5.4 Test

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VCE Specialist Mathematics ½

Graph Theory II [5.4]

Test

21 Marks. 1 Minute Reading. 17 Minutes Writing.

Results:

Test Questions	_____ / 21

Section A: Test Questions (21 Marks)

Question 1 (5 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. To find the number of possible walks with a length of 3, we consider A^3.	✓	
b. Euler trail is a walk where we pass all the vertices exactly once.		✓
c. A graph where exactly one vertex has an odd degree contains an Euler trail.		✓
d. For Euler circuits, we can go through the vertices multiple times.	✓	
e. A graph where all vertices have an even degree always contains an Euler circuit.	✓	
f. Hamiltonian path is a walk where we pass all the vertices exactly once.	✓	
g. Hamiltonian cycle does not have to use all the edges.	✓	
h. Trees must have a path that can visit all the vertices.	✓	
i. Trees must not have a cycle, meaning they cannot go through all the edges and come back to the original edge.	✓	
j. Spanning tree cannot be a subgraph of a graph that is not a tree.		✓

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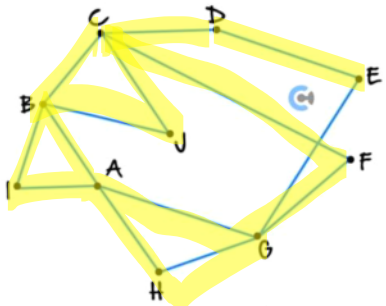
Question 2 (4 marks)

Identify an Eulerian trail and a Hamiltonian cycle in each of the following graphs (if they exist).

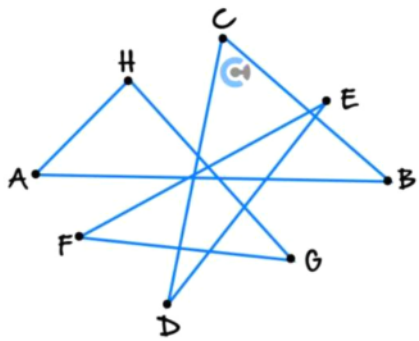
a. (2 marks)



ET. E D C B J C F G A B I A H G

H.C: No H.C

b. (2 marks)



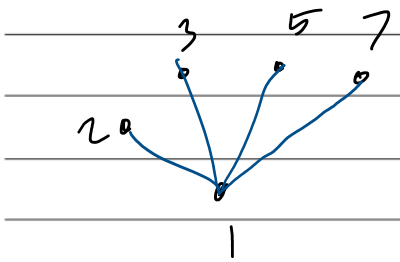
E.T: C D E F G H A B C

H.C:

Question 3 (4 marks)

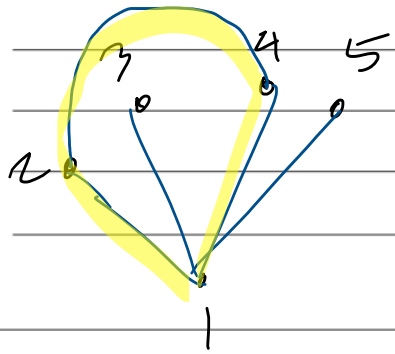
Which of the following graphs are trees? In each case, we insist that $m \neq n$.

a. Vertex set $\{1, 2, 3, 5, 7\}$ and an edge between m and n if m divides n or n divides m. (2 marks)



Yes!

b. Vertex set $\{1, 2, 3, 4, 5\}$ and an edge between m and n if m divides n or n divides m. (2 marks)



No!

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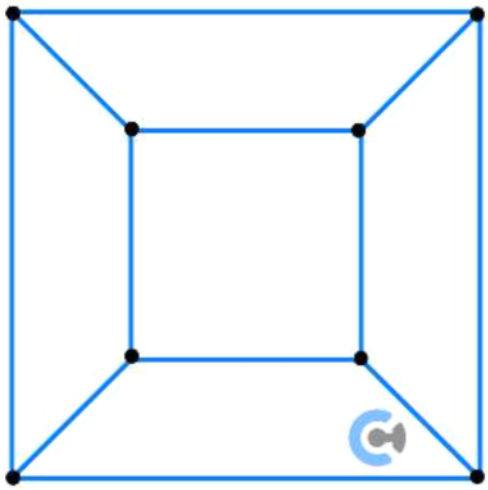
Question 4 (2 marks)

Find spanning trees of the following graph.



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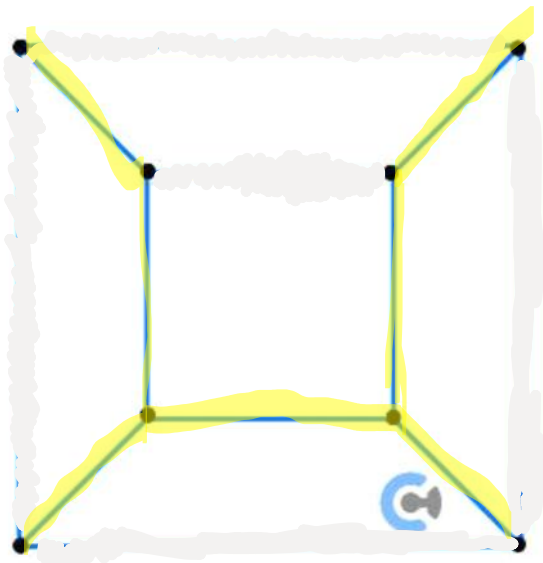
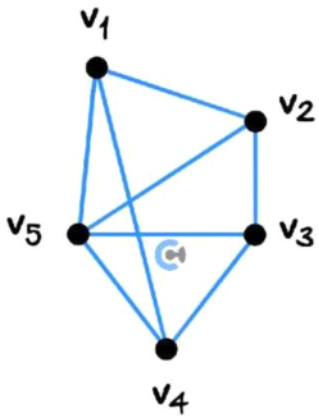


Question 5 (6 marks) **Tech-Active.**



a. Write down the adjacency matrix, A, for this graph. (2 marks)

$$A = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

b. Evaluate A^4. (1 mark)

$$A^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 24 & 16 & 24 & 16 & 24 \\ 16 & 24 & 16 & 24 & 24 \\ 24 & 16 & 24 & 16 & 24 \\ 16 & 24 & 16 & 24 & 24 \\ 24 & 24 & 24 & 24 & 32 \end{bmatrix} \end{array}$$

c. Find the number of different walks of length 4 from v_5 to v_5. (1 mark)

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d. Verify that the trace of A^3 is 6 times the number of triangles in the graph. (2 marks)

$A^3 =$

$$\begin{bmatrix} 4 & 8 & 4 & 8 & 8 \\ 8 & 4 & 8 & 4 & 8 \\ 4 & 8 & 4 & 8 & 8 \\ 8 & 4 & 8 & 4 & 8 \\ 8 & 8 & 8 & 8 & 8 \end{bmatrix}$$

$Tr(A^3) = 4+4+8+4+4$

$= 24$

4 triangles in graph

$\times 6$

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