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VCE Specialist Mathematics ½

Combinatorics II [5.2]

Test Solutions

26.5 Marks. 1 Minute Reading. 27 Minutes Writing.

Results:

Test Questions	_____ / 26.5

Section A: Test Questions (26.5 Marks)

Question 1 (2.5 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. Combination is used when the order matters.		✓
b. There are 10^2 ways to select any number of people from the pool of 10 people.	Its 2^{10}.	✓
c. The pigeonhole principle is used to show that some pairs or groups of objects have the same property.	✓	
d. The inclusion-exclusion principle allows us to count the number of elements in a union of sets.	✓	
e. The inclusion-exclusion principle formula for three sets is given by $\|A \cup B \cup C\| = \|A\| + \|B\| + \|C\| - \|A \cap B\| - \|B \cap C\| - \|A \cap C\| - \|A \cap B \cap C\|$.		✓

Space for Personal Notes

Question 2 (8 marks) **Tech-Active.**

Consider the word 'TRIANGLE'.

a. How many ways can the letters of the word TRIANGLE be arranged? (2 marks)

Here there are a total of eight choices for the first letter, seven for the second, six for the third, and so on. By the multiplication principle we multiply for a total of

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8! = 40,320$$

different ways.

b. How many ways can the letters of the word TRIANGLE be arranged if the first three letters must be RAN (in that exact order)? (2 marks)

The first three letters have been chosen for us, leaving us five letters. After RAN we have five choices for the next letter followed by four, then three, then two, then one. By the multiplication principle, there are

$$5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$$

ways to arrange the letters in a specified way.

c. How many ways can the letters of the word TRIANGLE be arranged if the first three letters must be RAN (in any order)? (2 marks)

Look at this as two independent tasks: the first arranging the letters RAN, and the second arranging the other five letters. There are

$$3! = 6$$

ways to arrange RAN and

$$5!$$

ways to arrange the other five letters. So there are a total of

$$3! \times 5! = 720$$

ways to arrange the letters of TRIANGLE as specified.

d. How many ways can the letters of the word TRIANGLE be arranged if the first three letters must be RAN (in any order) and the last letter must be a vowel? (2 marks)

Look at this as three tasks: the first arranging the letters RAN, the second choosing one vowel out of I and E, and the third arranging the other four letters. There are

$$3! = 6$$

ways to arrange RAN,

$$2$$

ways to choose a vowel from the remaining letters, and

$$4!$$

ways to arrange the other four letters. So there are a total of

$$3! \times 2 \times 4! = 288$$

ways to arrange the letters of TRIANGLE as specified.

Space for Personal Notes

Question 3 (2 marks)

Find the minimum number of students in a class such that three of them are born in the same month.

Solution.
Number of months $n = 12$
According to the given condition, $k + 1 = 3 \rightarrow k = 2$
$M = kn + 1 = 2 \times 12 + 1 = 25$

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Question 4 (3 marks)

Show that in a group of 20 people and friendship are mutual, showing that there exist two people who have the same number of friends.

Solution: Each person can have 0 to 19 friends. But if someone has 0 friends, then no one can have 19 friends and similarly you cannot have 19 friends and no friends. So, there are only 19 options for the number of friends and 20 people, so we can use pigeonhole.

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Question 5 (8 marks) **Tech-Active.**

How many seven-letter arrangements of the word CONTOUR,

a. Begin with a vowel? (2 marks)

1080

b. End with a vowel? (2 marks)

1080

c. Begin and end with a vowel? (2 marks)

360

d. Begin or end with a vowel? (2 marks)

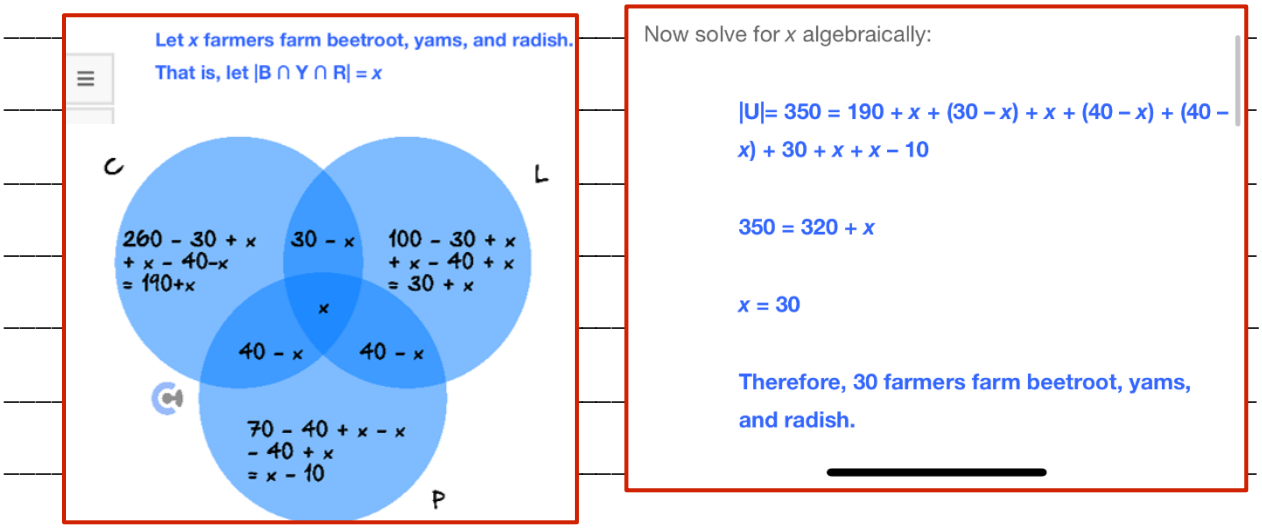
1800

Question 6 (3 marks)

There are 350 farmers in a large region. 260 farm beetroot, 100 farm yams, 70 farm radish, 40 farm beetroot and radish, 40 farm yams and radish and 30 farm beetroot and yams.

Let B, Y, and R denote the set of farms that farm beetroot, yams and radish respectively.

Determine the number of farmers who farm beetroot, yams, and radish.



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