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VCE Specialist Mathematics ½
Graph Theory I [5.3]
Homework Solutions

Admin Info & Homework Outline:

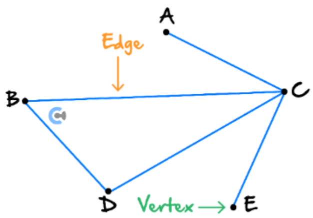
Student Name	
Questions You Need Help For	
Recap	Pg 2 - Pg 4
Basic (Compulsory) Questions	Pg 5 - Pg 13
Problem Solving Questions	Pg 14 - Pg 22
Supplementary Questions	Pg 23 - Pg 33

Section A: Recap

Cheat Sheet

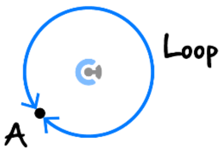
[5.3.1] - Graph theory fundamentals - vertices, edges, degree, adjacency lists and matrices

- Vertices and Edges



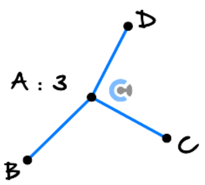
  - A graph consists of a set of points called vertices and set of unordered pairs of vertices, called edges.

- Loops



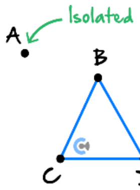
  - Loop is an edge which connects to the same vertex.

- Degree of a Vertex



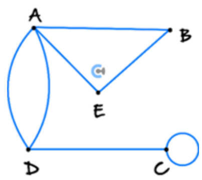
  - Degree of a vertex is the number of edges connected to the vertex.

- Isolated Vertex



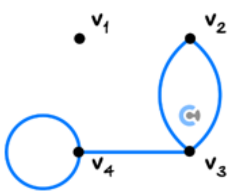
  - Isolated vertex has no edges connected to it.
  - Its degree is equal to zero.

- Adjacency Lists

Graph	Adjacency List
	$A \to (B, D, D, E)$ $B \to (A, E)$ $C \to (C, D)$ $D \to (A, A, C)$ $E \to (A, B)$

  - Adjacency list contains all the vertices a given vertex is connected to.
  - If the point is connected multiple times, we write the vertex multiple times.
  - If a point is looped with itself, we write the vertex to be adjacent to itself.

- Adjacency Matrix



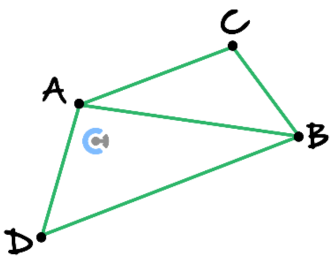
$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

  - A matrix that represents the vertices and edges that connect the vertices of a graph.

Cheat Sheet

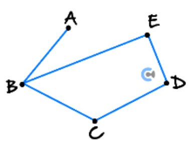
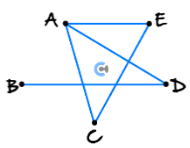
[5.3.2] - Types of graphs

- **Simple Graph**



- A simple graph is one in which pairs of vertices are connected by one edge at most.

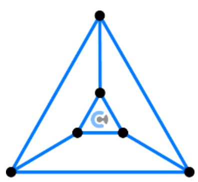
- **The Complement of a Simple Graph**

Simple Graph	The Complement
	

$$\text{Complement of } G = \overline{G}$$

- The complement of a simple graph contains the same set of vertices.
- But it contains complement set of edges. (Edges the original graph does not have.)

- **Regular Graphs**



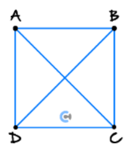
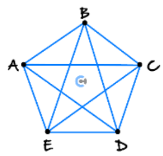
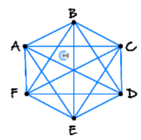
- Regular graph has all its vertices with the same degree.
- If each vertex has a degree r then the graph is "regular of degree r" or "r-regular".

- **Number of Edges and Degree of All Vertices of a Regular Graph**

$$\text{Number of Edges} \times 2 = \text{Total Degree of all Vertices}$$

- The above result is commonly known as the Handshake Lemma.

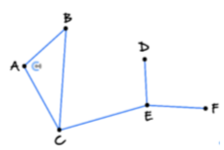
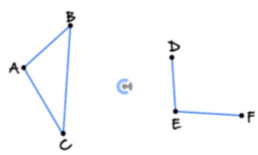
- **Complete Graph (K_n)**

K_4	K_5	K_6
		

- A complete graph is a simple graph in which each vertex is connected to every other vertex.
- A complete graph is denoted by K_n, where n is the number of vertices in the graph.
- Complete graph is a type of regular graph.
- For K_n:

$$\text{Number of Edges for Complete Graph} = \frac{n(n-1)}{2}$$

- **Connected Graphs**

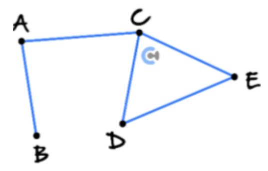
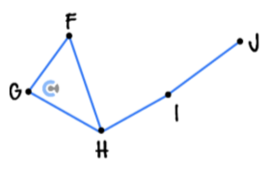
Connected Graph	Disconnected Graph
	

- A connected graph is a graph where it is possible to reach all vertices by moving along edges.
- A graph which is not connected is called a disconnected graph.

Cheat Sheet

[5.3.3] – Isomorphisms and subgraphs.

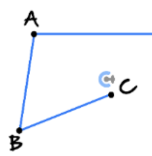
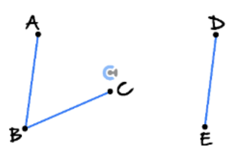
- Isomorphism

Isomorphic Graph	
	

- Two graphs are **isomorphic** if their vertices and edges differ only by the way in which they are named.

- **Checklist for Determining Isomorphism**
  - Are the number of vertices the same in each graph?
  - Are the number of edges the same in each graph?
  - Check that the degrees of each vertex match for both graphs.
  - Label each vertex on both graphs and check if there is a correspondence between the vertices.

- Subgraphs

Original Graph	Subgraph
	

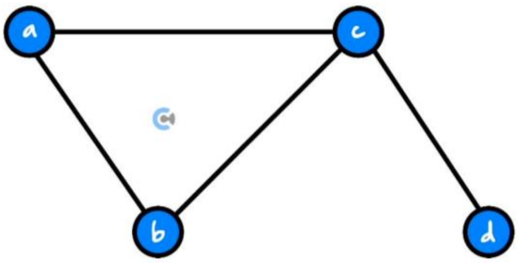
- A subgraph is a graph whose vertices and edges are all contained within the original graph.
- A subgraph can be created by removing edges and vertices from the original graph.

Section B: Basic (Compulsory) Questions

Sub-Section [5.3.1]: Graph Theory Fundamentals – Vertices, Edges, Degree, Adjacency Lists and Matrices

Question 1

Write down the vertex set and edge set of the given graph.



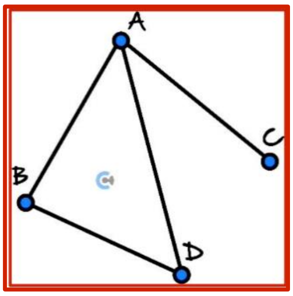
vertex set: $\{a, b, c, d\}$
edge set: $\{ac, ab, bc, cd\}$

Question 2

Draw pictures of 2 graphs with the following vertex and edge sets.

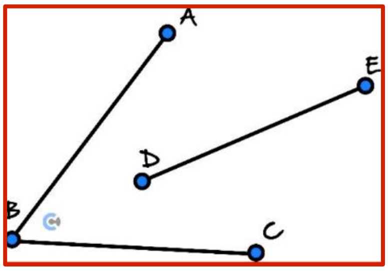
a. Vertex set: $\{A, B, C, D\}$

Edge set: $\{AB, AC, AD, BD\}$



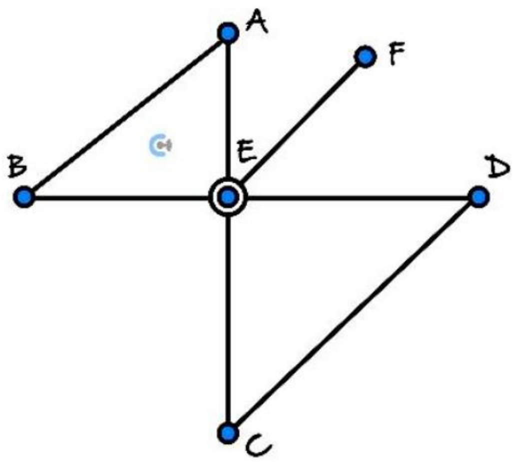
b. Vertex set: $\{A, B, C, D, E\}$

Edge set: $\{AB, BC, DE\}$



Question 3

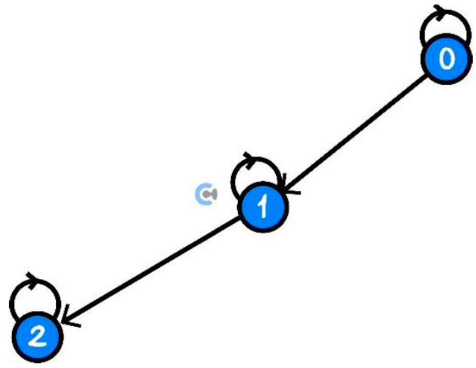
Fill in the following information for the graphs below.



Vertex	Degree of Vertex
A	1
B	2
C	4
D	1
E	1
F	1
Number of Edges: 5	Sum of Degrees: 10

Question 4

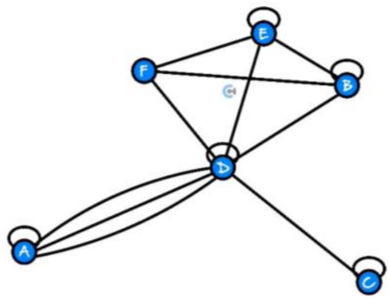
Create an adjacency list that describes the following graph.



$0 \rightarrow (0,1)$
$1 \rightarrow (0,1,2)$
$2 \rightarrow (1,2)$

Question 5

a. Construct the adjacency matrix for the given graphs.



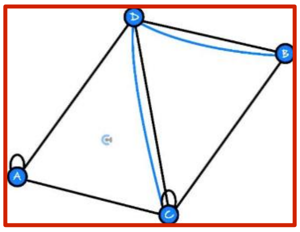
Adjacency Matrix:

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 3 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

b. Draw graphs to represent the following adjacency matrices.

Adjacency Matrix:

[1 0 1 1]
[0 0 1 2]
[1 1 1 2]
[1 2 2 0]



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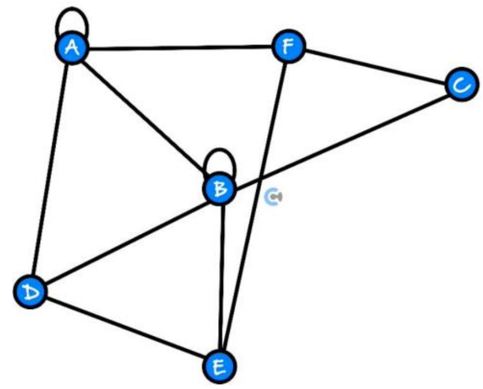
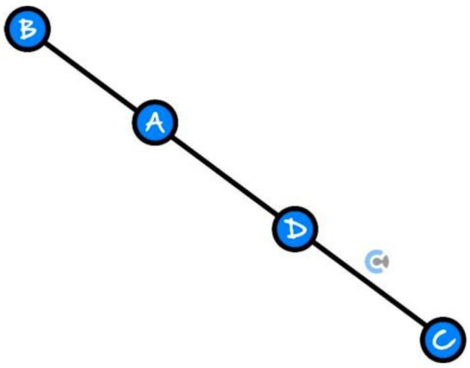
Sub-Section [5.3.2]: Types of Graphs

Question 6

State whether the following graphs are simple graphs or not.

 NO, there is a loop at vertex B and vertex A.	 YES, there are no loops or repeating edges.

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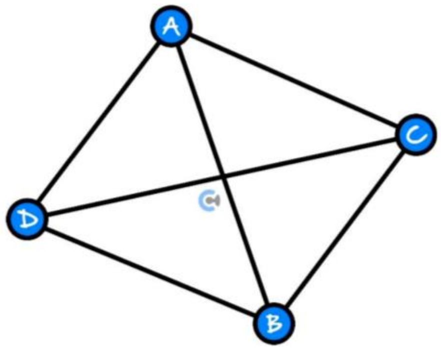
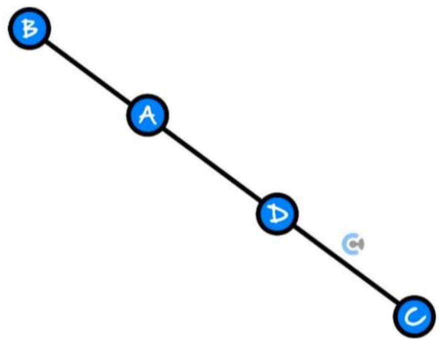
Question 7

State whether the following graphs are complete graphs or not.



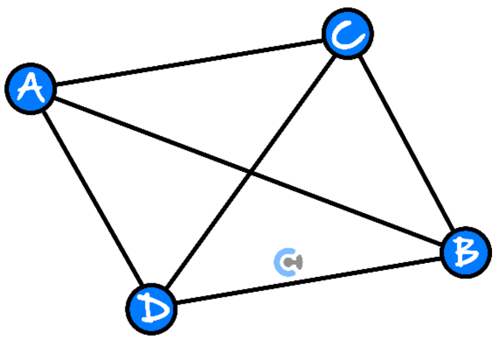
Yes, each vertex is connected to every other vertex.



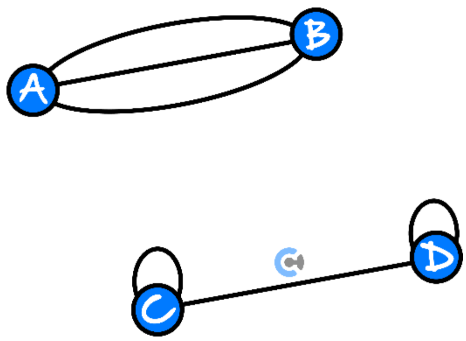
No, no one vertex is connected to every other.

Question 8

State whether the following graphs are regular graphs or not.



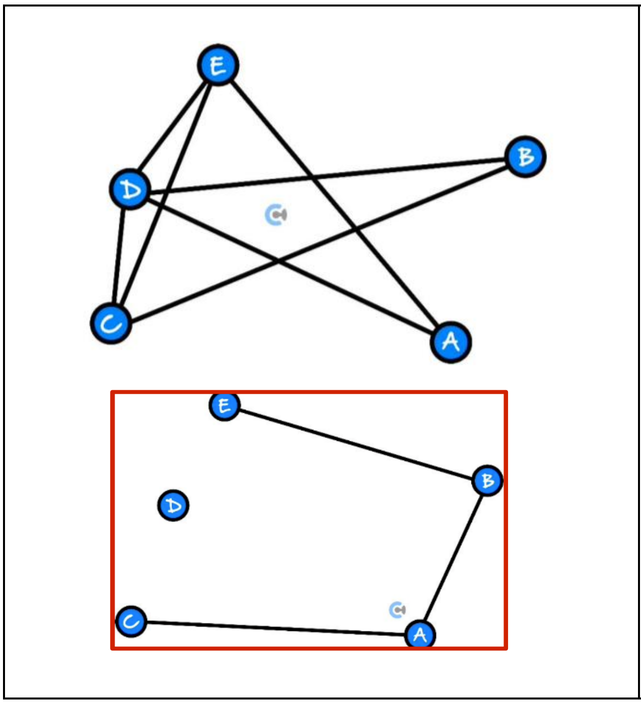
Yes, since every vertex has degree 3.

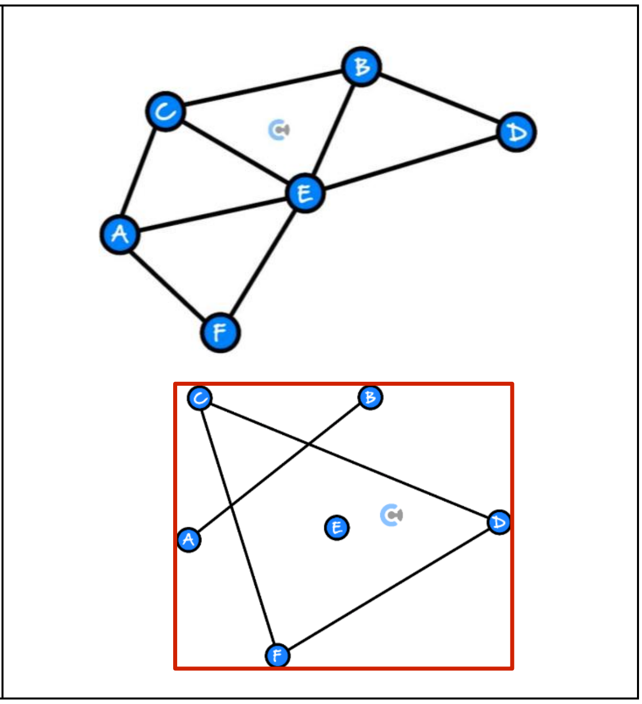


Yes, since every vertex has degree 3.

Question 9

Consider simple graphs below. Sketch the complement of each simple graph.





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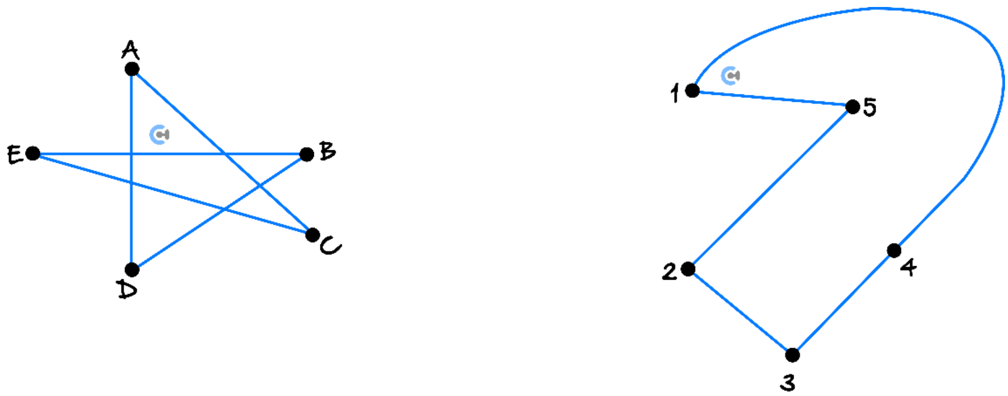
Sub-Section [5.3.3]: Isomorphisms and Subgraphs

Question 10

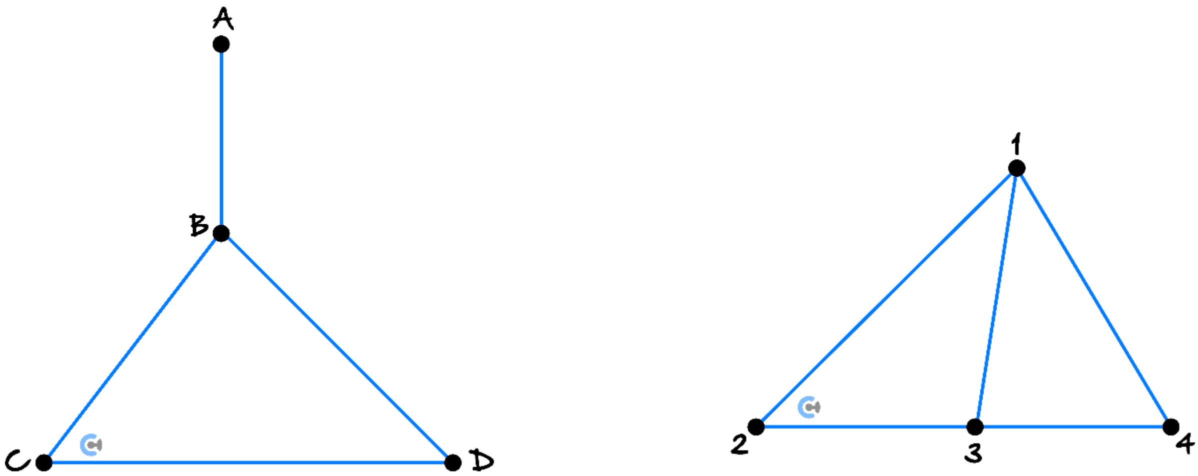
State whether the following graphs are isomorphic or not.

a.



Yes

b.



No

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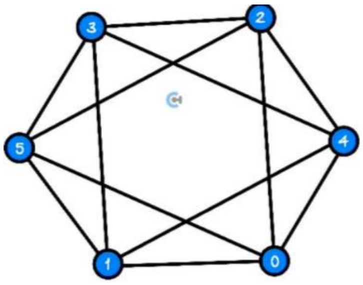
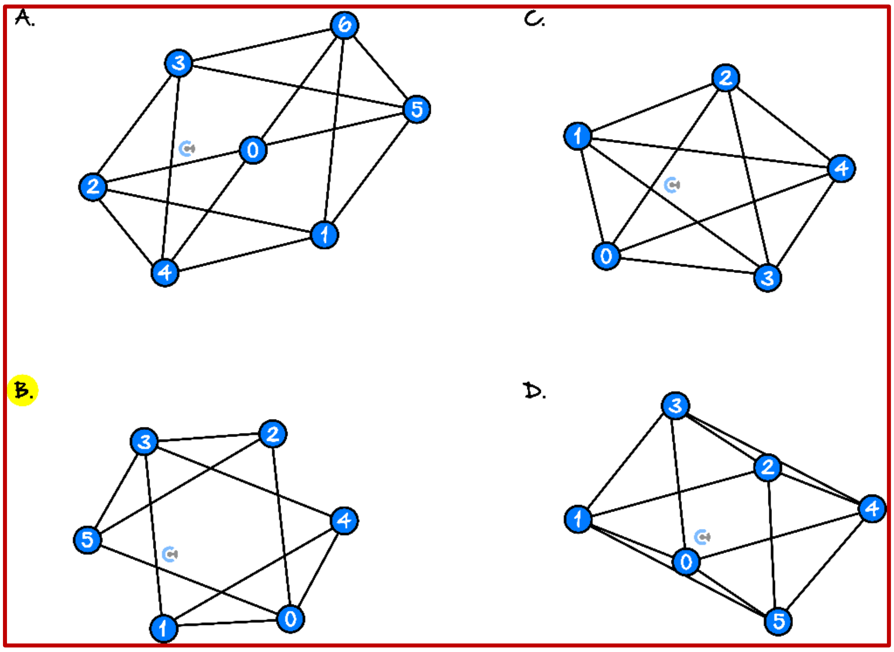
Question 11

Consider this network graph below.



A subgraph of this graph is:



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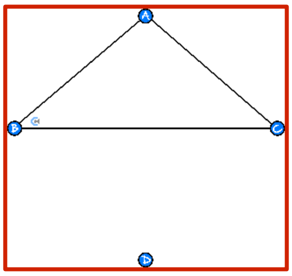
Section C: Problem Solving Questions

Question 12

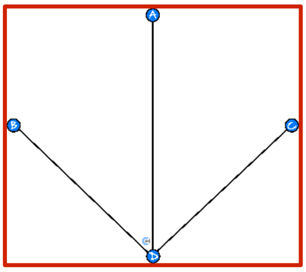
A town has 4 intersections labelled A, B, C and D. Roads exist between:

- A and B.
- A and C.
- B and C.

a. Draw the graph for this road network.



b. Draw the complement graph (i.e., connect all pairs that were not connected).



c. What does the complement tell you about missing road connections in the town?

The complement graph shows which intersections do not currently have roads between them. In this case, it highlights that intersection D is not connected to any other intersections in the original graph.

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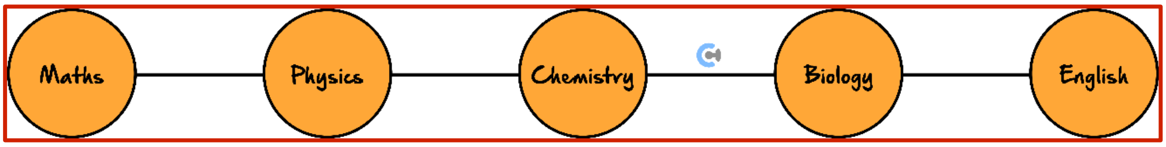
Question 13

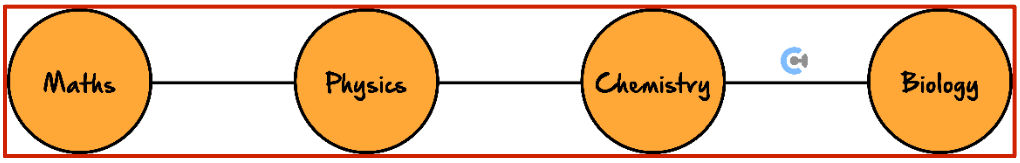
A graph shows connections between subjects that share common students:

- Maths ↔ Physics
- Physics ↔ Chemistry
- Chemistry ↔ Biology
- Biology ↔ English

a. Draw the graph.



b. Identify a subgraph that contains only science subjects.



c. What is the degree of Physics?

The degree of Physics is 2.

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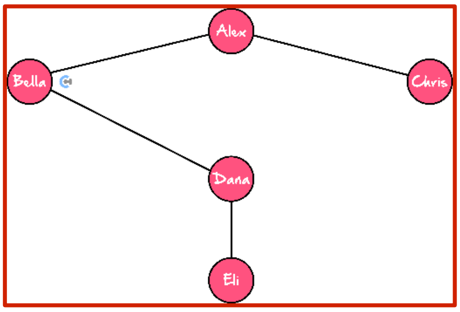
Question 14

Imagine a mini social network with five students: Alex, Bella, Chris, Dana, and Eli. The following pairs follow each other:

- Alex ↔ Bella
- Alex ↔ Chris
- Bella ↔ Dana
- Dana ↔ Eli

a. Draw a simple graph to represent the network.



b. List the degree of each student.

Alex: Degree 2 (Bella, Chris)
Bella: Degree 2 (Alex, Dana)
Chris: Degree 1 (Alex)
Dana: Degree 2 (Bella, Eli)
Eli: Degree 1 (Dana)

c. Write the adjacency list and adjacency matrix.

Adjacency list:
Alex: Bella, Chris
Bella: Alex, Dana
Chris: Alex
Dana: Bella, Eli
Eli: Dana
Adjacency Matrix:

	Alex	Bella	Chris	Dana	Eli
Alex	0	1	1	0	0
Bella	1	0	0	1	0
Chris	1	0	0	0	0
Dana	0	1	0	0	1
Eli	0	0	0	1	0

d. Is this graph regular? Why or why not?

Since not all vertices have the same degree, the graph is not regular.

Question 15

Let G be a simple graph where every vertex has odd degree.

Prove that the number of vertices with odd degree must be even.

Hint: Use the Handshake Lemma.

The Handshake Lemma indicates that the sum of all vertex degrees in any graph is even because it equals twice the number of edges.
Let's assume that k vertices have odd degree.

So we are summing k odd numbers and possibly some even numbers.

- The sum of an even number of odd numbers is even.
- The sum of an odd number of odd numbers is odd.

So if k is odd, the total degree sum would be odd,
which contradicts the Handshake Lemma (which says the sum must be even).
Therefore, the number of vertices with odd degree must be even.

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Question 16

Let K_n be a complete graph on n vertices.

a. Prove that each vertex in K_n has degree $n-1$.

By definition, a complete graph K_n is a simple graph where every pair of distinct vertices is connected by an edge.

So, for any single vertex in K_n :
- It connects to every other vertex.
- There are $n-1$ other vertices (since the graph has n vertices in total).

Therefore, the degree of each vertex is $n-1$.

b. How many edges does K_n have? Prove your answer.

In K_n , every pair of vertices is connected by an edge.

- The number of unique vertex pairs (i.e., edges) in a graph of n vertices is:

$$\binom{n}{2} = \frac{n(n-1)}{2}$$

So, the number of edges in K_n is: $\frac{n(n-1)}{2}$ (Because each edge connects 2 distinct vertices and there are no loops or multiple edges.)

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Question 17

A small friendship network of 6 students has the following number of friends:

- A has 2 friends.
- B has 3 friends.
- C has 2 friends.
- D has 1 friend.
- E has 2 friends.
- F has an unknown number of friends.

a. Use the Handshake Lemma to find how many friends F may have.

The sum of degrees in any graph must be even, because: $\sum \text{degrees} = 2 \times \text{number of edges}$

Let's calculate the known degree sum: $2 + 3 + 2 + 1 + 2 = 10$

Now include F: Total degree sum $= 10 + x \Rightarrow 10 + x$ must be even, so x must be an even number.
Since there are only 6 students, the maximum number of friends anyone can have is 5.
So: $x \in \{0,2,4\}$

b. How many total friendship pairs (edges) are there?

If F has 0 friends:

Total degree sum $= 10 + 0 = 10$, Edges $= \frac{10}{2} = 5$

If F has 2 friends:
Total degree sum $= 10 + 2 = 12$, Edges $= \frac{12}{2} = 6$

If F has 4 friends:
Total degree sum $= 10 + 4 = 14$, Edges $= \frac{14}{2} = 7$

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Question 18

In a classroom, students are asked to draw lines to represent if they've ever worked with another student on a group project. The resulting graph has 10 vertices and 21 edges.

a. What is the sum of the degrees of all students?

We apply the Handshake Lemma:

Sum of degrees $= 2 \times$ number of edges $= 2 \times 21 = 42$

b. Can every student have degree 5 in this graph? Explain your answer.

If every student had degree 5: Sum of degrees $= 10 \times 5 = 50$

But from **part a.**, we know the actual sum is 42, not 50. Therefore, it's not possible for all 10 students to have degree 5 in this graph.

c. What is the minimum possible degree of a vertex in this graph?

Let's assume we want to maximise the degrees of the other 9 students to minimise the degree of 1 student.
Let's say 9 students each have degree 5:
$9 \times 5 = 45 \rightarrow$ this already exceeds 42
Try 8 students with degree 5: $8 \times 5 = 40$
Remaining degree sum for 2 students $= 2$
So, the remaining two degrees must sum to 2.
To minimise one of them (the 10^{th} student), give it degree 0, and the other gets degree 2.
Final answer: The minimum possible degree is: 0 (This means it is possible one student hasn't worked with anyone at all.)

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Sub-Section: The Tech-Free "Final Boss"

Question 19

Two schools are studying student friendship patterns. In each school, students are represented as vertices, and a friendship between two students is an edge. The graphs are simple.

School A's friendship graph has:

- 6 students.
- The degrees are: $3, 2, 3, 1, 2, 1$.

School B's friendship graph is given as an adjacency matrix:

$$\begin{array}{c|cccccc} & A & B & C & D & E & F \\ \hline A & 0 & 1 & 1 & 0 & 0 & 0 \\ B & 1 & 0 & 1 & 0 & 1 & 0 \\ C & 1 & 1 & 0 & 0 & 0 & 0 \\ D & 0 & 0 & 0 & 0 & 1 & 0 \\ E & 0 & 1 & 0 & 1 & 0 & 1 \\ F & 0 & 0 & 0 & 0 & 1 & 0 \end{array}$$

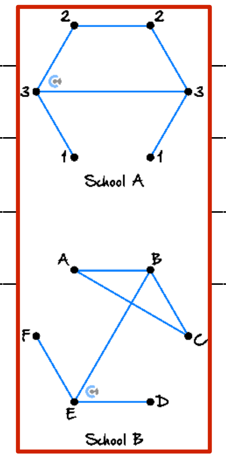
a. Use the Handshake Lemma to verify the number of edges in each school's graph.

From School B's degree sum:

Sum of degrees$= 2 + 3 + 2 + 1 + 3 + 1 = 12 \Rightarrow$ Number of edges $= \frac{12}{2} = 6$

School A's degrees: $[3, 2, 3, 1, 2, 1]$
Sum $= 12 \Rightarrow 6$ edges
Both graphs have 6 edges, consistent with the Handshake Lemma.

b. Are the two graphs necessarily be isomorphic? Justify your answer.

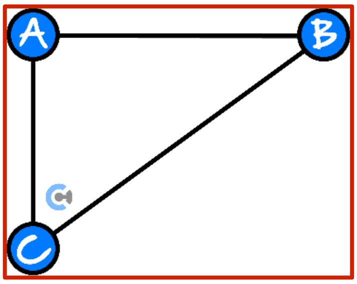


The graphs are not isomorphic, because although they have the same degree sequence and number of edges, School B contains a complete subgraph (triangle) that School A may not, see above graph as a counter example.

c. Represent School B's graph as an adjacency list.

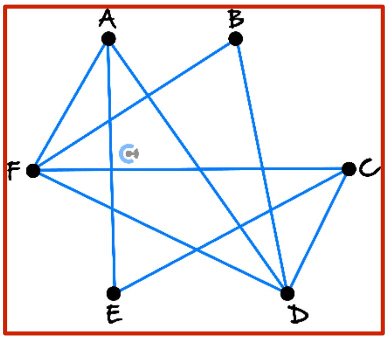
$A: B, C$
$B: A, C, E$
$C: A, B$
$D: E$
$E: B, D, F$
$F: E$

d. Identify and draw a subgraph in School B that forms a complete graph.



All are connected to each other: $A-B, B-C, C-A$.

e. Draw the complement of School $B's$ graph, and what does it represent in the friendship context?



The complement shows all the missing friendships.
If two students are connected in the complement, they have never worked together.

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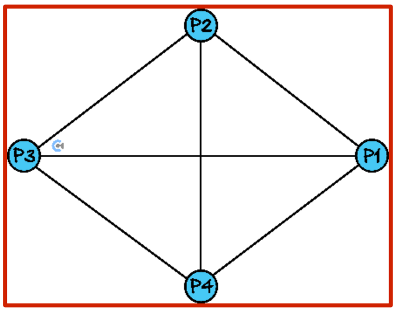
Section D: Supplementary Questions

Sub-Section: Exam 1 (Tech-Free)

Question 20

In a table tennis tournament, each player must play every other player exactly once.

a. If there are 4 players, draw the graph representing all matches.

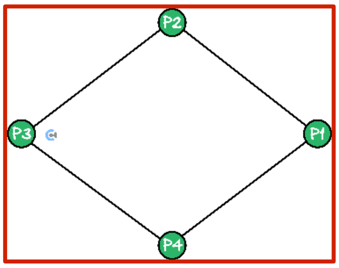


b. What type of graph is this? (eg., Simple, complete, regular.)

This is a:
- Simple graph (no loops or multiple edges).
- Complete graph (every vertex connected to all others).
- Regular graph (each vertex has degree 3).

Answer: Simple, complete, regular.

c. If 4 players each play 2 matches, can the graph be regular but not complete? Draw and explain.



Yes, this graph is regular (degree 2) but not complete (not all players play each other).

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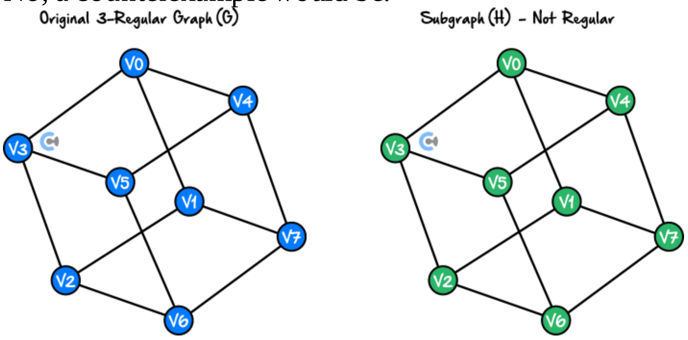
Question 21

Let G be a **3**-regular graph (each vertex has degree 3). Let H be a subgraph of G.

Is H necessarily regular? If not, provide a reason (or a counterexample).

No, a counterexample would be:



This subgraph is not regular, because not all vertices have the same degree.

Question 22

At a party, each person shakes hands with some others. There are 8 people at the party, and together they exchanged 20 handshakes.

a. What is the sum of the degrees of all the people (vertices)?

We apply the Handshake Lemma, which states:

Sum of degrees $= 2 \times$ number of handshakes
So, sum of degrees $= 2 \times 20 = 40$

b. Is it possible that everyone shook hands with exactly 5 people? Why or why not?

If each of the 8 people shook hands with exactly 5 people:
Sum of degrees $= 8 \times 5 = 40$
This matches the actual sum of degrees from **part a.**
Yes, it is possible.
If every person shook hands with 5 others, the total degree sum would be 40, which results in 20 handshakes - exactly as given in the question.

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Question 23

A graph has 7 vertices. The degrees of six of them are: 3, 2, 4, 5, 3, 2.

a. Can you find the smallest possible degree of the 7^{th} vertex using the Handshake Lemma?

Given degrees of 6 vertices: $3,\ 2,\ 4,\ 5,\ 3,\ 2$
Sum of known degrees: $3+2+4+5+3+2=19$
The Handshake Lemma tells us the sum of all degrees must be even, and let the 7^{th} vertex have degree x.
So: $19+x$ must be even $\Rightarrow x$ must be odd.
Possible values: $x \in \{1,3,5\ \ldots\}$
The smallest possible value for x is 1.

b. Is it possible for all vertices to have odd degree in this graph? Why?

If all 7 vertices had odd degrees $\rightarrow$ there would be 7 odd numbers, which is odd, contradicting the rule.
No, it is not possible for all 7 vertices to have odd degree, because the number of vertices with odd degree in any graph must be even.

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Question 24

A group of 7 students- A, B, C, D, E, F and G- are working together on various class projects. Your task is to draw a simple graph based on the following conditions:

1. The graph has 7 vertices, one for each student.

2. Student A has worked with exactly 2 others.

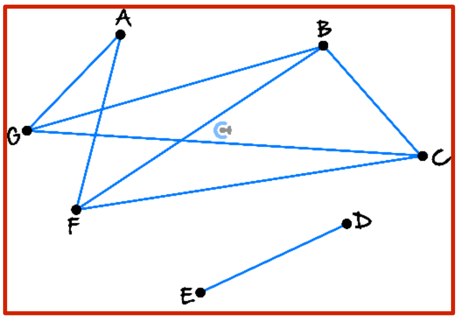
3. Students B and C have the same number of connections (same degree).

4. Students D and E are only connected to each other (and no one else).

5. Students F and G are each connected to 3 other students, but not to each other.

6. The graph has a total of 8 edges.

a. Draw the graph satisfying all the conditions. Label all vertices clearly.



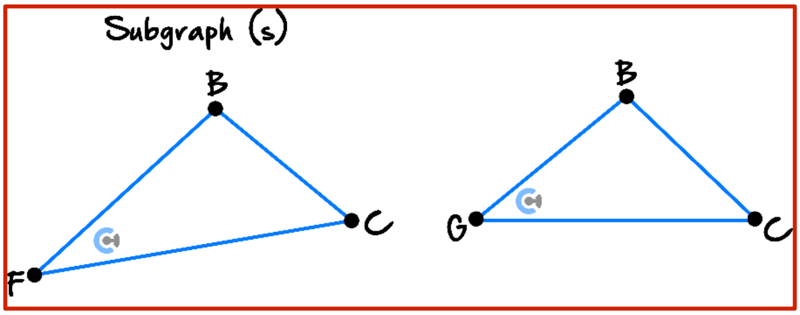
b. Write the degree of each student.

degrees:
A: 2
B: 3
C: 3
D: 1
E: 1
F: 3
G: 3

c. Create an adjacency list for the graph.

adjacency list:
A: G, F
B: G, F, C
C: G, F, B
D: E
E: D
F: A, B, C
G: A, B, C.

d. Identify any subgraph(s) that form complete graph(s) with at least 3 vertices.



e. Is the graph regular? Why or why not?

No, the graph is not regular because the vertices have different degrees.

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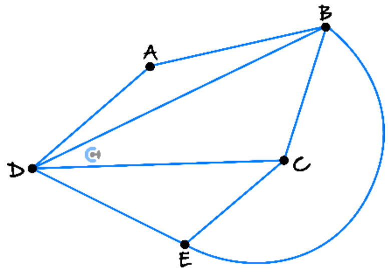
Sub-Section: Exam 2 (Tech-Active)

Question 25

Which vertices in the network below have the largest degree?



A. A and D.

B. B and D.

C. C and B.

D. D and E.

Question 26

If the sum of the degrees in a graph is 8, how many edges does the graph have?

A. 2

B. 4

C. 8

D. 16

The sum of the degrees in a graph is double the number of edges.

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Question 27

Let M be a complete graph. Let N be its complement.

Prove that N has no edges.

Hint: Use the definition of complement and complete graph.

In a complete graph M, every pair of distinct vertices is connected by an edge.
That means all possible edges between the vertices exist in M.
By definition, the complement N of a graph M includes only those edges that are not in M.
But since M already has all possible edges, there are no edges left to put in N.
Therefore, the complement N has no edges.

Question 28

Let G be a simple graph with n vertices.

Prove that the maximum number of edges in G is $n(n-1)/2$.

Hint: Think about complete graphs again.

A simple graph:

- Has no loops (no edge connects a vertex to itself).
- Has no multiple edges between any pair of vertices.

The maximum number of edges occurs when every pair of distinct vertices is connected by exactly one edge, which is a complete graph with $n(n-1)/2$ edges. (Proved in compulsory section.)

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Question 29

A town is designing a road system between 7 intersections. They want the road network to be:

- A regular graph of degree 4.
- Have no loops or multiple edges.

Then, the town planner mistakenly adds a road that connects an intersection to itself (a loop) and creates a multiple edge between two intersections.

a. How many edges does the graph planned to have if it's 4-regular (A regular graph of degree 4.) on 7 vertices?

Use the Handshake Lemma:
Sum of degrees $= 7 \times 4 = 28 \Rightarrow$ Number of edges $= \frac{28}{2} = 14$

b. After adding the loop and multiple edge, does the Handshake Lemma still apply? Why or why not?

Yes, the Handshake Lemma still applies, since a loop adds 2 to the degree of a vertex. In this situation, Sum of degrees $= 2 \times$ number of edges still holds while sum of degrees increases 4 and number of edges increases 2.

c. Is the resulting graph still simple? Explain.

No, the graph is not simple because it contains a loop and a multiple edge.

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Question 30

A company creates a communication graph where:

- Each department is a vertex.
- A connection (edge) indicates direct communication between departments.
- The graph has 8 vertices and is not complete, but each department is connected to at least 4 others.
- One department (vertex v) is only connected to departments that themselves all connect with each other.

a. Prove that the subgraph formed by v's neighbours is a complete graph.

Yes, the subgraph formed by v's neighbours is a complete graph, because by definition, all of v's neighbours are connected to each other.

b. Can the overall graph be regular? Why or why not?

No, the graph cannot be guaranteed to be regular, because although each vertex has at least 4 neighbours, some may have more.

c. What is the minimum possible number of edges in this graph?

$8 \times 4 = 32 \Rightarrow$ Minimum number of edges $= 32/2 = 16$

d. If you take the complement of the graph, what does a connected component represent in this context?

So, a connected component in the complement graph is a group of departments that are mutually disconnected in the original graph.

Question 31

In a university, each student joins a club. A graph is formed where:

- Each vertex represents a student.
- An edge connects two students if they are in the same club.
- The graph has 10 vertices, and the degrees of the vertices are: $3,4,3,4,5,4,3,4,5,x$.
- The complement of the graph represents students who do not share any club.

a. Use the Handshake Lemma to determine the possible value(s) for x.

Handshake Lemma:

Sum of degrees $= 2 \times$ Number of edges
First, sum the known degrees: $3+4+3+5+4+5+3+4+3 = 34 \Rightarrow 34+$
x must be even $\Rightarrow x$ must be even

$$x \in \{0,2,4,6,8\}$$

($x = 10$ is not possible in a simple graph with only 10 vertices.)

b. What is the total number of clubs (edges) if students only form pairs?

For each possible value of x, total degrees $= 34 + x$, and:
Edges $= \frac{34+x}{2} \Rightarrow$Possible number of edges:

x	Degree Sum	Number of Edges
0	34	17
2	36	18
4	38	19
6	40	20
8	42	21

The number of clubs (edges) could be any of: $17, 18, 19, 20, 21$.

c. Identify whether this graph could be regular. Justify.

No, the graph is not regular because the degrees vary.

d. One club has 4 members and they are all connected to each other. Identify the subgraph type formed and explain why.

Complete graph on 4 vertices.

Because every pair of students in that group is connected.

e. Suppose another university has the same degree sequence but different connections. Can the two graphs still be isomorphic? Why?

Same degree sequence doesn't guarantee isomorphism — structure (which vertices are connected) also matters.

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