**Website:** contoureducation.com.au | **Phone:** 1800 888 300
**Email:** hello@contoureducation.com.au

# VCE Specialist Mathematics ½
## Graph Theory II [5.4]
## Workbook

**Outline:**

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**Walk of a Graph (12D)** Pg 5-14
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- Length of a Walk
- Counting the Number of Walks

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- Identifying Whether a Graph has an Euler Trail
- Finding Euler Trails Efficiently
- Definition of Euler Circuit
- Identifying Whether a Graph has an Euler Circuit

**Hamiltonian Paths and Cycles (12C)** Pg 29-33
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- Definition of Hamiltonian Cycles

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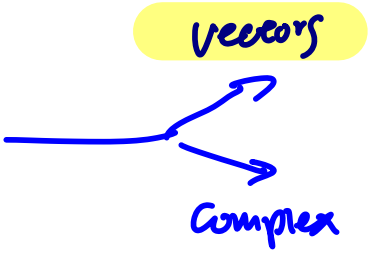
- Spanning Tree

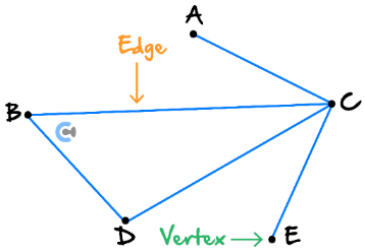


# Section A: Recap of Graph Theory I

## Cheat Sheet

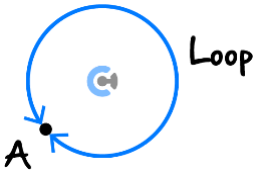
### [5.3.1] – Graph theory fundamentals – vertices, edges, degree, adjacency lists and matrices

#### Vertices and Edges



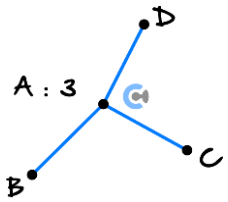

- A graph consists of a set of points called vertices and a set of unordered pairs of vertices, called edges.

#### Loops



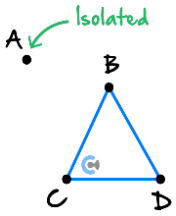

- Loop is an edge which connects to the same vertex.

#### Degree of a Vertex



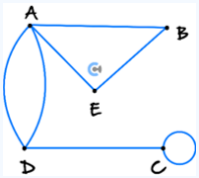

- Degree of a vertex is the number of edges connected to the vertex.

#### Isolated Vertex




- Isolated vertex has no edges connected to it.
- Its degree is equal to _____.

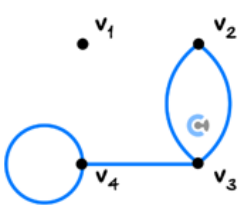
#### Adjacency Lists

| Graph | Adjacency List |
|---|---|
|  | $A \to (B, D, D, E)$<br>$B \to (A, E)$<br>$C \to (C, D)$<br>$D \to (A, A, C)$<br>$E \to (A, B)$ |



- Adjacency list contains all the vertices a given vertex is connected to.
- If the point is connected multiple times, we write the vertex multiple times.
- If a point is looped with itself, we write the vertex to be adjacent to itself.

#### Adjacency Matrix



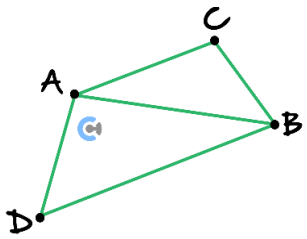

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

- A matrix that represents the vertices and edges that connect the vertices of a graph.

# Cheat Sheet

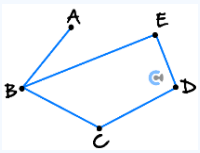
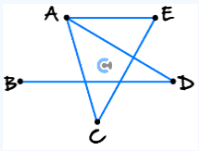
## [5.3.2] – Types of graphs

### Simple Graph




- A simple graph is one in which pairs of vertices are connected by _____.

### The Complement of a Simple Graph

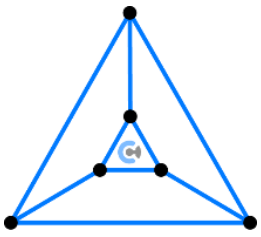
| Simple Graph | The Complement |
|---|---|
|  |  |



**Complement of $G = \overline{G}$**

- The complement of a simple graph contains the same set of vertices.
- But it contains a complementary set of edges. (Edges that the original graph does not have.)

### Regular Graphs



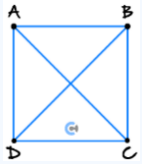
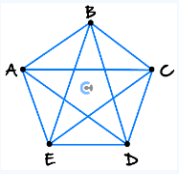
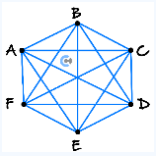
- Regular graph has all its vertices with the same _____.
- If each vertex has a degree $r$ then the graph is "regular of degree $r$" or "$r$-regular".

### Number of Edges and Degree of All Vertices of a Regular Graph

**Number of Edges $\times$ 2 = Total Degree of all Vertices**

- The above result is commonly known as the Handshake Lemma.

### Complete Graph $(K_n)$

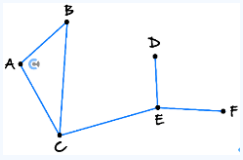
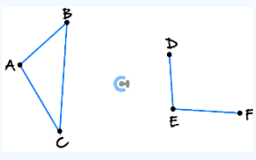
| $K_4$ | $K_5$ | $K_6$ |
|---|---|---|
|  |  |  |



- A complete graph is a simple graph in which each vertex is connected to every other vertex.
- A complete graph is denoted by $K_n$, where $n$ is the number of vertices in the graph.
- A complete graph is a type of regular graph.
- For $K_n$:

**Number of Edges for Complete Graph = _____**

### Connected Graphs

| Connected Graph | Disconnected Graph |
|---|---|
|  |  |

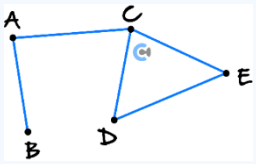
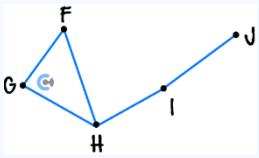


- A connected graph is a graph where it is possible to reach all vertices by moving along edges.
- A graph that is not connected is called a disconnected graph.

# Cheat Sheet

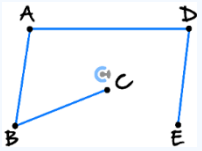
## [5.3.3] – Isomorphisms and subgraphs

### Isomorphism

| Isomorphic Graph | |
|---|---|
|   |   |

- Two graphs are **isomorphic** if their vertices and edges differ only by how they are named.
- **Checklist for Determining Isomorphism**
  - Are the number of vertices the same in each graph?
  - Are the numbers of edges the same in each graph?
  - Check that the degrees of each vertex match for both graphs.
  - Label each vertex on both graphs and check if there is a correspondence between the vertices.

### Subgraphs

| Original Graph | Subgraph |
|---|---|
|   | |

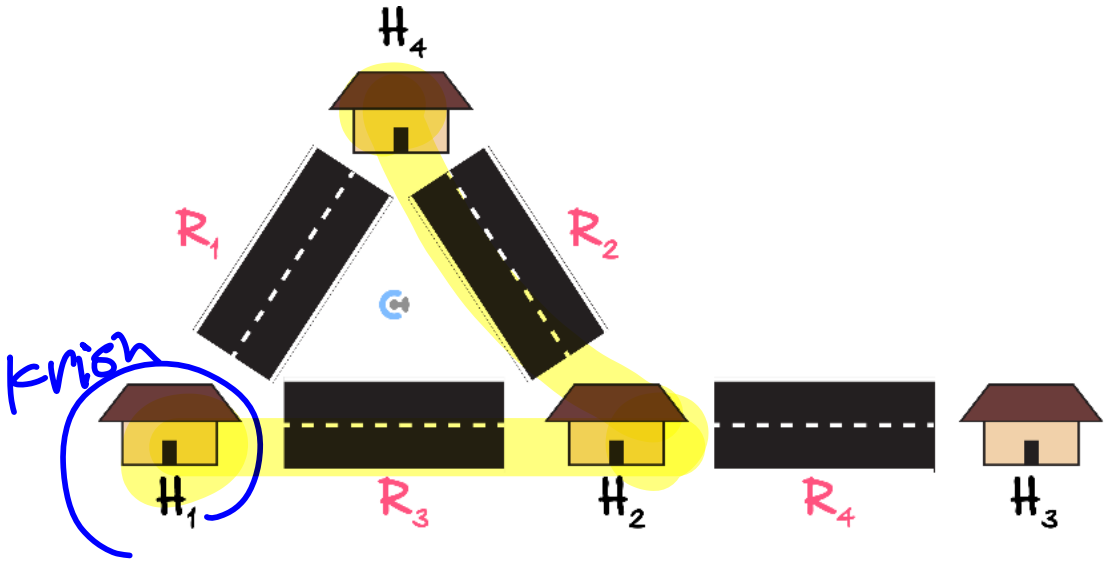
- A subgraph is a graph whose vertices and edges are all contained within the original graph.
- A subgraph can be created by removing edges and vertices from the original graph.

## Section B: Walk of a Graph (12D)

### Sub-Section: Walk of a Graph

**Analogy: Walk of a Graph**

- Krish goes around the Contour village.




- He starts from $H_1$ and visits $H_2$ then $H_4$.

**How can we describe Krish's path?**

- We can describe his path with the houses he visited and the road he took.

$$H_1, R_3, H_2, R_2, H_4$$

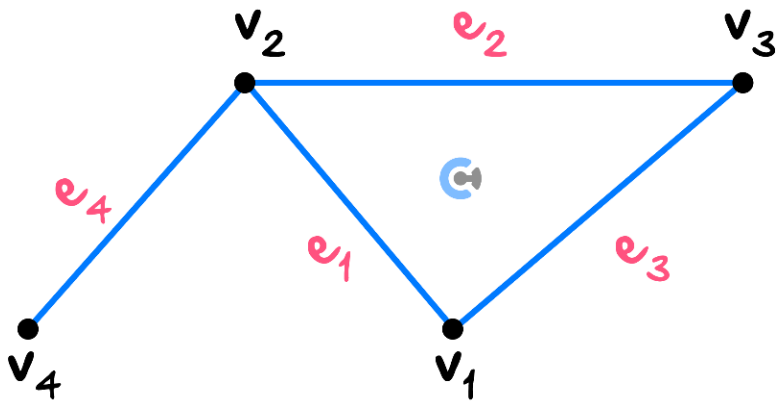
**Walk of a Graph**

$$v_1, e_1, \cdots, v_{n-1}, e_1, v_n$$

- Order of the vertices (points) and edges (lines) we are visiting.
- Alternates between a vertex and an edge.
- If there is only one edge between two vertices, stating an edge can be abbreviate.

**Question 1**

Consider the graph below:




State the walk that visits all vertices once, starting with $v_1$ and ending with $v_4$. (Vertices do not have to be in an ascending order.)

$v_1, e_3, v_3, e_2, v_2, e_4, v_4$

**Discussion:** Did we need to write the edges for the previous question?

NO

**Space for Personal Notes**

## Sub-Section: Length of a Walk

**Discussion:** How many edges did we have to go through for the previous question?

3.

*This is called the length of a walk!*

**Length of a Walk**

**Length of a Walk = Number of Edges in a Walk**

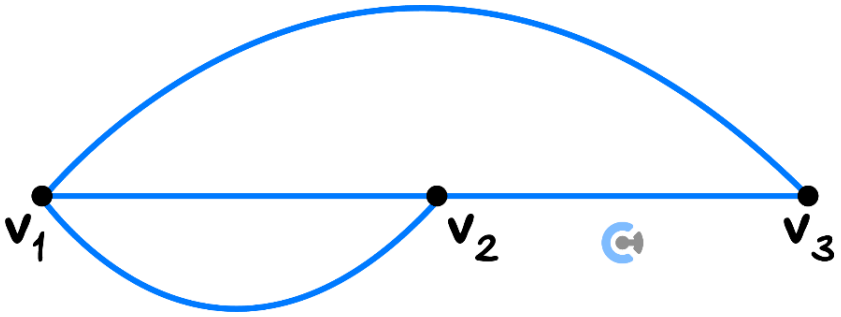
- Number of edges in a walk.
- If we use the same edge multiple times, we count it multiple times for the length.

**Space for Personal Notes**

## Sub-Section: Counting the Number of Walks

**Question 2 Walkthrough.**

Consider a graph and its adjacency matrix.




Adjacency matrix $A = \begin{matrix} \\ v_1 \\ v_2 \\ v_3 \end{matrix} \begin{matrix} \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \\ \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{matrix}$

**a.** State the number of walks of length 1 from $v_1$ to $v_2$.

2.

**b.** State the element of the adjacency matrix: $A_{1,2}$.

2

**c.** Hence, what do you notice?

adjacency matrix = no. of walks of length 1.

Space for Personal Notes

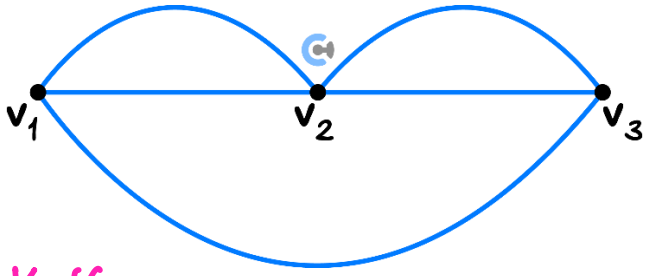
## Number of Walks with the Length of 1

$$A = \begin{bmatrix} v_1, v_1 & v_1, v_2 & v_1, v_3 \\ v_1, v_2 & v_2, v_2 & v_2, v_3 \\ v_1, v_3 & v_3, v_2 & v_3, v_3 \end{bmatrix}$$

- The element gives the number of walks of length 1 between two vertices.

**Question 3**

Consider a graph and its adjacency matrix:




Adjacency matrix $A = \begin{matrix} \\ v_1 \\ v_2 \\ v_3 \end{matrix} \begin{matrix} \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \\ \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \end{matrix}$

Find the number of walks of length 1 between:

**a.** $v_1$ and $v_2$.

2

**b.** $v_2$ and $v_3$.

2

**c.** $v_1$ and $v_3$.

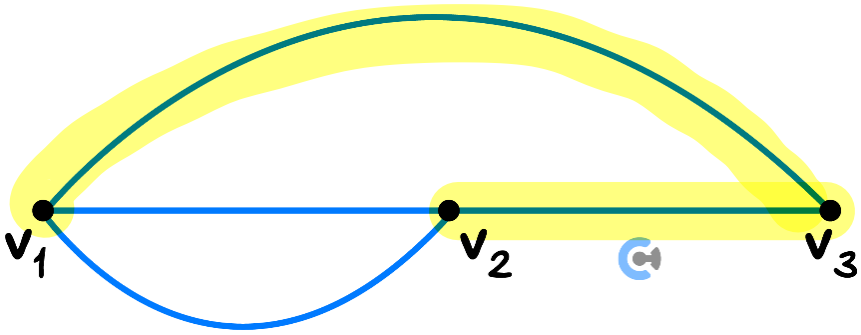
1

Space for Personal Notes

*Let's now consider walks of length 2.*

**Question 4**

Consider a graph and its adjacency matrix:




Adjacency matrix $A = \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

**a.** State the number of walks of length 2 from $v_1$ to $v_2$.

1 walk (length 2)

**b.** Find $A^2$.

$$\begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{matrix} \\ v_1 \\ v_2 \\ v_3 \end{matrix} \begin{matrix} \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \\ \begin{bmatrix} 5 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix} \end{matrix} \quad A^2$$

**c.** State the element of the adjacency matrix: $A^2_{1,2}$.

Row, Column

1

**d.** Hence, what do you notice?

Same

$A^n$ — n is the length of the walk

Definition

**Number of Walks with the Length of $n$**

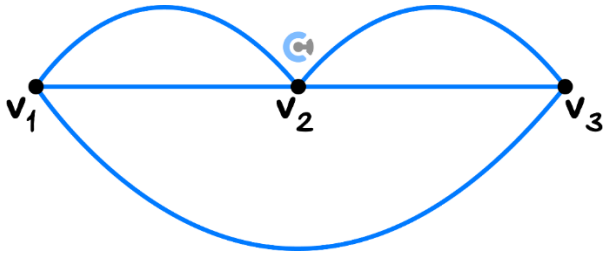
$$A^n = \begin{bmatrix} v_1, v_1 & v_1, v_2 & v_1, v_3 \\ v_1, v_2 & v_2, v_2 & v_2, v_3 \\ v_1, v_3 & v_3, v_2 & v_3, v_3 \end{bmatrix}$$

- For a walk of length $n$, we power the adjacency matrix by _____.
- The element gives the number of walks of length $n$ between two vertices.

**Space for Personal Notes**

**Question 5**

Consider a graph and its adjacency matrix:




Adjacency matrix $A = \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$

**a.** Find $A^2$ length 2

$$\begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} = \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} \begin{bmatrix} 5 & 2 & 4 \\ 2 & 8 & 2 \\ 4 & 2 & 5 \end{bmatrix}$$

$v_1$ $v_2$ $v_3$

**b.** Find the number of walks of length 2 between $v_1$ and itself.

5

**c.** Find the number of walks of length 2 between $v_1$ and $v_2$.

2

**d.** Find the number of walks of length 2 between $v_2$ and $v_3$.

2

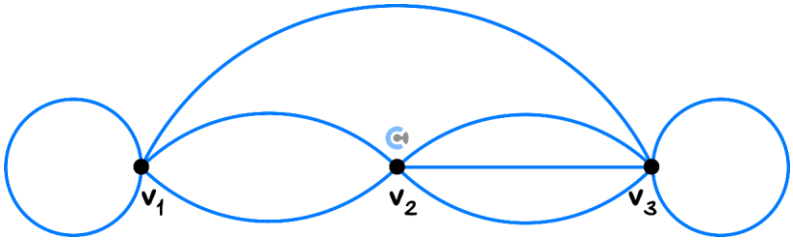
**e.** Find the number of walks of length 2 between $v_1$ and $v_3$.

4

*How does this work?*

**Exploration: Counting the Number of Walks from the Adjacency Matrix**

- Consider the following graph with the following adjacency matrix:




$$\textbf{Adjacency Matrix} = \begin{bmatrix} n(v_1 \to v_1) & n(v_1 \to v_2) & n(v_1 \to v_3) \\ n(v_2 \to v_1) & n(v_2 \to v_2) & n(v_2 \to v_3) \\ n(v_3 \to v_1) & n(v_3 \to v_2) & n(v_3 \to v_3) \end{bmatrix}$$

- How many ways are there to go from $v_1$ to $v_3$ with the length of 2?

  State the possible order of the vertices we can go through.

$v_1 \to v_1 \to v_3$

$v_1 \to v_2 \to v_3$

$v_1 \to v_3 \to v_3$

- The number of ways to achieve each path would be a multiple of what two numbers?

$n(v_1 \to v_1 \to v_3) = n(v_1 \to v_1) \times n(v_1 \to v_3)$

$n(v_1 \to v_2 \to v_3) = n(v_1 \to v_2) \times n(v_2 \to v_3)$

$n(v_1 \to v_3 \to v_3) = n(v_1 \to v_3) \times n(v_3 \to v_3)$

- Now let's consider our adjacency matrix **squared.**

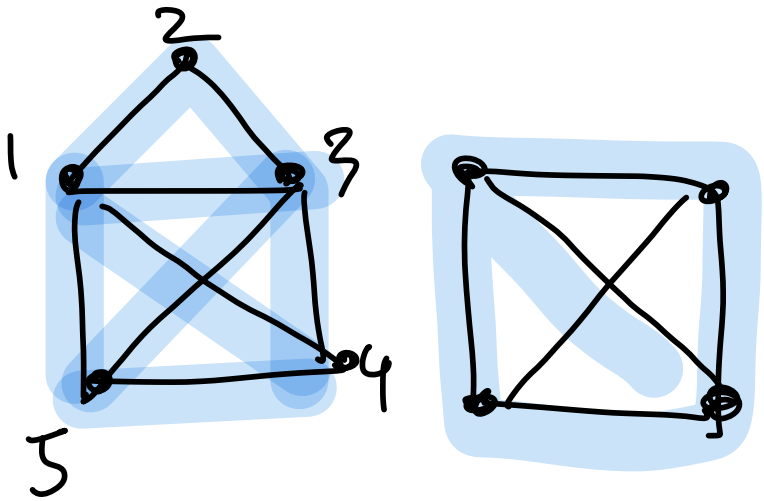
$$\begin{bmatrix} n(v_1 \to v_1) & n(v_1 \to v_2) & n(v_1 \to v_3) \\ n(v_2 \to v_1) & n(v_2 \to v_2) & n(v_2 \to v_3) \\ n(v_3 \to v_1) & n(v_3 \to v_2) & n(v_3 \to v_3) \end{bmatrix} \cdot \begin{bmatrix} n(v_1 \to v_1) & n(v_1 \to v_2) & n(v_1 \to v_3) \\ n(v_2 \to v_1) & n(v_2 \to v_2) & n(v_2 \to v_3) \\ n(v_3 \to v_1) & n(v_3 \to v_2) & n(v_3 \to v_3) \end{bmatrix}$$

- What do we get when we combine the first row of the first matrix $(v_1)$ and the third column of the second matrix $(v_3)$?

$$n(\quad \rightarrow \quad) \times n(\quad \rightarrow \quad) + n(\quad \rightarrow \quad) \times n(\quad \rightarrow \quad) + n(\quad \rightarrow \quad) \times n(\quad \rightarrow \quad)$$

- What do you notice?

**Space for Personal Notes**

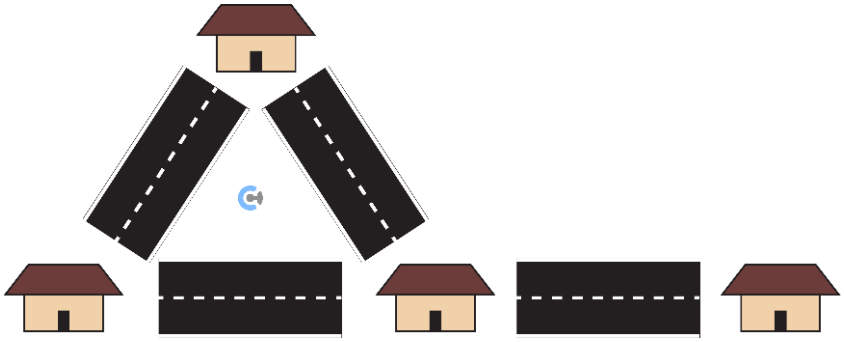

2
1
3
4
5

## Section C: Euler Trail and Circuits (12B)

### Sub-Section: Definition of Euler Trails

**Analogy: Understanding Euler Trails**

- Krish is faced with a dilemma.

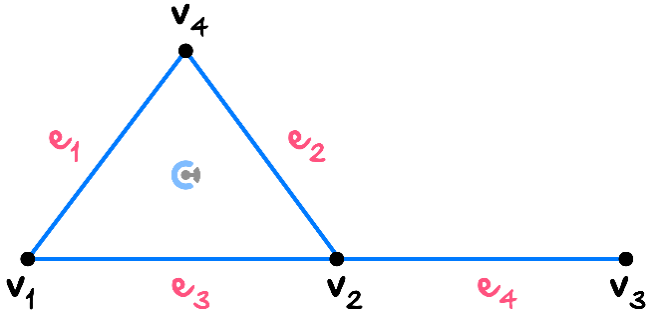


How do I cover all the roads exactly once?

Is that even possible?

- What Krish is trying to achieve is called Euler train.
- Euler Trail is a walk of a graph where all the edges (roads) are used exactly once.

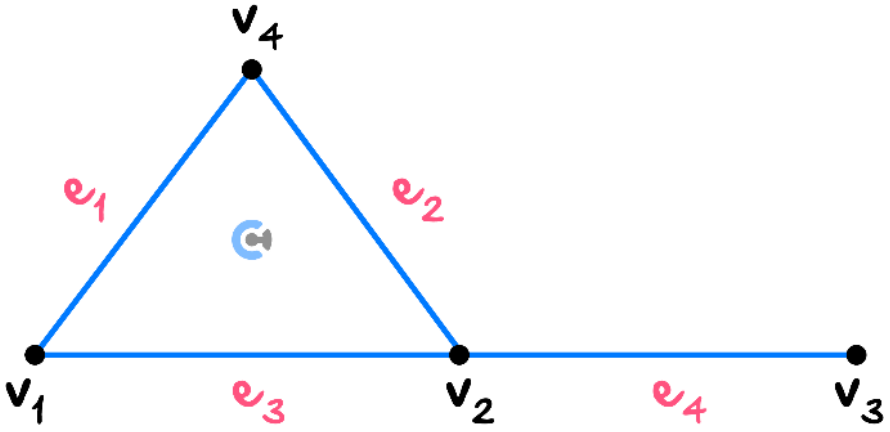
**Euler Trail**




- Euler Trail is a walk of a graph where all the edges are used exactly once.
- Whether a vertex is used exactly once is irrelevant.

**Question 6 Walkthrough.**

Consider a graph with an Euler trail.




one

State all possible Euler trails.

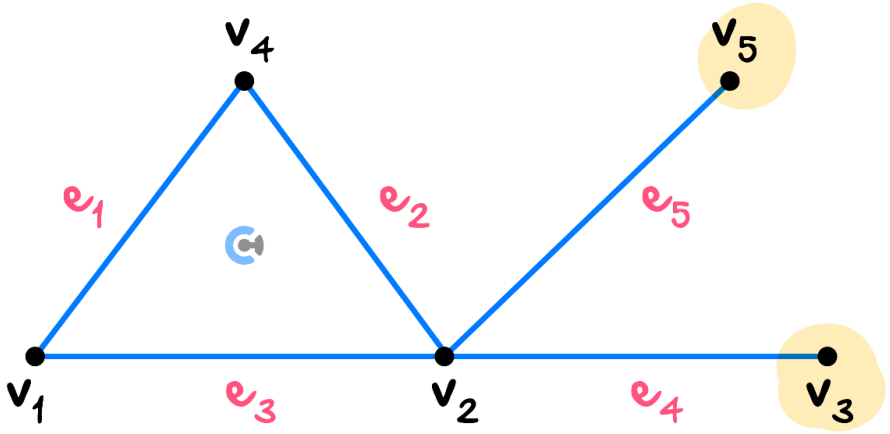
$v_3 e_4 v_2 e_3 v_1 e_1 v_4 e_2 v_2$

**NOTE:** A vertex can be visited twice. We are focusing on the edge being used exactly once.

**Space for Personal Notes**

**Question 7**

Consider a graph with an Euler trail.




one

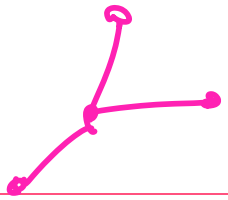
State all possible Euler trails.

$v_5 v_2 v_4 v_1 v_2 v_3$

Space for Personal Notes

## Sub-Section: Identifying Whether a Graph has an Euler Trail

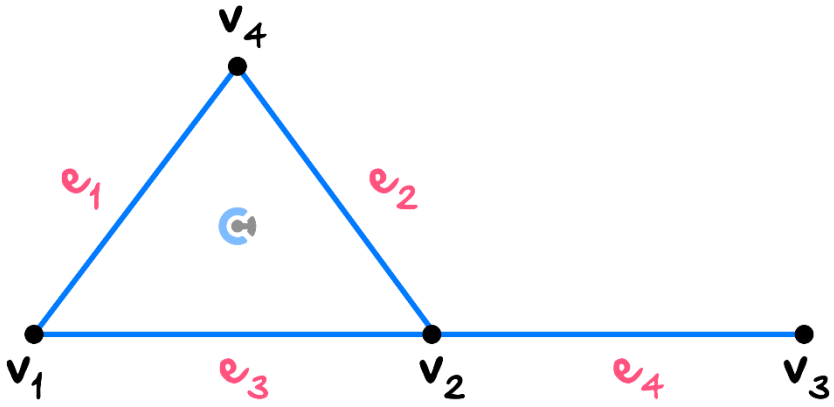
**Discussion:** Does every graph have a valid Euler trail?



No!

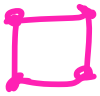
*Let's take a look at how we can tell if a graph has an Euler trail!*

### Existence of an Euler Trail




- A graph needs to satisfy one of the following two rules:

1. Every vertex has an even degree.



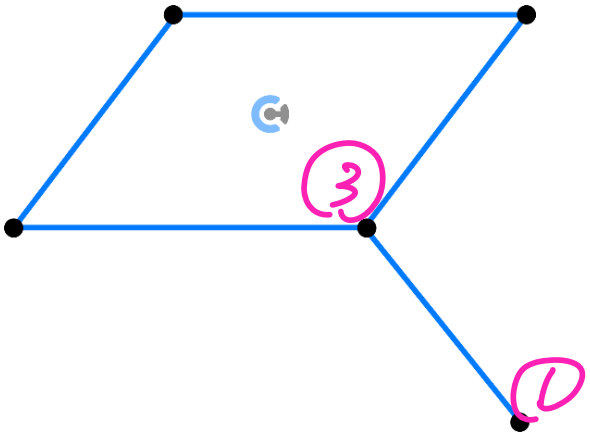
OR

2. Exactly two vertices have an odd degree. (end points)

**Space for Personal Notes**

**Question 8 Walkthrough.**

Consider the graph below and state whether or not it will have an Euler trail.




Euler trail

**TIP:** This can get very confusing. Always write down the degree of each vertex first.

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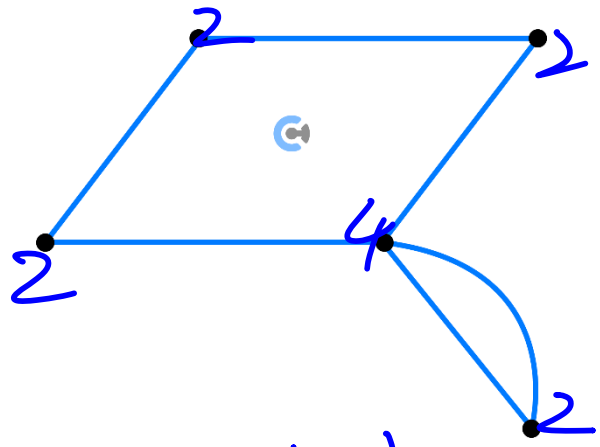
**Question 9**

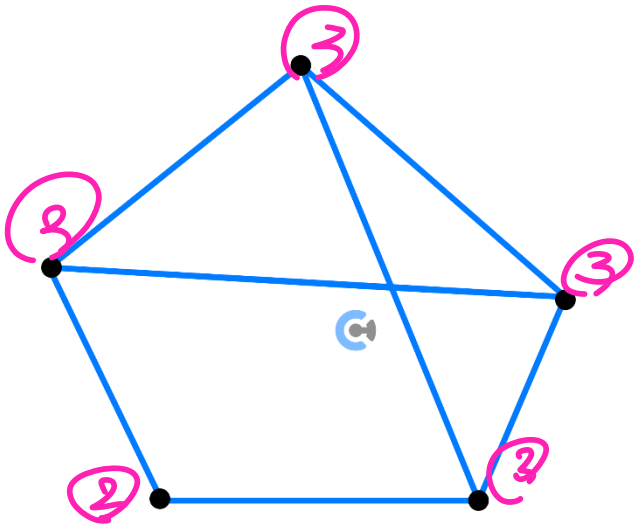
Consider the graphs below and state whether or not they will have an Euler trail.

**a.**




all even ⇒ Yes!

**b.**




4 odd ⇒ No!

Space for Personal Notes

## Sub-Section: Finding Euler Trails Efficiently

*Is there any more efficient way of finding the Euler trail?*

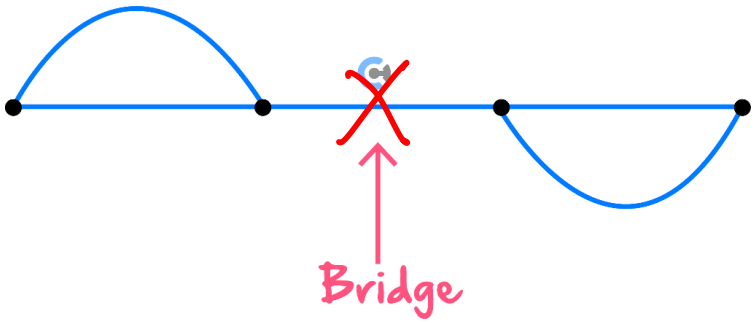
### Fleury's Algorithm

- Allows us to find Euler's trail efficiently in graphs with Euler trails.
- The steps are as follows:

1. If there are two vertices of odd degrees, then start from one of them. Otherwise, start from any vertex.
2. Move from the current vertex across an edge to an adjacent vertex. Always choose a non-bridge edge unless there is no alternative.
3. Delete the edge that you have just traversed.
4. Repeat from Step 2 until there are no edges left.

*Wait, what's a bridge?*

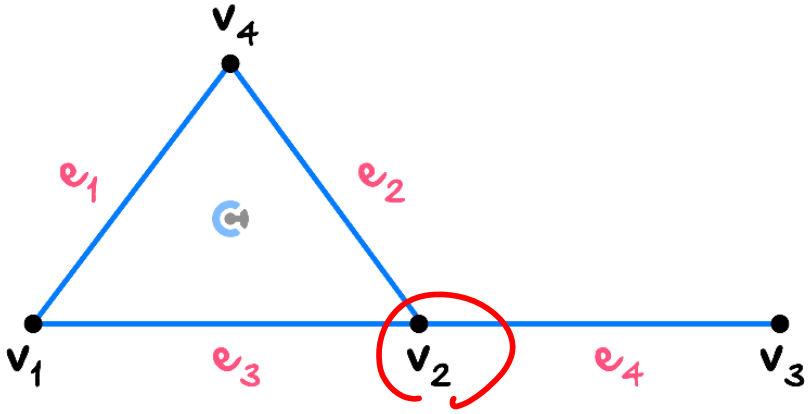
### Bridge




- A bridge is an edge that, if you delete it, will cause a graph to be cut into two graphs.

**Question 10 Walkthrough.**

Consider a graph with an Euler trail.




State any possible Euler trail using Fleury's algorithm.

① Odd ? → start there

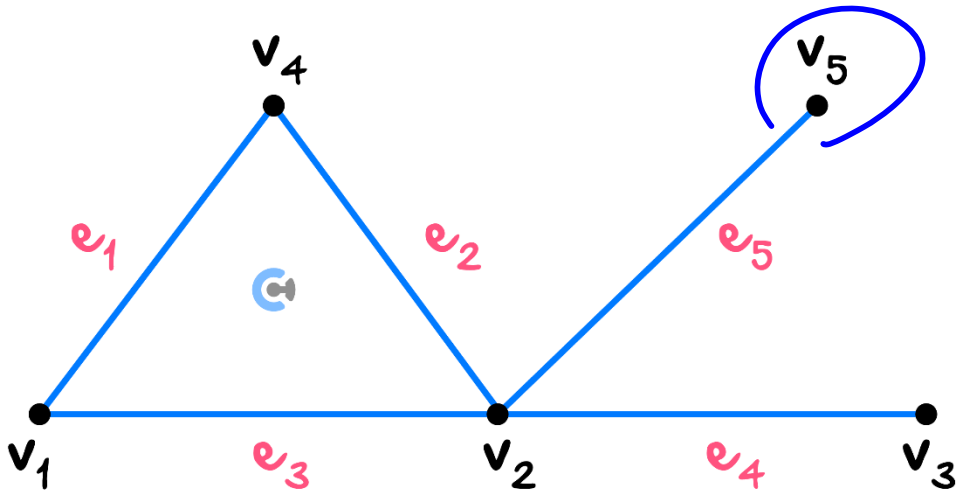
$V_2\ V_4\ V_1\ V_2\ V_3$

**NOTE:** A vertex can be visited twice. We are focusing on the edge being used exactly once.

**Space for Personal Notes**

**Question 11**

Consider a graph with an Euler trail.




State any possible Euler trail using Fleury's algorithm.

odd

$v_5\ v_2\ v_4\ v_1\ v_2\ v_3$

**NOTE:** Hope you got the same answer as before!

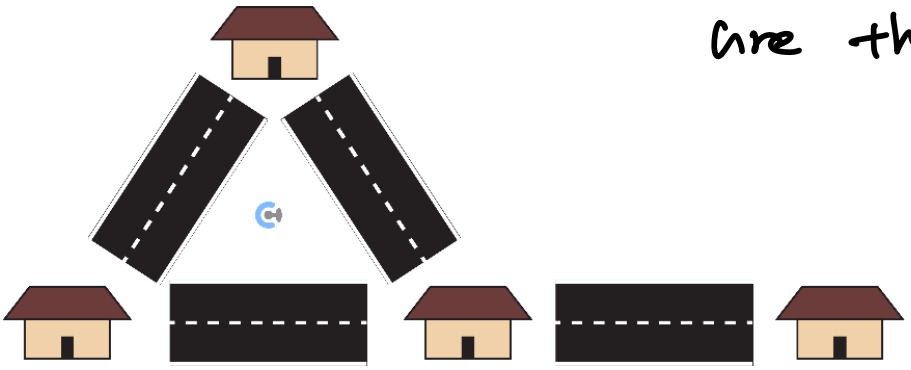
**Space for Personal Notes**

## Sub-Section: Definition of Euler Circuit

Start & end are the same vertex

**Analogy: Understanding Euler Circuits**

- Krish is back with his dilemma.



*How do I cover all the roads exactly once?*

*Is that even possible?*

- Krish realises his mum wants him to also come back home!

  And now asks the question,

**Is there a way to cover the roads exactly once**

**AND come back to the same position at the start?**

- Krish now wants to form an Euler trail that also comes back to the same starting point.

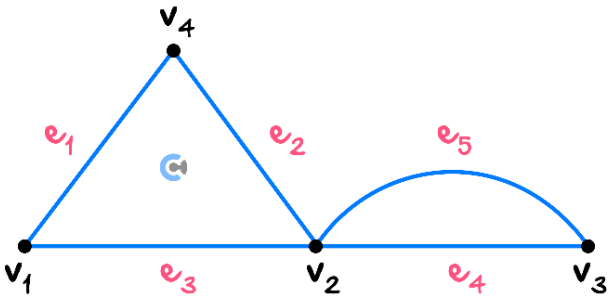
  We call this Euler circuit.

**Discussion: Is the Euler circuit still an Euler trail?**

Trail

Circuit

## Euler Circuits



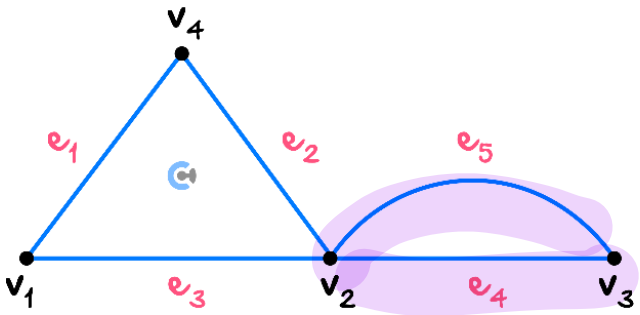

- Euler Circuit is a walk of a graph where all the edges are used exactly once.

  Starting vertex and ending vertex must be the same ⇒ vertices are all even.

- Whether a vertex is used exactly once is irrelevant.
- Euler circuits are a subset of Euler trails.

**Question 12 Walkthrough.**

Consider a graph with an Euler circuit:




State a possible Euler circuit.

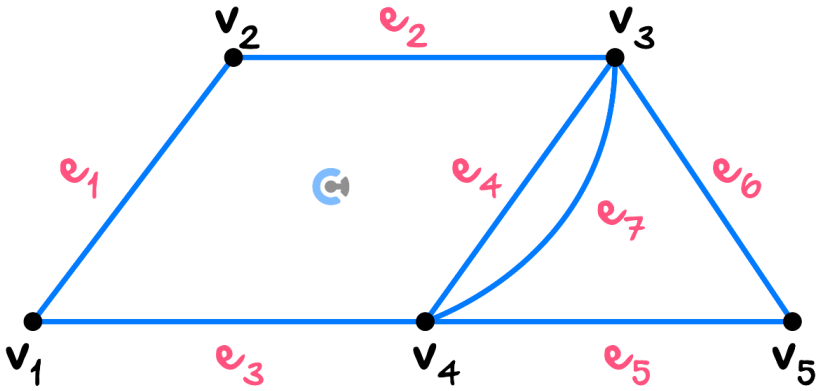
$v_1, v_4, v_2, e_1, v_3, e_4, v_2, v_1$

Same

**NOTE:** We must come back to the same position.

**Question 13**

Consider a graph with an Euler circuit:




State a possible Euler circuit.

$V_5 V_3, e_7, V_4, e_4, V_3, V_2, V_1, V_4, V_5$

Space for Personal Notes

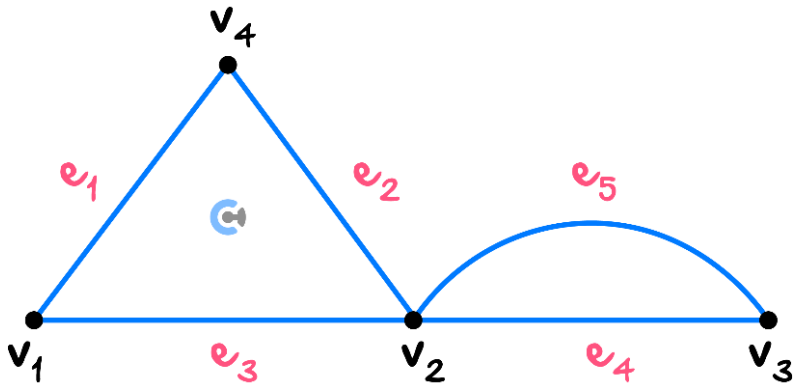
## Sub-Section: Identifying Whether a Graph has an Euler Circuit

**Discussion:** Does every graph have a valid Euler circuit?

No.

*Let's take a look at how we can tell if a graph has an Euler circuit!*

### Existence of an Euler Circuit




- A graph needs to satisfy the following rule:
  - Every vertex has an even degree.

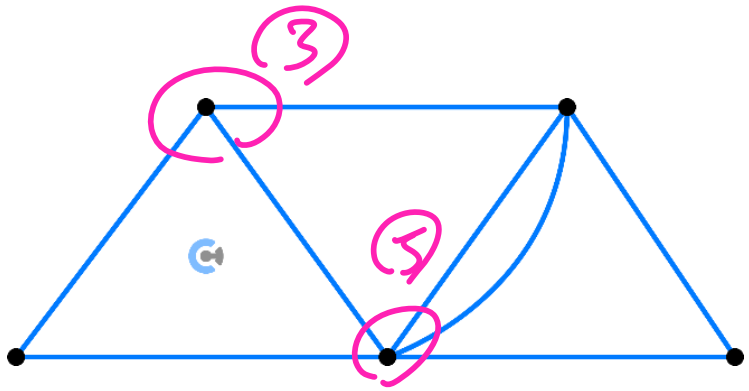
**NOTE:** It's simply the first condition of the Euler trail (but not the second).

**Space for Personal Notes**

**Question 14**

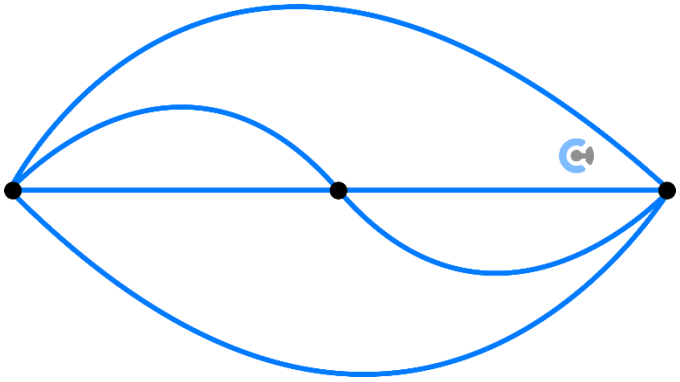
Consider the graphs below and state whether or not they will have an Euler circuit.

**a.**




No!

**b.**



even

Yes!!

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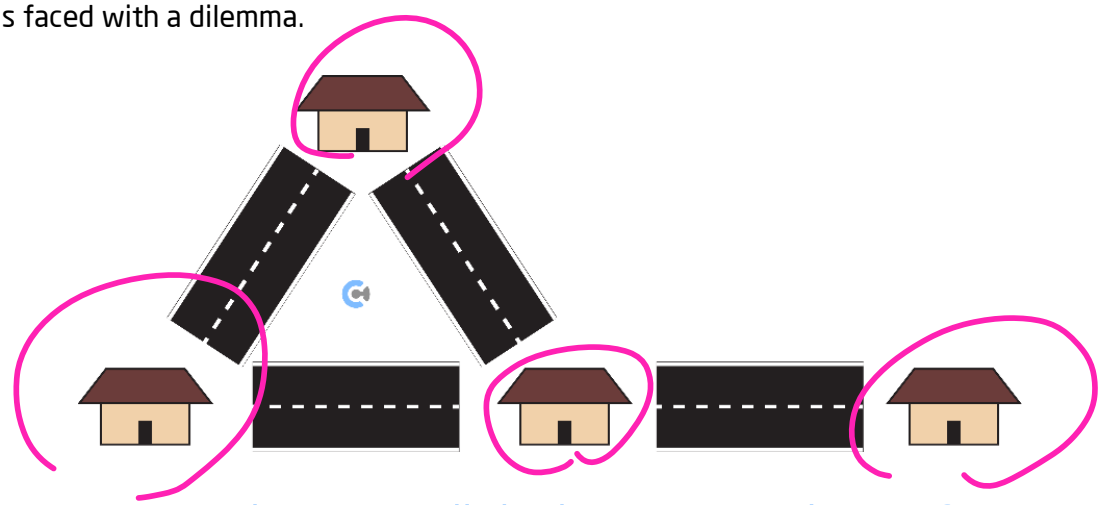
# Section D: Hamiltonian Paths and Cycles (12C)

## Sub-Section: Definition of Hamiltonian Paths

**Analogy: Understanding Hamiltonian Paths**

- Krish is faced with a dilemma.



How do I cover all the houses exactly once?

Is that even possible?

- What Krish is trying to achieve is called Hamiltonian Path.
- Hamiltonian path is a walk of a graph where all the vertices(houses) are used exactly once.

**NOTE:** Similar to Euler, but we are now looking at vertices instead of edges.

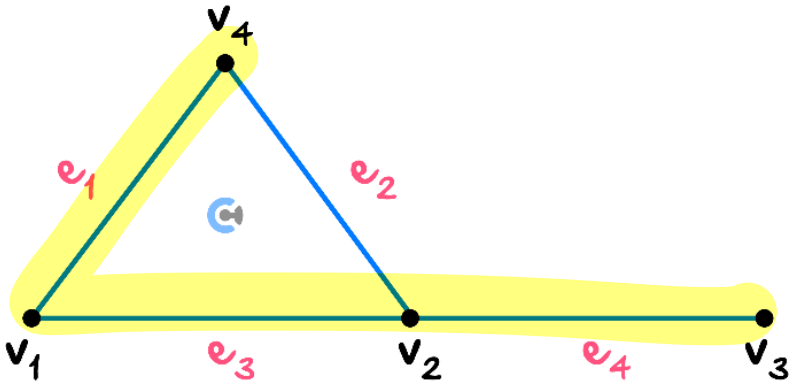
**Hamiltonian Path**

- A path that covers all the vertices exactly once.

**Space for Personal Notes**

**Question 15 Walkthrough.**

Consider a graph with a Hamiltonian path:




State any possible Hamiltonian path.

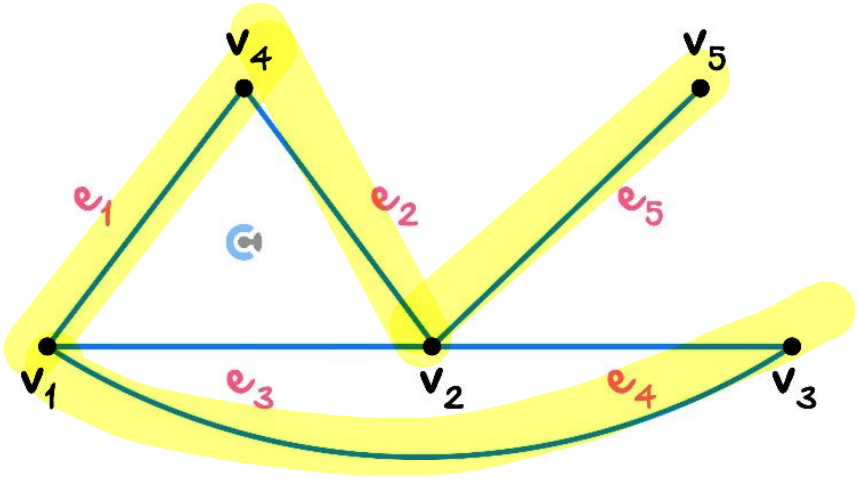
$v_4, v_1, v_2, v_3$

**NOTE:** An edge can be visited twice. We are focusing on the vertex being visited exactly once.

Space for Personal Notes

**Question 16**

Consider a graph with a Hamiltonian path:




State any possible Hamiltonian path.

$v_5, v_4, v_4, v_1, v_3$

Space for Personal Notes

## Sub-Section: Definition of Hamiltonian Cycles

**Discussion:** What do you think the Hamiltonian cycle means?

Every vertex once, Start & end is the same

↳ This one is twice

**Hamiltonian Cycle**

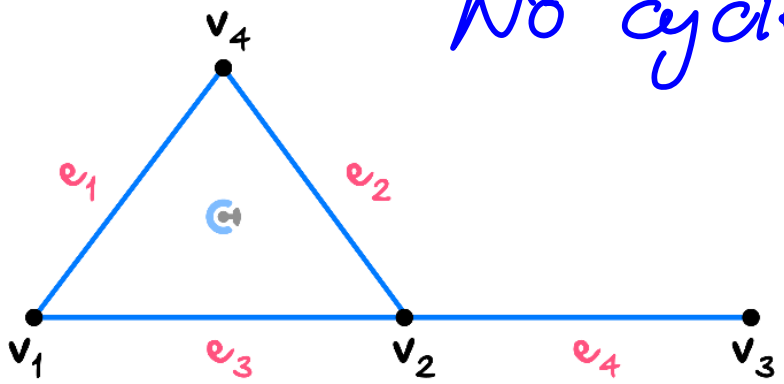
- A path which covers all the vertices exactly once (Except the starting one).

  The starting position and ending vertex must be identical.

**Question 17 Walkthrough.**

Consider the graph below:

No cycle.

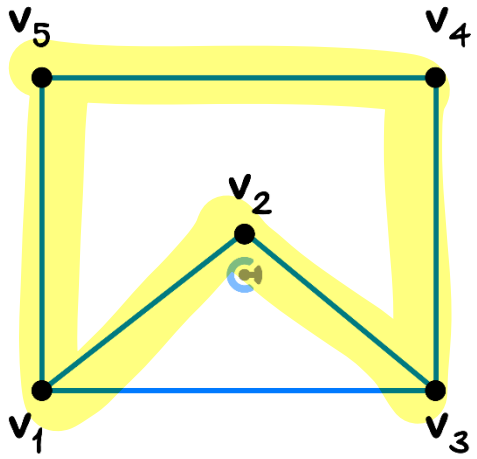



Does it have a Hamiltonian cycle? If so, state below.

**Space for Personal Notes**

**Question 18**

Consider the graph below:




Does it have a Hamiltonian cycle? If so, state below.

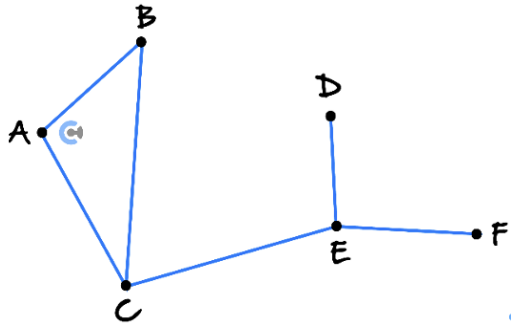
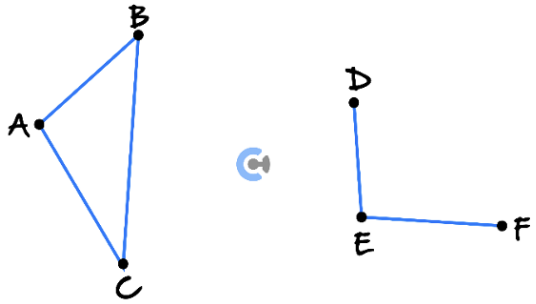
Yes $V_3, V_2, V_1, V_5, V_4, V_3$

Space for Personal Notes

# Section E: Trees

## Sub-Section: Definition of Trees

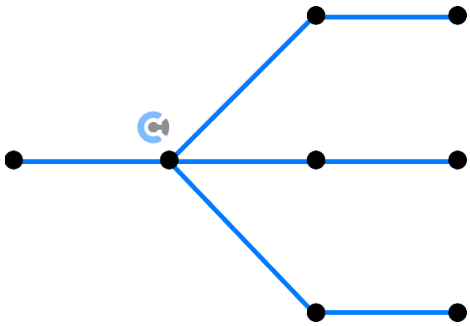
REMINDER: Connected Graphs

| Connected Graph | Disconnected Graph |
|---|---|
|  |  |



- A connected graph is a graph where it is possible to reach all vertices by moving along edges.

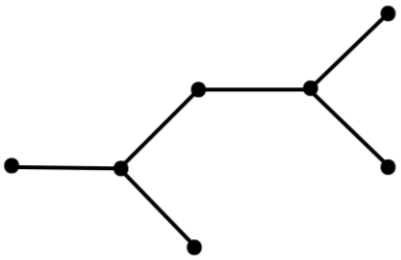
Trees

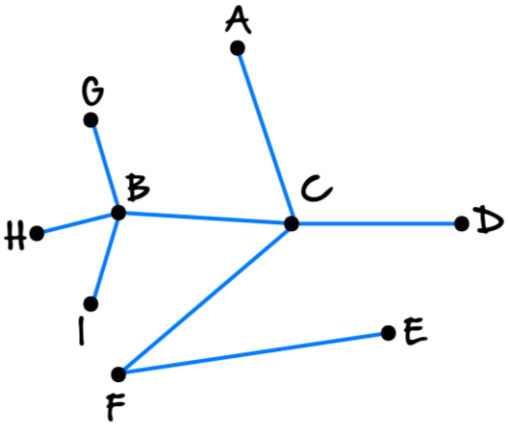


- A tree is a connected graph (can go from one vertex to any other vertex) without any cycle.
- We cannot come back to the same vertex by not repeating the edges.

Space for Personal Notes

## Trees



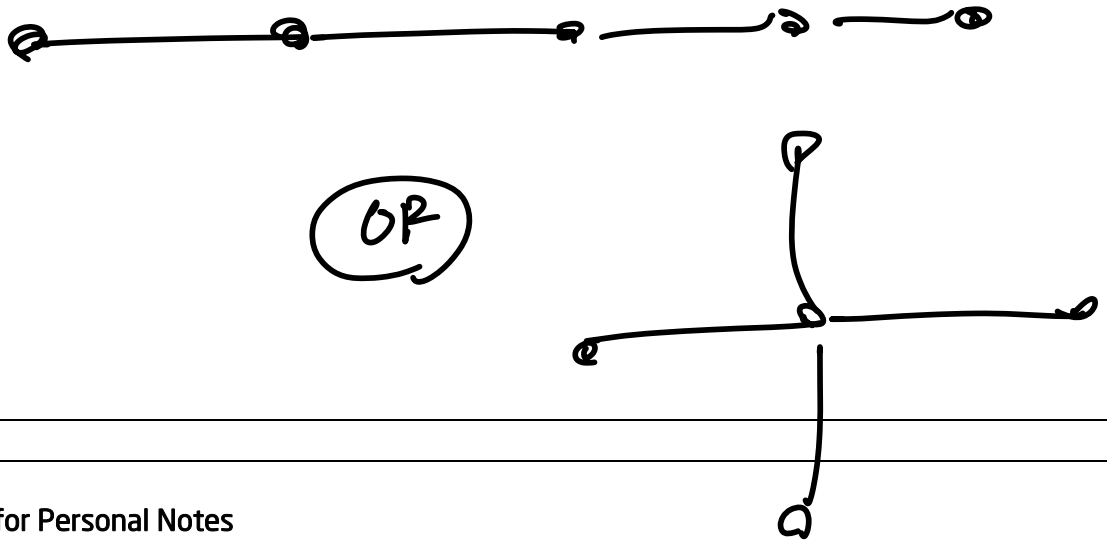



- A tree is a connected graph (can go from one vertex to any other vertex) without any cycle.
- We cannot come back to the same vertex by not repeating the edges.
- **Equivalent conditions for a tree:**
  - A simple graph with $n$ vertices is a tree if ANY of the following conditions are met:
    - It is connected and contains no cycles. ← def
    - It is connected and has $(n-1)$ edges
    - It has no cycles and has $(n-1)$ edges.
    - It is connected but would become disconnected if any edge is removed.
    - It is connected and would form a cycle if any edge is added
    - Any 2 vertices are connected by only a single path.

**Space for Personal Notes**

**Question 19 Walkthrough.**

Sketch any tree that contains five vertices.

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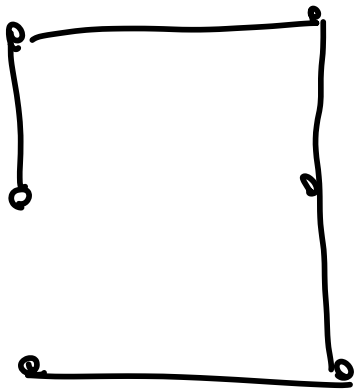
**Question 20**

Sketch any tree that contains six vertices.



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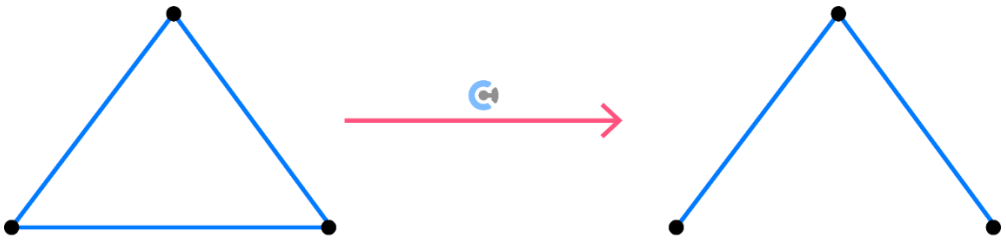
## Sub-Section: Spanning Tree

### *What is a spanning tree?*

**Spanning Trees**



- Spanning trees are Sub-graph (trees) and retain all of the vertices of the original graph.
  - The original graph does not have to be a tree.

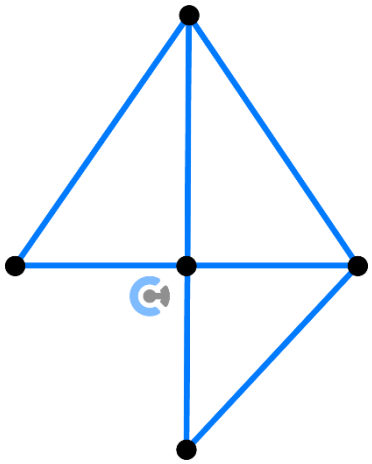
Spanning tree

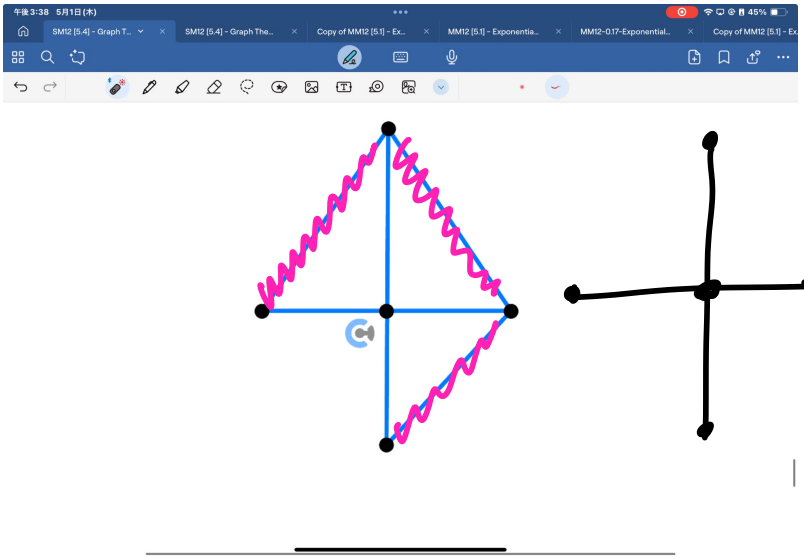
**NOTE:** The cycle goes through all the points and comes back to the same point.

**Space for Personal Notes**

**Question 21 Walkthrough.**

Find the spanning tree for the given graph below:





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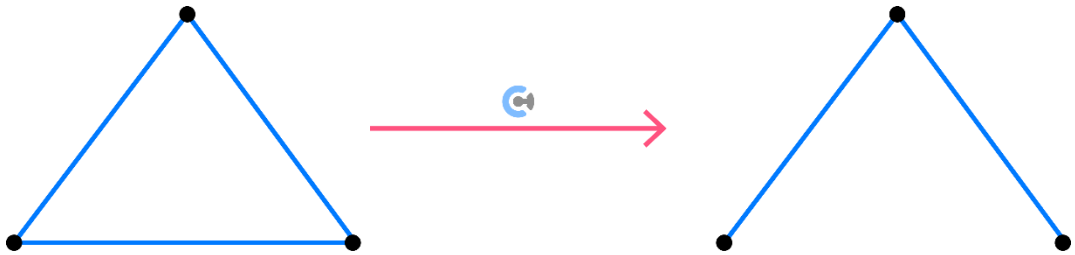
*Is there a systematic way to find a spanning tree for harder questions?*

Definition

## Algorithm for Finding a Spanning Tree



Spanning tree

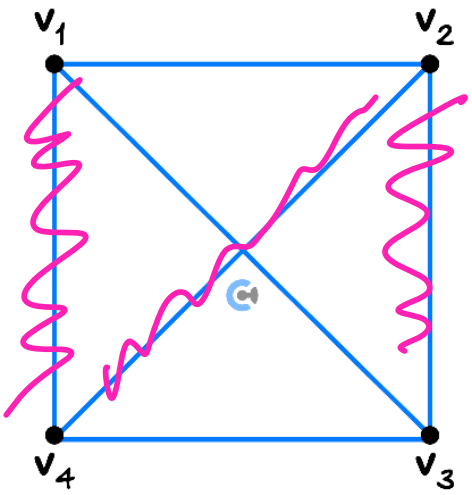
- The steps in finding a spanning tree:

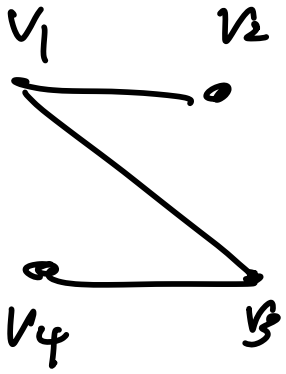
1. If the graph has no cycles, then stop.
2. Choose **any** edge that belongs to a cycle, and delete the chosen edge.
3. Repeat from 1.

**Space for Personal Notes**

**Question 22 Walkthrough.**

Using the algorithm, find a spanning tree for the given graph below:

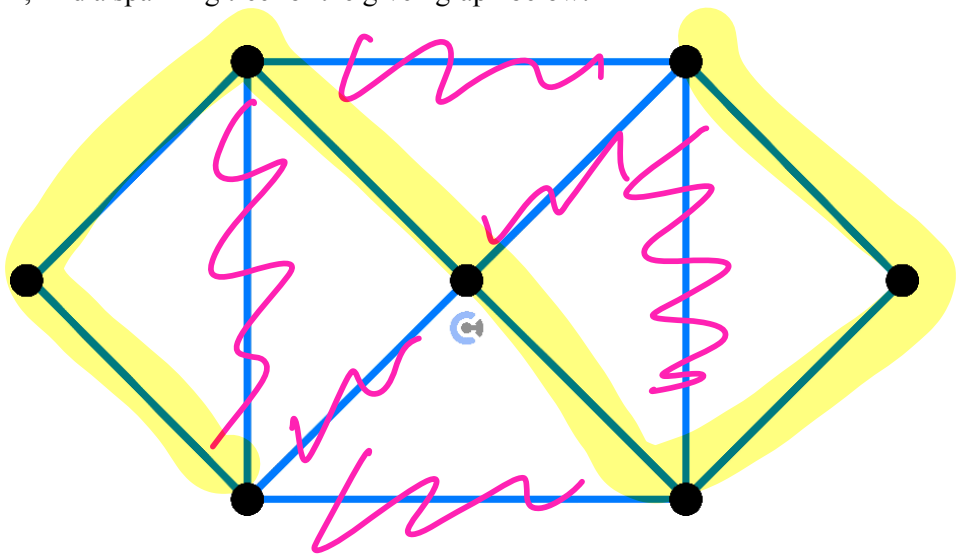



Space for Personal Notes

**Question 23** (4 marks) **Walkthrough.**

Using the algorithm, find a spanning tree for the given graph below:



Space for Personal Notes

# Cheat Sheet

## [5.4.1] – Walk of a graph

### Walk of a Graph

$$v_1, e_1, \cdots, v_{n-1}, e_1, v_n$$

- Order of the vertices (points) and edges (lines) we are visiting.
- Alternates between a vertex and an edge.
- If there is only one edge between two vertices, stating that an edge can be abbreviated.

### Length of a Walk

$$\textbf{Length of a Walk} = \textbf{Number of Edges in a Walk}$$

- Number of edges in a walk.
- If we use the same edge multiple times, we count it multiple times for the length.

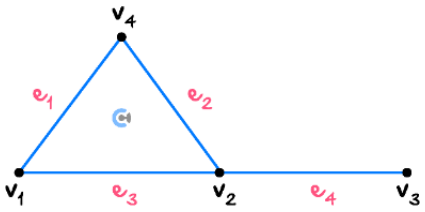
### Number of Walks with the Length of $n$

$$A^n = \begin{bmatrix} v_1, v_1 & v_1, v_2 & v_1, v_3 \\ v_1, v_2 & v_2, v_2 & v_2, v_3 \\ v_1, v_3 & v_3, v_2 & v_3, v_3 \end{bmatrix}$$

- For a walk of length $n$, we power the adjacency matrix by $n$.
- The element gives the number of walks of length $n$ between two vertices.

## [5.4.2] – Euler trail and circuits

### Euler Trail




- Euler Trail is a walk of a graph where all the edges are used exactly once.
- Whether a vertex is used exactly once is irrelevant.

### Existence of an Euler Trail

- A graph needs to satisfy one of the following two rules:

1. Every vertex has an even degree.

OR

2. Exactly two vertices have an odd degree.

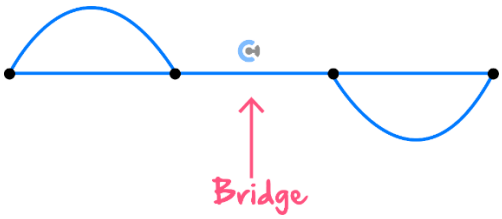
### Fleury's Algorithm

- Allows us to find Euler's trail efficiently in graphs with Euler trails.
- The steps are as follows:

1. If there are two vertices of odd degrees, start from one. Otherwise, start from any vertex.
2. Move from the current vertex across an edge to an adjacent vertex. Always choose a non-bridge edge unless there is no alternative.
3. Delete the edge that you have just traversed.
4. Repeat from Step 2 until there are no edges left.

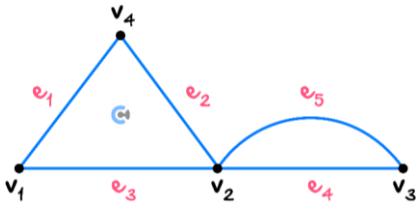
# Cheat Sheet

## Bridge




- A bridge is an edge that, if you delete it, will cause a graph to be cut into two graphs.

## Euler Circuits




- Euler Circuit is a walk of a graph where all the edges are used exactly once.

  The starting vertex and ending vertex must be the same.

- Whether a vertex is used exactly once is irrelevant.

- Euler circuits are a subset of Euler trails.

## Existence of an Euler Circuit

- A graph needs to satisfy the following rule:
  - Every vertex has an even degree.

## [5.4.3] – Hamiltonian Paths and Cycles

## Hamiltonian Path

- A path that covers all the vertices exactly once.

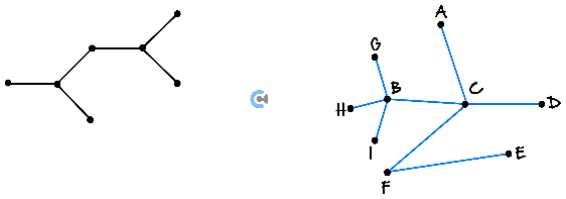
## Hamiltonian Cycle

- A path that covers all the vertices exactly once. (Except the starting one.)

  The starting position and ending vertex must be identical.

## [5.4.4] – Trees

## Trees




- A tree is a connected graph (can go from one vertex to any other vertex) without any cycle.

- We cannot come back to the same vertex by not repeating the edges.

## Equivalent Conditions for a Tree

- A simple graph with $n$ vertices is a tree if ANY of the following conditions are met:
  - It is connected and contains no cycles.
  - It is connected and has $(n-1)$ edges.
  - It has no cycles and has $(n-1)$ edges.
  - It is connected but would become disconnected if any edge is removed.
  - It is connected and would form a cycle if any edge is added.
  - Any two vertices are connected by only a single path.

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