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VCE Specialist Mathematics ½

Graph Theory II [5.4]

Test Solutions

21 Marks. 1 Minute Reading. 17 Minutes Writing.

Results:

Test Questions	_____ / 21

Section A: Test Questions (21 Marks)

Question 1 (5 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. To find the number of possible walks with a length of 3, we consider A^3.	✓	
b. Euler trail is a walk where we pass all the vertices exactly once.		✓
It's the edges.		
c. A graph where exactly one vertex has an odd degree contains an Euler trail.		✓
It must be exactly two vertices OR all the vertices have an even degree.		
d. For Euler circuits, we can go through the vertices multiple times.	✓	
e. A graph where all vertices have an even degree always contains an Euler circuit.	✓	
f. Hamiltonian path is a walk where we pass all the vertices exactly once.	✓	
g. Hamiltonian cycle does not have to use all the edges.	✓	
h. Trees must have a path that can visit all the vertices.	✓	
i. Trees must not have a cycle, meaning they cannot go through all the edges and come back to the original edge.	✓	
They cannot go through all the vertices and come back to the original vertex.		
j. Spanning tree cannot be a subgraph of a graph that is not a tree.		✓

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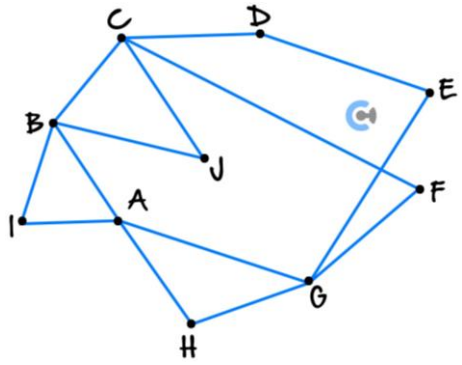
Question 2 (4 marks)

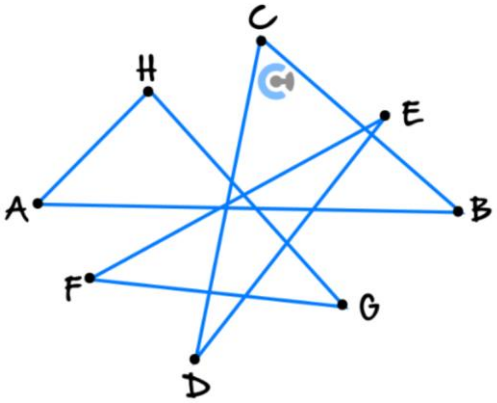
Identify an Eulerian trail and a Hamiltonian cycle in each of the following graphs (if they exist).

a. (2 marks)



Eulerian trail: AIBAHGFCJBCDEGA; Hamiltonian cycle: none exist

b. (2 marks)

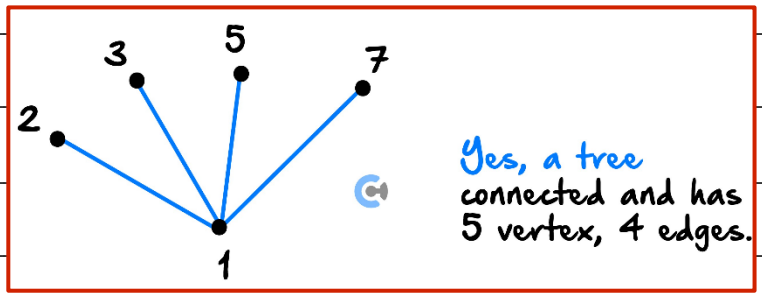


Eulerian trail: ABCDEFGHA (others exist); Hamiltonian cycle: HABCDEFGH (others exist)

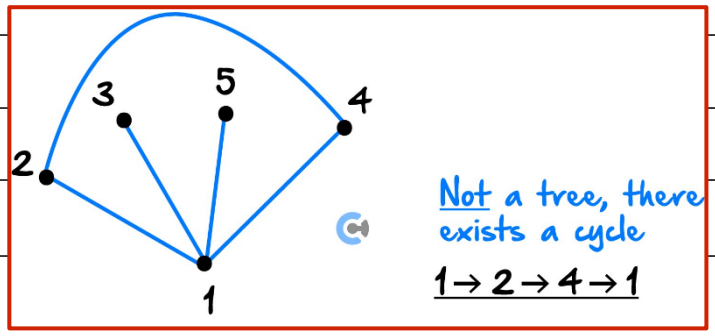
Question 3 (4 marks)

Which of the following graphs are trees? In each case, we insist that $m \neq n$.

a. Vertex set $\{1, 2, 3, 5, 7\}$ and an edge between m and n if m divides n or n divides m. (2 marks)



b. Vertex set $\{1, 2, 3, 4, 5\}$ and an edge between m and n if m divides n or n divides m. (2 marks)



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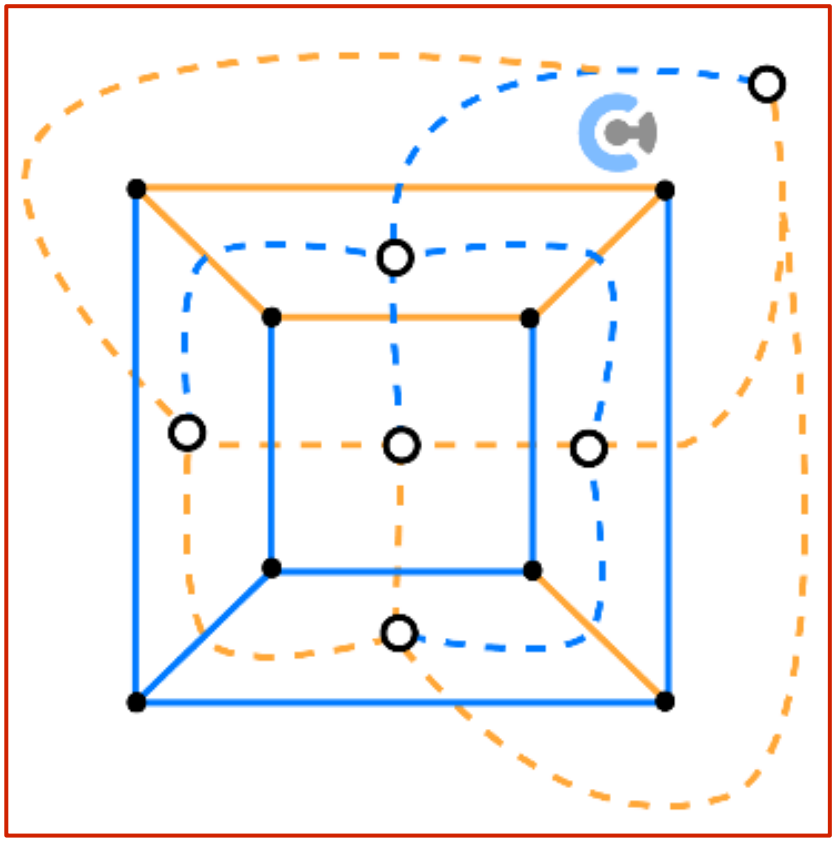
Question 4 (2 marks)

Find spanning trees of the following graph.



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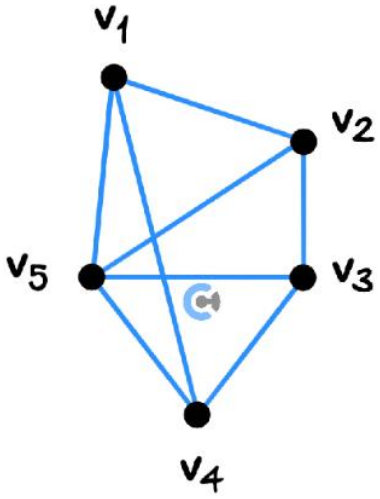
Question 5 (6 marks) **Tech-Active.**



a. Write down the adjacency matrix, A, for this graph. (2 marks)

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

b. Evaluate A^4. (1 mark)

$$\begin{bmatrix} 24 & 16 & 24 & 16 & 24 \\ 16 & 24 & 16 & 24 & 24 \\ 24 & 16 & 24 & 16 & 24 \\ 16 & 24 & 16 & 24 & 24 \\ 24 & 24 & 24 & 24 & 32 \end{bmatrix}$$

c. Find the number of different walks of length 4 from v_5 to v_5. (1 mark)

The number of different walks of length 4 from v_5 to v_5 is the $(5,5)$ entry in A^4. The $(5,5)$ entry in A^4 is equal to the dot product of row 5 of A^2 and column 5 of A^2. That is, $(2\ 2\ 2\ 2\ 4) \cdot (2\ 2\ 2\ 2\ 4) = 32$.

d. Verify that the trace of A^3 is 6 times the number of triangles in the graph. (2 marks)

There are 4 triangles in the graph. The trace of A^3 is the sum of its diagonal entries. Multiplying A^2 by A we find that the entries on the main diagonal of A^3 are $4, 4, 4, 4, 8$, so that the trace of $A^3 = 4+4+4+4+8 = 24 = 6 \times 4$. (Note that a triangle in a graph is a walk of length 3 from a vertex to itself. The element $(A^3)_{ii}$ on the main diagonal of A^3 is the number of walks of length 3 from vertex i to itself. Adding all the diagonal elements counts each triangle 6 times, and so the number of triangles in a graph (without loops) is $\frac{1}{6}\times$ the trace of A^3.)

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