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VCE Specialist Mathematics ½

Vectors I [6.1]

Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 02-Pg 18
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Section A: Compulsory Questions

Sub-Section [6.1.1]: Basics of Vectors

Question 1

The vector $\boldsymbol{u} = \begin{bmatrix} a \\ b \end{bmatrix}$ is defined by the directed line segment from $(-2, 6)$ to $(4, -2)$. Find a and b.

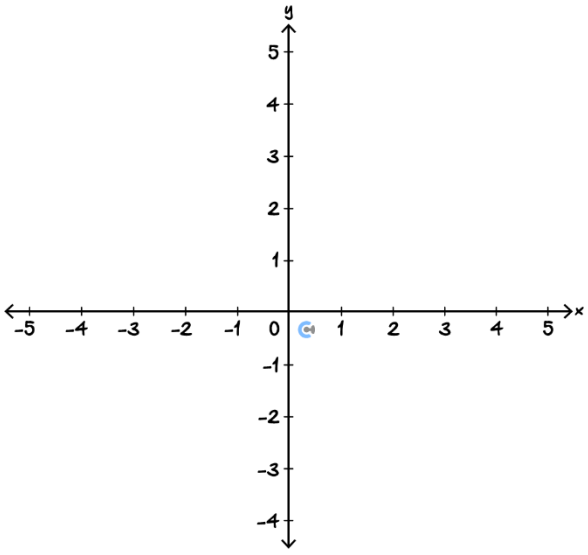
Question 2

$A = (1, 3)$, $B = (4, 1)$ and O is the origin. Sketch the following vectors:

a. $\overrightarrow{OA}$

b. $\overrightarrow{OB}$

c. $\overrightarrow{AB}$



Question 3

If $\boldsymbol{a} = \boldsymbol{i} - 5\boldsymbol{j}$ and $\boldsymbol{b} = -3\boldsymbol{i} + 6\boldsymbol{j}$ find in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$:

a. $a + b$

b. $2a - 3b$

Question 4

In the triangle OAB, $\overrightarrow{OA} = 4\boldsymbol{i} + 2\boldsymbol{j}$ and $\overrightarrow{OB} = \boldsymbol{i} + 3\boldsymbol{j}$. If M is the midpoint of AB, find $\overrightarrow{OM}$ in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.

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Sub-Section [6.1.2]: Magnitude and Unit Vectors

Question 5

a. Find the length of vector $\boldsymbol{v} = 3\boldsymbol{i} - 2\boldsymbol{j}$.

b. Find the unit vector parallel to $\boldsymbol{v}$.

Question 6

Let $\boldsymbol{a} = -\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{b} = 3\boldsymbol{i} + 5\boldsymbol{j}$ .Vector $\boldsymbol{c}$ is parallel to $2\boldsymbol{a} + \boldsymbol{b}$, and has a magnitude of 10.

Find $\boldsymbol{c}$ in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.

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Question 7

$A(2,3)$, $B(4,5)$ and $C(7,2)$ are the vertices of a triangle ABC.

a. Find

i. $\left|\overrightarrow{AB}\right|$

ii. $\left|\overrightarrow{BC}\right|$

iii. $\overrightarrow{AC}$

b. Identify the type of triangle.

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Sub-Section [6.1.3]: Dot Product

Question 8

If $\boldsymbol{u} = -3\boldsymbol{i} + \boldsymbol{j}$ and $= 3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$, find $\boldsymbol{u} \bullet \boldsymbol{v}$.

Question 9

If $|\boldsymbol{u}| = 5$ and $|\boldsymbol{v}| = 6$, and the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is 60°, find $\boldsymbol{u} \bullet \boldsymbol{v}$.

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Question 10 Tech-Active.

Find the angle of the vector $\boldsymbol{a} = 2\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ makes with the positive direction of the z-axis in degrees, correct to two decimal places.

Question 11

A position vector in two dimensions has a magnitude of 4 and a direction of 120° measured anticlockwise from the x-axis. Find the vector.

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Sub-Section: Problem Solving

Question 12

If $\overrightarrow{OA} = 5\boldsymbol{i} + 2\boldsymbol{j} + 7\boldsymbol{k}$ and $\overrightarrow{OB} = 9\boldsymbol{i} + 6\boldsymbol{j} + 7\boldsymbol{k}$. Find $\overrightarrow{AB}$ and hence, show that $\overrightarrow{AB}$ is parallel to the x-y plane.

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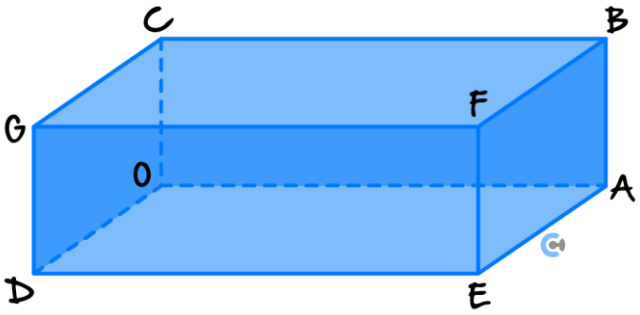
Question 13

Suppose $OABCDEFG$ is a cuboid and that $\overrightarrow{OA} = 5\boldsymbol{j}$, $\overrightarrow{OC} = 2\boldsymbol{k}$, and $\overrightarrow{OD} = 3\boldsymbol{i}$.



a. Express, in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$:

i. $\overrightarrow{OE}$

ii. $\overrightarrow{OF}$

iii. $\overrightarrow{GA}$

b. Let M be the midpoint of face $FEAB$. Find $\overrightarrow{OM}$ in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.

Question 14

Let $\boldsymbol{u} = 4\boldsymbol{i} - 3\boldsymbol{j}$ and $\boldsymbol{v} = 2\boldsymbol{i} - \boldsymbol{j}$.

a. Find the length of vector $\boldsymbol{u}$.

b. Find unit vector parallel to $\boldsymbol{u}$.

c. Find a vector of length 10 parallel to $\boldsymbol{u}$.

d. Find the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$.

e. Write $\boldsymbol{u}$ as the sum of two vectors, one parallel to $\boldsymbol{v}$ and the other perpendicular to $\boldsymbol{v}$.

Question 15

Let $t \in \mathbb{R}$ and suppose $\overrightarrow{OA} = t\boldsymbol{i} + t\boldsymbol{j} + 8\boldsymbol{k}$ and $\overrightarrow{OB} = t\boldsymbol{i} - 2\boldsymbol{j} - 3\boldsymbol{k}$. Find the values of t for which $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$.

Question 16

A, B, C, and D are the vertices of a parallelogram.

Given that $A = (2, 3)$, $B = (7, 6)$ and $C = (10, 1)$, find the coordinates of D in vector form.

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Question 17

The points A and B have position vectors $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}$ respectively.

a.

i. Find the vector $\overrightarrow{AB}$.

ii. Find $\left|\overrightarrow{AB}\right|$.

The point D has a position vector $\begin{bmatrix} d \\ 0 \end{bmatrix}$.

b. Find the vectors $\overrightarrow{AD}$ and $\overrightarrow{DB}$ in terms of d.

c. If angle $\angle ADB$ is 90°, find the two possible values of $\boldsymbol{d}$.

d. For the smaller value of $\boldsymbol{d}$ find the area of the triangle ADB.

e. For the larger value of $\boldsymbol{d}$ show that the triangle is isosceles.

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Sub-Section: The Tech-Free "Final Boss" [VCAA Level]

Question 18

Points A, B, and C have position vectors:

$$\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j}, \boldsymbol{b} = 5\boldsymbol{i} + 2\boldsymbol{j}, \boldsymbol{c} = 3\boldsymbol{i} + 6\boldsymbol{j}$$

Let point D lie on line segment AC, and suppose its position vector is $\boldsymbol{d} = (1 - k)\boldsymbol{a} + k\boldsymbol{c}$, where $0 \leq k \leq 1$.

a.

i. Find the vectors $\overrightarrow{AD}$ and $\overrightarrow{DB}$ in terms of k.

ii. Hence, write an expression of $\overrightarrow{AD} \bullet \overrightarrow{DB}$.

b. If angle $\angle ADB = 90^{\circ}$, use your expression from **part a.** to find the exact value(s) of k that satisfies this condition.

c. For the value(s) of k found in **part b.**, compute the lengths of $\overrightarrow{AD}$ and $\overrightarrow{DB}$, and hence find the area of triangle ADB.

d. Let $\boldsymbol{m} = \boldsymbol{b} - \boldsymbol{a}$ and $\boldsymbol{n} = \boldsymbol{c} - \boldsymbol{a}$.

Use the dot product to find the angle between vectors $\boldsymbol{m}$ and $\boldsymbol{n}$, in degrees.

e. Suppose point E lies on the line AB, with position vector $\boldsymbol{e} = \boldsymbol{a} + t(\boldsymbol{b} - \boldsymbol{a})$.

If the angle between $\boldsymbol{e}$ and $\boldsymbol{c}$ is 60°, find the value of t.

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Sub-Section: The Tech-Active "Final Boss" [VCAA Level]

Question 19

A runner sets off on a bearing of 120° (assume east is in the direction of $\boldsymbol{i}$ and north is in the direction of $\boldsymbol{j}$).

a. Find a unit vector for the direction the runner goes.

b. If the runner runs in this direction for 3 km, find the position of the runner with respect to their starting point.

c. If the runner now turns and runs 6 km south, find the position of the runner with respect to the original starting point.

d. Find the distance of the runner from the starting point.

e. Find the bearing of the runner from their starting point, correct to 2 decimal places.

f. Another runner sets off from the same starting point and runs directly **east** for 4 km.

Find the **angle between the two runners' final displacement vectors**. Round your answer to **2 decimal places**.

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Section B: Supplementary Questions

Sub-Section: Exam 1 (Tech-Free)

Question 20

P is the point $(-1, 3)$, Q is the point $(12, 4)$ and R is the midpoint of PQ.

a. Calculate the lengths of OP and OQ.

b. Find $\overrightarrow{PQ}$ and hence determine the length PQ.

c. Show that ΔPOQ is a right-angled triangle.

d. Find $\overrightarrow{OR}$ and hence show R is equidistant from the three vertices of ΔPOQ.

Question 21

Points A, B, C, and D are defined by position vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$, and $\boldsymbol{d}$ respectively. If $\overrightarrow{AB} + \overrightarrow{CD} = 0$:

a. Express $\boldsymbol{d}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$.

b. Show that AC and BD bisect each other.

c. Prove that $ABCD$ is a rhombus if $|\boldsymbol{a}| = |\boldsymbol{c}|$ and angles AOB and BOC are equal.

Question 22

A pyramid $ABCDV$ has a square base $ABCD$, with vertices A, B, and D having position vectors: $\mathbf{i} - \boldsymbol{j} + \boldsymbol{k}$, $11\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{i} + 5\boldsymbol{j} + 9\boldsymbol{k}$.

a. Verify that sides AB and AD are equal in length and are perpendicular to each other.

b. Determine the coordinates of C, the fourth vertex of the square base.

c. Find the coordinates of P, the point where the diagonals of the square $ABCD$ intersect.

d. If V is defined by $x\boldsymbol{i} + y\boldsymbol{j} + 2\boldsymbol{k}$, and if $\overrightarrow{VP}$ is perpendicular to the two diagonals $\overrightarrow{AC}$ and $\overrightarrow{BD}$ of the base, find x and y.

e. Let M be the midpoint of side AB. Calculate the angle VMP (in degrees).

f. Find the exact volume of the pyramid.

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Sub-Section: Exam 2 (Tech-Active)

Question 23

If vector $\overrightarrow{AB} = \boldsymbol{u}$ and vector $\overrightarrow{BC} = \boldsymbol{v}$ then vector $\overrightarrow{AC}$ is equal to:

A. $\boldsymbol{u} + \boldsymbol{v}$

B. $\boldsymbol{v} - \boldsymbol{u}$

C. $\boldsymbol{u} - \boldsymbol{v}$

D. $\boldsymbol{u} \times \boldsymbol{v}$

Question 24

$ABCD$ is a parallelogram. If $\overrightarrow{AB} = \boldsymbol{a}$ and $\overrightarrow{BC} = \boldsymbol{b}$, then in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$, $\overrightarrow{CA}$ equals:

A. $\boldsymbol{a} + \boldsymbol{b}$

B. $\boldsymbol{a} - \boldsymbol{b}$

C. $-\boldsymbol{b} - \boldsymbol{a}$

D. $\boldsymbol{b} \bullet \boldsymbol{a}$

Question 25

If $\boldsymbol{a} = 3\boldsymbol{i} + 4\boldsymbol{j}$, then the unit vector parallel to $\boldsymbol{a}$ is:

A. $3\boldsymbol{i} + 4\boldsymbol{j}$

B. $\frac{1}{5}(3\boldsymbol{i} + 4\boldsymbol{j})$

C. $\frac{1}{\sqrt{5}}(3\boldsymbol{i} + 4\boldsymbol{j})$

D. $\frac{1}{\sqrt{3}}(3\boldsymbol{i} + 4\boldsymbol{j})$

Question 26

If $\boldsymbol{a} = 3\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k}$ then $\hat{\boldsymbol{a}}$ is:

A. $\frac{1}{6}(3\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k})$

B. $\frac{1}{\sqrt{17}}(3\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k})$

C. $\frac{1}{7}(3\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k})$

D. $\frac{1}{\sqrt{13}}(3\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k})$

Question 27

If vector $a = 3\boldsymbol{i} + 5\boldsymbol{j} - 2\boldsymbol{k}$ is parallel to vector $\boldsymbol{b} = a\boldsymbol{i} + b\boldsymbol{j} - 4\boldsymbol{k}$, then:

A. $a = 3$ and $b = 5$

B. $a = 5$ and $b = 10$

C. $a = 6$ and $b = 10$

D. $a = 1$ and $b = 2$

E. $a = 9$ and $b = 15$

Question 28

A and B are points on a plane such that $\overrightarrow{OA} = 4\boldsymbol{i} + 3\boldsymbol{j}$ and $\overrightarrow{OB} = 2\boldsymbol{i} - 5\boldsymbol{j}$. If M is the midpoint of the line segment B, then $\overrightarrow{MO}$ equals:

A. $\frac{3}{2}\boldsymbol{i} - 4\boldsymbol{j}$

B. $-3\boldsymbol{i} + \boldsymbol{j}$

C. $-\frac{3}{2}\boldsymbol{i} + 4\boldsymbol{j}$

D. $3\boldsymbol{i} - \boldsymbol{j}$

Question 29

Given $|\boldsymbol{a}| = 3, |\boldsymbol{b}| = 4$ and $\boldsymbol{a} \bullet \boldsymbol{b} = 5$, the value of $|\boldsymbol{a} - \boldsymbol{b}|$ is:

A. $\sqrt{7}$

B. $\sqrt{15}$

C. 1

D. 15

Question 30

If vector $\boldsymbol{b} = 4\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$, the angle $\boldsymbol{b}$ makes with the z-axis is closest to:

A. 38°

B. 52°

C. 101°

D. 54°

Question 31

A two-dimensional unit vector that is perpendicular to $3\boldsymbol{i} + 4\boldsymbol{j}$, is:

A. $3\boldsymbol{i} - 4\boldsymbol{j}$

B. $\frac{4}{5}\boldsymbol{i} + \frac{3}{5}\boldsymbol{j}$

C. $\frac{4}{5}\boldsymbol{i} - \frac{3}{5}\boldsymbol{j}$

D. $\frac{3}{5}\boldsymbol{i} + \frac{4}{5}\boldsymbol{j}$

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Question 32

If $(2x\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k}) \bullet (-3\boldsymbol{i} + 2x\boldsymbol{j} - 4\boldsymbol{k}) = 8$, then x is equal to:

A. $\frac{4}{3}$

B. 2

C. 3

D. -2

Question 33

Two drones depart from the same base.

Drone M moves in the direction $4\boldsymbol{i} + 3\boldsymbol{j}$ and Drone N moves in the direction $-6\boldsymbol{i} + 8\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the East and North directions, respectively (with 1 unit representing 1 kilometre).

a. Find the unit vector representing the direction of:

i. Drone M.

_____

_____

ii. Drone N.

_____

_____

b. Using the vector method, find the distance between the two drones if Drone M travels for $12\ km$ and Drone N travels for $16\ km$.

c. Find the angle between the directions of the two drones.

d. On a different mission, Drone M flies in the direction $5\boldsymbol{i} - \sqrt{7}\boldsymbol{j}$.

Drone N must fly at $90°$ to Drone M's direction.

Find two possible unit vectors for Drone N's direction in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.

Question 34

Two ships leave from the same port.
Ship X sails in the direction of $5\boldsymbol{i} + 12\boldsymbol{j}$, and Ship Y sails in the direction of $7\boldsymbol{i} - 24\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors pointing East and North, respectively (with 1 unit representing 1 kilometre).

a. Find the unit vector representing the direction of:

i. Ship X.

ii. Ship Y.

b. Using a vector method, find the distance between the two ships if Ship X travels for 15 km and Ship Y travels for 10 km. Round your answer to one decimal place.

c. Use a vector method to find the angle between the directions of the two ships. Round your answer to one decimal place.

d. On another journey, Ship X again sails in the direction $2\boldsymbol{i} + \sqrt{5}\boldsymbol{j}$. Ship Y departs from and sails at 60° in the direction of Ship X. Find two possible unit vectors for Ship Y's direction in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$.

Question 35

Two cyclists leave from the same place, O. Cyclist A heads in a direction of $3\boldsymbol{i} + 4\boldsymbol{j}$ and Cyclist B in a direction of $2.5\boldsymbol{i} - 6\boldsymbol{j}$ where $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the East and North directions, respectively with 1 unit representing 1 kilometre.

a. Find the unit vector representing the direction of:

i. Cyclist A.

ii. Cyclist B.

b. Find the **midpoint** between the two cyclists after they finish their journey where each cyclist ends up after travelling for 2.5 km (Cyclist A) and 26 km (Cyclist B) respectively. Express the coordinates.

c. Use a vector method to find the angle between the direction of travel of the two cyclists. Round to one decimal place.

d. Another time, the cyclists again leave from the same place, O. Cyclist A heads in the direction $3\boldsymbol{i} + \sqrt{3}\boldsymbol{j}$.

Cyclist B sets off at 120° to Cyclist A. Give two possible unit vectors in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$ for the direction of Cyclist B.

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