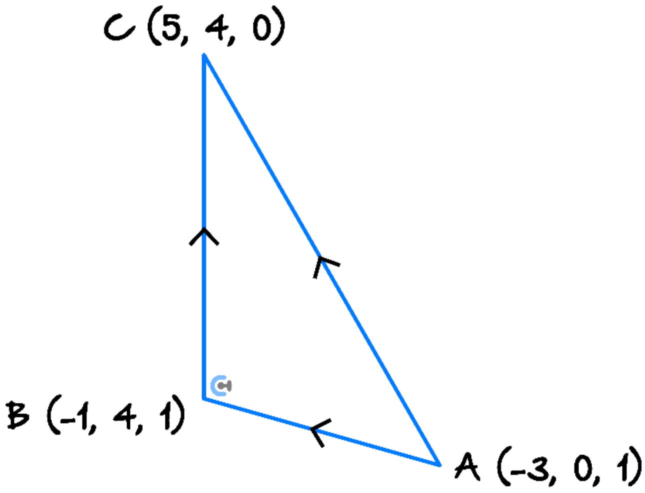
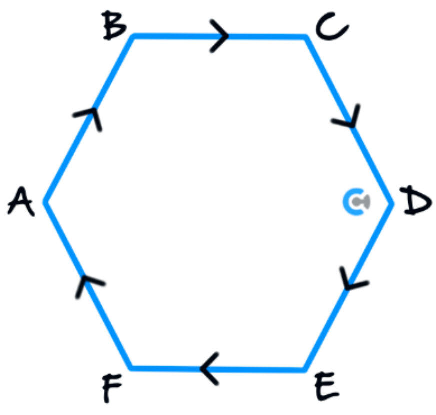
Question 10

Find the sum of the following vectors:

a. $\overrightarrow{AB} + \overrightarrow{BC}$



b. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} - \overrightarrow{AF}$



Section D: Magnitude and Unit Vectors

Sub-Section: Magnitude

How do we find the length of the vector?

Magnitude of a Vector

$$|\boldsymbol{v}| = \sqrt{x^2 + y^2 + z^2}$$

- The **magnitude** of a vector is simply the ________________________________ of the vector.

Question 11

For vector $\boldsymbol{v} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$, find the size of the vector.

Discussion: Where does the formula come from?

Calculator Commands: Finding the Size of Vectors

Mathematica

- $Norm[v]$

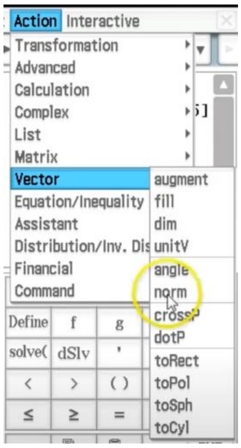
TI-Nspire

- Menu 771 or simply type norm from the keyboard.

$$norm(v)$$

Casio Classpad

- $norm(v)$



Question 12 Tech-Active.

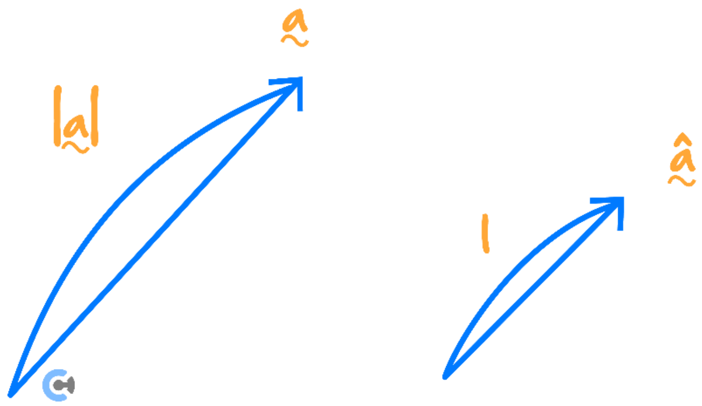
For vector $\boldsymbol{v} = \boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$, find the size of the vector using technology.

Discussion: Is a magnitude of a vector, a vector or a scalar?

Sub-Section: Unit Vector

Discussion: How would you make any vector have the length of 1?

Unit Vector



$$\hat{a} = \frac{a}{|a|}$$

- A unit vector is simply a vector with a ____________________.
- The unit vector of a vector a is denoted by $\hat{a}$ ("a-hat").

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Question 13

Consider the vector $\boldsymbol{v} = 2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$.

a. Find $|\boldsymbol{v}|$.

b. Hence, find the unit vector $\hat{\boldsymbol{v}}$.

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Calculator Commands: Finding Unit Vectors

Mathematica

- Either line works.

$$\boldsymbol{Normalise}[\boldsymbol{v}]$$

$$\frac{\boldsymbol{v}}{\boldsymbol{Norm}[\boldsymbol{v}]}$$

TI-Nspire

- Either line works.
- UnitV: Menu $7C1$ (Or simply type using the keyboard).

$$\boldsymbol{unitV}(\boldsymbol{v})$$

$$\frac{\boldsymbol{v}}{\boldsymbol{norm}(\boldsymbol{v})}$$

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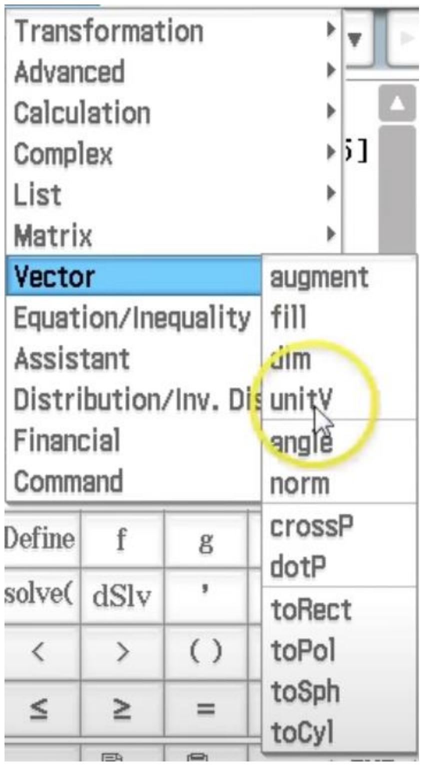
- $unitV(v)$ or $\frac{v}{norm(v)}$



Question 14 Tech-Active.

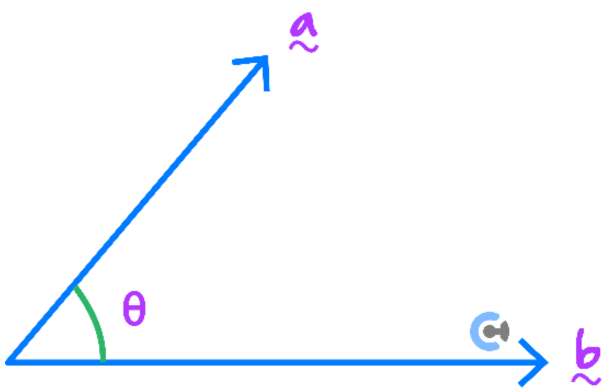
For vector $v = 4\boldsymbol{i} - 2\boldsymbol{j} + 4\boldsymbol{k}$, find the unit vector using technology.

Section E: Dot Product

Sub-Section: Dot Product

Discussion: How do we multiply two vectors?

Dot Product (Scalar Product)



$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Or

$$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}|\, |\boldsymbol{b}| \cos(\theta)$$

Where θ *= Angle between the two vectors*

- Dot(scalar) product of two vectors $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$ and $\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$.

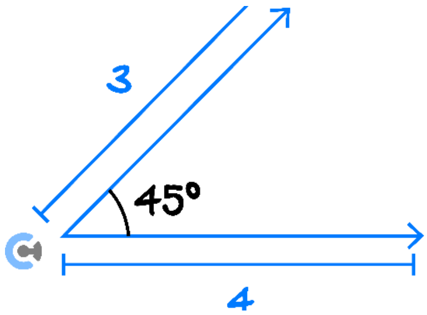
Discussion: Why is a dot product also called a scalar product?

Question 15 Walkthrough.

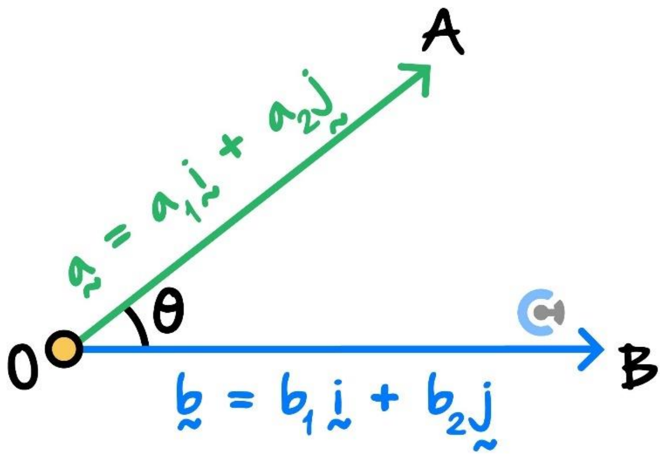
Find the dot product of the following vectors:

a. $\boldsymbol{i} - 5\boldsymbol{j}$ and $2\boldsymbol{i} + \boldsymbol{k}$

b.



<u>Exploration</u>: Proving Dot Products Formula via Cosine Rule



- Step 1: Draw and label $\overrightarrow{AB}$ on the diagram above write it in $\underset{\sim}{i}, \underset{\sim}{j}$ notation in terms of a_1, b_1, a_2, b_2.

- Step 2: Recall the Cosine Rule; write an expression for $|\overrightarrow{AB}|$ in terms of $|\overrightarrow{BA}|$, $|\overrightarrow{OB}|$ & θ.

- Step 3: Use the fact that $\left|x\underset{\sim}{i} + y\underset{\sim}{j}\right|^2 = x^2 + y^2$ to prove the dot product formula:

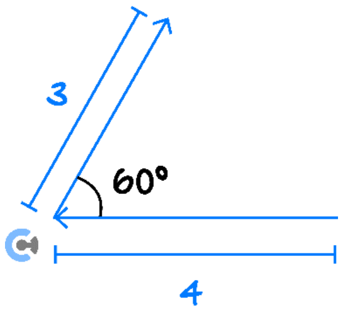
$$\boldsymbol{\underset{\sim}{a} \cdot \underset{\sim}{b} = a_1 b_1 + a_2 b_2 = \left|\underset{\sim}{a}\right| \left|\underset{\sim}{b}\right| \cos(\theta)}$$

Question 16

Find the dot product of the following vectors:

a. $-\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$ and $2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}$

b.



NOTE: Angle is always measured when two vectors' heads or tails are touching.

Question 17

Relative to a fixed origin O, the respective position vectors of three points A, B and C are $3\boldsymbol{i} + 2\boldsymbol{j} + 9\boldsymbol{k}$, $-5\boldsymbol{i} + 11\boldsymbol{j} + 6\boldsymbol{k}$ and $4\boldsymbol{i} - 8\boldsymbol{j}$ respectively. Determine, in component form, the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

Calculator Commands: Dot Products

Mathematica

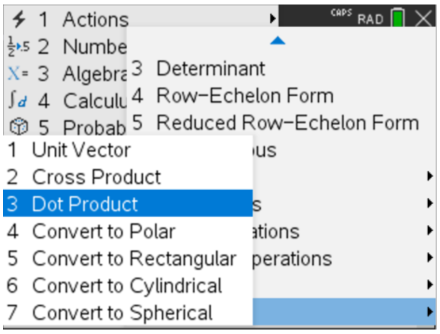
$\boldsymbol{a} \cdot \boldsymbol{b}$

- Put a full stop between two vectors.

TI-Nspire

- Menu $7C3$ or type dot$p$

$\boldsymbol{dotP}(\boldsymbol{a}, \boldsymbol{b})$



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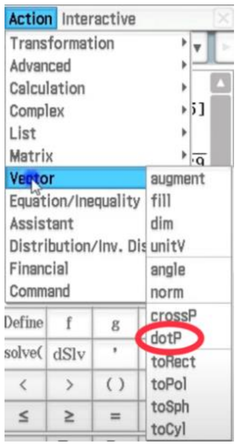
$dotp(a, b)$



Question 18 Tech-Active

Find the dot product of the following vectors using technology.

$$2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k} \text{ and } 4\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$$

Discussion: Okay cool, what is dot product useful for?

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Section F: Parallel and Perpendicular Vectors

Sub-Section: Parallel Vectors

Discussion: Does scalar multiplication change the slope of the vector?

Discussion: Hence, $\boldsymbol{a}$ and $k\boldsymbol{a}$ will be what to each other?

Parallel Vectors

- Vectors along the line must be parallel, hence:

$$\boldsymbol{a} = k\boldsymbol{b}, \qquad k \in \mathbb{R}\backslash\{\mathbf{0}\}$$

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Question 19 Walkthrough.

$$\boldsymbol{a} = 2\boldsymbol{i} - 3m\boldsymbol{j}$$

$$\boldsymbol{b} = -\boldsymbol{i} - 3\boldsymbol{j}$$

Find the value of m such that $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel to each other.

Question 20

$$\boldsymbol{a} = 3\boldsymbol{i} - 5m\boldsymbol{j}$$

$$\boldsymbol{b} = 6\boldsymbol{i} + \boldsymbol{j}$$

Find the value of m such that $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel to each other.

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Sub-Section: Perpendicular Vectors

REMINDERS: Dot Product

$$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}|\cos(\theta)$$

Where θ = Angle between the two vectors

Discussion: What does the dot product equal to if $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular to each other?

Perpendicular Vectors

- Vectors along the line must be perpendicular, hence:

$$\boldsymbol{a} \cdot \boldsymbol{b} = 0$$

Question 21 Walkthrough.

Find the value of m if $\boldsymbol{a} = -2\boldsymbol{i} + \boldsymbol{j}$ and $\boldsymbol{b} = \boldsymbol{i} + m\boldsymbol{j}$ perpendicular to each other.

Question 22

Find the value of m if $\boldsymbol{a} = -3\boldsymbol{i} + 2\boldsymbol{j} + m\boldsymbol{k}$ and $\boldsymbol{b} = \boldsymbol{i} + m\boldsymbol{j} - \boldsymbol{k}$ perpendicular to each other.

Space for Personal Notes

Section G: Angles

Sub-Section: Angle between Two Vectors

How can we find the angle between two vectors?

Exploration: Angle between Two Vectors

- Consider the dot product formula.

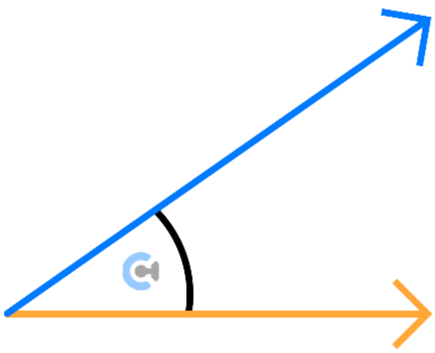
$$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}|\ |\boldsymbol{b}| \cos(\theta)$$

$$\textit{Where } \theta = \textit{Angle between the two vectors}$$

- Try to make the angle theta the subject,

$$\theta = \underline{\hspace{6cm}}$$

  - Which angle did we just find? Try marking where the angle is on the diagram below:



Angle between Vectors

$$\theta = \cos^{-1}\left(\frac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}\right)$$

- When solving for the angle, make sure to align the vectors such that they either face outwards or inwards, but not both.

Question 23 Walkthrough. Tech-Active.

If $\boldsymbol{a} = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = -\boldsymbol{i} + \boldsymbol{k}$, find the cosine of the angle between the vectors and find the angle.

Question 24

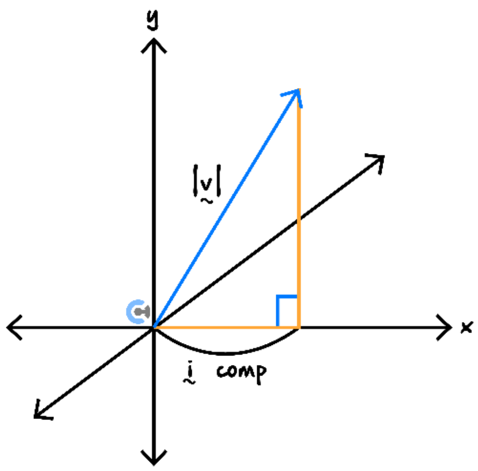
If $\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k}$ and $\boldsymbol{b} = 2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$. Let θ be the angle between the vectors. Find $\sin(\theta)$.

Sub-Section: Angle between a Vector and an Axis

Discussion: Now how do we find the angle between a vector and an axis?

Exploration: Finding the Angle between a Vector and an Axis

- We can simply turn the axis into a vector.
  - The x-axis can be thought of as $\boldsymbol{i}$.
  - The y-axis can be thought of as $\boldsymbol{j}$.
  - The z-axis can be thought of as $\boldsymbol{k}$.



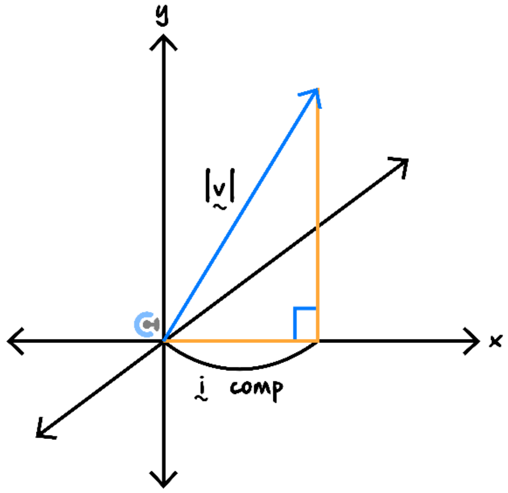
- Try to derive the formulas below for $\boldsymbol{v} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$.

$$\text{Angle between } \boldsymbol{v} \text{ and } x\text{-axis} = \_\_\_\_\_\_\_\_$$

$$\text{Angle between } \boldsymbol{v} \text{ and } y\text{-axis} = \_\_\_\_\_\_\_\_$$

$$\text{Angle between } \boldsymbol{v} \text{ and } z\text{-axis} = \_\_\_\_\_\_\_\_$$

Finding the Angle between a Vector and an Axis



$$\textit{Angle between } \boldsymbol{v} \textit{ and x-axis} = \cos^{-1}\left(\frac{\boldsymbol{v}\cdot\boldsymbol{i}}{|\boldsymbol{v}||\boldsymbol{i}|}\right) = \cos^{-1}\left(\frac{x}{|\boldsymbol{v}|}\right)$$

$$\textit{Angle between } \boldsymbol{v} \textit{ and y-axis} = \cos^{-1}\left(\frac{\boldsymbol{v}\cdot\boldsymbol{j}}{|\boldsymbol{v}||\boldsymbol{j}|}\right) = \cos^{-1}\left(\frac{y}{|\boldsymbol{v}|}\right)$$

$$\textit{Angle between } \boldsymbol{v} \textit{ and z-axis} = \cos^{-1}\left(\frac{\boldsymbol{v}\cdot\boldsymbol{k}}{|\boldsymbol{v}||\boldsymbol{k}|}\right) = \cos^{-1}\left(\frac{z}{|\boldsymbol{v}|}\right)$$

Question 25 Walkthrough. Tech-Active.

Find the angles in degrees correct to two decimal places between the following vector and the y-axis.

$$\boldsymbol{v} = \boldsymbol{i} - 2\boldsymbol{j} - 3\boldsymbol{k}$$

Question 26 Tech-Active.

Find the angles in degrees correct to two decimal places between the following vector and the x-axis.

$$\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$$

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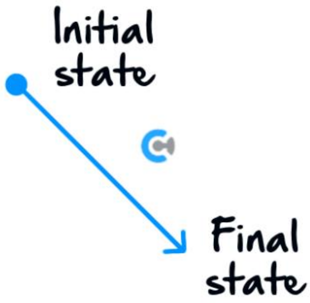
Cheat Sheet

[6.1.1] - Basics of vectors

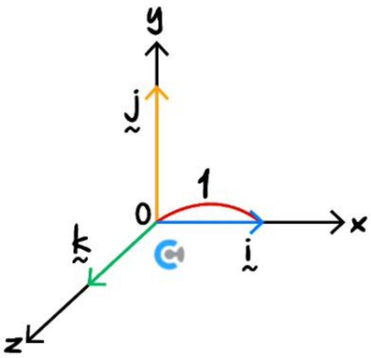
- **Vectors**



- Vectors are mathematical objects with a size and direction.
- They are used to represent an overall change.
- Vectors are best visualised with an arrow between the initial state and the final state.

- **Scalars**
  - Only have a size.

- **Vectors in 3D Space**



$$v = xi + yj + zk$$

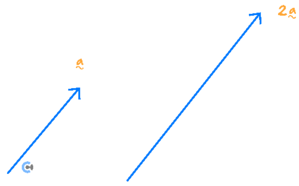
- $\boldsymbol{i}$ represents movement in the x-axis by 1 unit.
- $\boldsymbol{j}$ represents movement in the y-axis by 1 unit.
- $\boldsymbol{k}$ represents movement in the z-axis by 1 unit.

- **Column Vector Notation**

$$v = xi + yj + zk = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

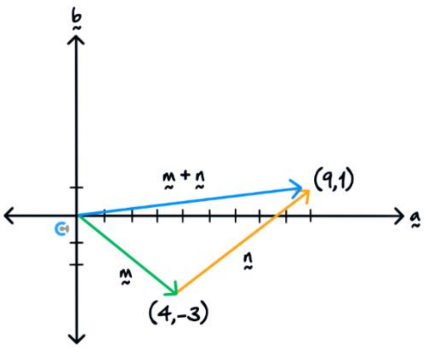
- Column vector can be used to represent vectors.

- **Scalar Multiplication**



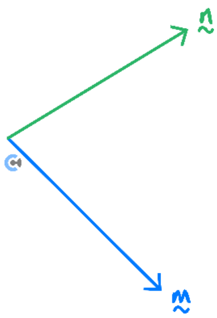
- Scalar multiplication does not change the slope of the vector.
  - It results in the stretching or shrinking of the vector.

- **Addition of Vectors**



- When two vectors are added, the **sum vector** represents the overall (total) change.
- Visually, we line up the vectors and find the sum vector by joining the starting point to the final endpoint.

- **Subtraction of Vectors**



$$\boldsymbol{m} - \boldsymbol{n} = \boldsymbol{m} + (-\boldsymbol{n})$$

- Subtracting a vector is best thought of as adding a negative vector,
  - i.e., if we have to subtract $\boldsymbol{m} - \boldsymbol{n}$, we can instead **add the vectors $\boldsymbol{m}$ and $-\boldsymbol{n}$**.

Cheat Sheet

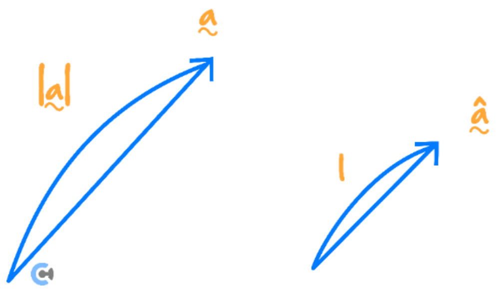
[6.2.2] – Magnitude and unit vectors

- **Magnitude of a Vector**

$$|\boldsymbol{v}| = \sqrt{x^2 + y^2 + z^2}$$

- The **magnitude** of a vector is simply the size or length of the vector.

- **Unit Vector**



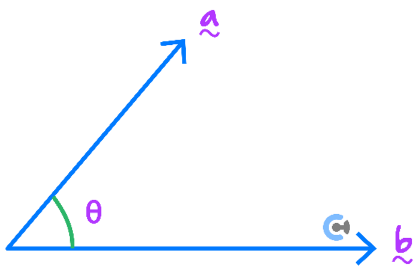
$$\hat{\boldsymbol{a}} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|}$$

- A unit vector is simply a vector with a magnitude of 1.

- The unit vector of a vector a is denoted by $\hat{a}$ ("*a*-hat").

[6.1.3] – Dot Product and angles between vectors

- **Dot Product (Scalar Product)**



$$\boldsymbol{a} \cdot \boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$$

Or

$$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}|\ |\boldsymbol{b}| \cos(\theta)$$

$$\textit{Where } \theta = \textit{Angle between the two vectors}$$

- Dot(scalar) product of two vectors $\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k}$ and $= b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k}$.

- **Angle between Vectors**

$$\boldsymbol{\theta} = \cos^{-1}\left(\frac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}\right)$$

- When solving for the angle, make sure to align the vectors such that they either face outwards or inwards, but not both.

- **Parallel Vectors**

- Vectors along the line must be parallel, hence:

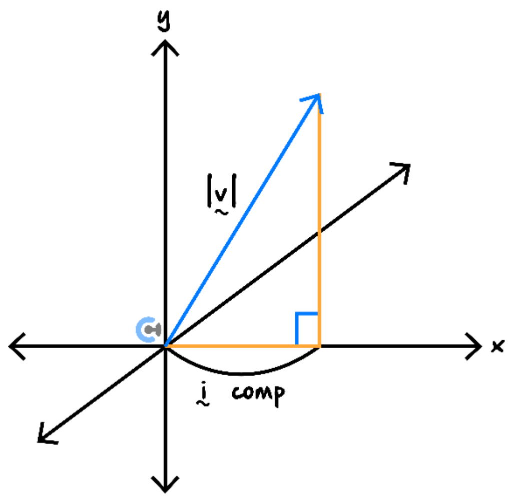
$$\boldsymbol{a} = k\boldsymbol{b}, \qquad k \in \mathbb{R}\backslash\{\boldsymbol{0}\}$$

- **Perpendicular Vectors**

- Vectors along the line must be perpendicular, hence:

$$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{0}$$

- **Finding the Angle between a Vector and an Axis**



$$\textit{Angle between } \boldsymbol{v} \textit{ and x-axis} = \cos^{-1}\left(\frac{\boldsymbol{v} \cdot \boldsymbol{i}}{|\boldsymbol{v}||\boldsymbol{i}|}\right) = \cos^{-1}\left(\frac{x}{|\boldsymbol{v}|}\right)$$

$$\textit{Angle between } \boldsymbol{v} \textit{ and y-axis} = \cos^{-1}\left(\frac{\boldsymbol{v} \cdot \boldsymbol{j}}{|\boldsymbol{v}||\boldsymbol{j}|}\right) = \cos^{-1}\left(\frac{y}{|\boldsymbol{v}|}\right)$$

$$\textit{Angle between } \boldsymbol{v} \textit{ and z-axis} = \cos^{-1}\left(\frac{\boldsymbol{v} \cdot \boldsymbol{k}}{|\boldsymbol{v}||\boldsymbol{k}|}\right) = \cos^{-1}\left(\frac{z}{|\boldsymbol{v}|}\right)$$

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