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VCE Specialist Mathematics ½
Combinatorics II [5.2]
Homework

Admin Info & Homework Outline:

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Questions You Need Help For	
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Section A: Recap

Contour Check

Learning Objective: [5.2.1] – Selections of any size and Pascal's triangle

Key Takeaways

- **Pascal's Triangle and ${}^nC_r$:**

  - A new entry in Pascal's triangle is found by adding the two entries above it from the previous row.

- **Symmetrical Property:**

$$^nC_r = {}^nC_{n-r}$$

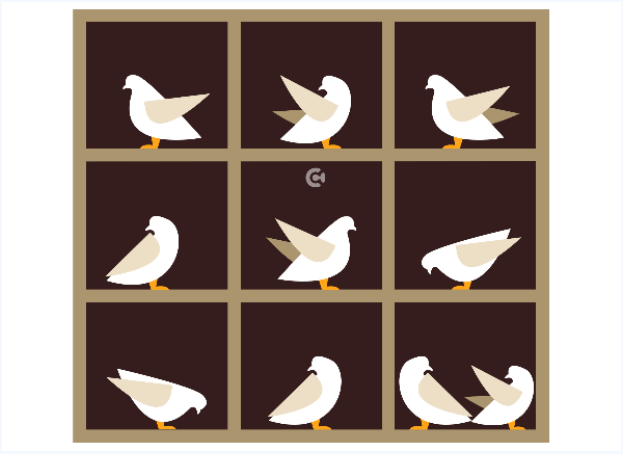
- **Selection of any Size:**

$$^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n$$

Learning Objective: [5.2.2] – Pigeonhole principle

Key Takeaways

Pigeonhole Principle:



"If $n+1$ pigeons are placed into n holes, then some hole contains at least two pigeons."

- If $n+1$ or more objects are placed into n places, then someplace contains at least two objects.

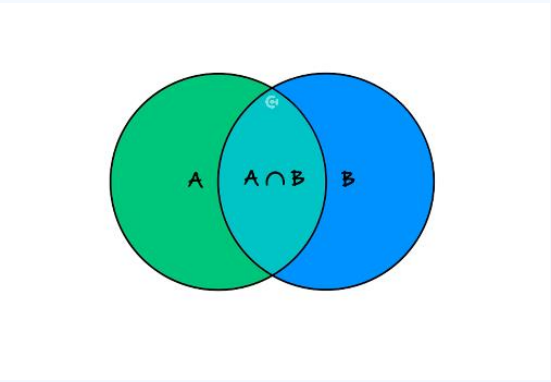
Generalised Pigeonhole Principle:

"If $mn+1$ pigeons are placed into n holes, then some hole contains at least $m+1$ pigeons."

Learning Objective: [5.2.3] – Inclusion-Exclusion principle

Key Takeaways

- **Inclusion-Exclusion Principle for Two Sets:**



$$|\boldsymbol{A} \cup \boldsymbol{B}| = |\boldsymbol{A}| + |\boldsymbol{B}| - |\boldsymbol{A} \cap \boldsymbol{B}|$$

- **Inclusion-Exclusion Principle for Three Sets:**
  - If A, B, and C are three finite sets of objects:

$$|\boldsymbol{A} \cup \boldsymbol{B} \cup \boldsymbol{C}| = |\boldsymbol{A}| + |\boldsymbol{B}| + |\boldsymbol{C}| - |\boldsymbol{A} \cap \boldsymbol{B}| - |\boldsymbol{A} \cap \boldsymbol{C}| - |\boldsymbol{B} \cap \boldsymbol{C}| + |\boldsymbol{A} \cap \boldsymbol{B} \cap \boldsymbol{C}|$$

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Section B: Basics (Compulsory)

NOTE: The notation $\binom{n}{r}$ is equivalent to ${}^{n}C_{r}$.

Question 1

a. Evaluate $\binom{7}{3}$.

b. State and use the symmetrical property of combinations to show that $\binom{9}{2} = \binom{9}{7}$.

c. Find the value of the following.

$$\binom{8}{0} + \binom{8}{1} + \binom{8}{2} + \cdots + \binom{8}{8}$$

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Question 2

a. There are 6 pigeonholes and 13 pigeons. Explain why some pigeonholes must contain at least 3 pigeons.

b. If $5n + 1$ pigeons are placed into n holes, explain why some holes contain at least 6 pigeons.

c. There are 28 socks in a drawer, each sock is either red, blue, or green. What is the minimum number of socks you must pull out to guarantee that you have at least 9 socks of the same colour?

Question 3

a. In a group of students:

- 23 play netball.
- 17 play soccer.
- 9 play both sports.

How many students play at least one of the two sports?

b. In a school of 60 students:

- 25 study French.
- 30 study Japanese.
- 18 study Indonesian.
- 12 study both French and Japanese.
- 10 study both French and Indonesian.
- 8 study both Japanese and Indonesian.
- 5 study all three languages.

How many students study at least one language?

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Section C: Problem Solving (Compulsory)

Question 4

At a party, every guest leaves their shoes at the entrance. The shoes are mixed randomly in a pile. There are 10 different types of shoes (e.g., boots, sneakers, sandals, etc.), and each guest owns exactly one pair from a type. Suppose there are 41 individual shoes (not pairs) in the pile.

Prove that at least five shoes of the same type must be present in the pile.

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Question 5

A tech company surveys 120 employees on their skillsets:

- 72 know Python.
- 65 know Java.
- 58 know C + +.
- 38 know both Python and Java.
- 31 know both Python and C + +.
- 28 know both Java and C + +.
- 18 know all three.

How many employees know at least one of the three programming languages?

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Question 6

A school has 1000 lockers, each secured by a 3-digit code using digits 0-9. Each code must contain at most two distinct digits (e.g., 111, 112, 212 are valid; 123 is not).

If there are 350 students, show that at least two students must have the same locker code.

Question 7

A drawer contains 18 pairs of gloves in 4 colours: red, blue, green, and black. All gloves are mixed together (left and right gloves are not paired).

What is the minimum number of gloves that must be pulled out to guarantee at least one matching pair of the same colour and hand (i.e., left and right glove of the same colour)?

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Question 8

In a group of 90 people:

- 45 like tea.
- 50 like coffee.
- 20 like both.

How many people like neither tea nor coffee?

Question 9

A bowl contains 30 fruit pieces: bananas, oranges, and apples.

What is the minimum number of fruit you must pick to ensure that you have at least 11 pieces of the same type?

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Question 10

A list of 13 integers is selected from the set $\{1, 2, 3, \ldots, 24\}$ (without replacement).

Prove that at least two of the selected integers differ by a multiple of 12.

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Question 11

In a school of 100 students:

- 45 play netball.
- 52 play volleyball.
- 40 play basketball.
- 20 play both netball and volleyball.
- 25 play both netball and basketball.
- 18 play both volleyball and basketball.
- 10 play all three sports.

How many students play none of the three sports?

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Question 12 Tech-Active.

Consider all the arrangements of the letters in the word CALCULATE.

a. How many distinct arrangements of the letters are there?

b. Calculate the number of arrangements where at least one repeated pair (AA, CC, or LL) occurs adjacently.

c. Hence, how many of these arrangements have no repeated letters next to each other?

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Section D: Exam 1

Question 13

A teacher is giving away coloured pens to students as part of a game. The students can choose from 5 unique colours of pens.

a. Each student writes down **at least one** of the colours of pen that they want to win.

How many possible selections of colours are there?

_____

_____

_____

_____

b. A student will win the colour of pen that they wrote down as long as nobody else wrote down the exact same colour of pen.

What is the least number of students that must be in the class, to guarantee that nobody will win?

_____

_____

_____

_____

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Question 14

In a group of 40 students:

- 20 like tennis.
- 18 like basketball.
- 15 like soccer.
- 8 like both tennis and basketball.
- 6 like both tennis and soccer.
- 5 like both basketball and soccer.
- 3 like all three sports.

a. How many students like at least one of the three sports?

b. How many students like at least two of the three sports?

Question 15

Consider all 8 letters of the word TANZANIA.

a. How many different ways can all the letters be arranged if the three A's are kept together?

b. How many arrangements are there where consonants (T, N, Z) and vowels (A, I) alternate, beginning with a consonant?

From the 8 letters of TANZANIA, 4 are selected.

c. How many different selections include exactly one N and one A?

d. How many different selections include exactly one N?

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Section E: Exam 2

Question 16

Maya owns three types of collectables: 6 unique wooden carvings, 4 different sea shells, and 3 handmade pottery birds.

a. She allows her niece, Lily, to choose 5 items to play with. Lily must select at least one item of each type. How many different selections can Lily make?

Maya arranges 10 of the 13 items in a row on a shelf.

b. How many arrangements are possible if she places a bird at each end and no birds appear anywhere else?

c. How many arrangements are possible if she places a bird at each end, and wooden carvings and sea shells are alternated in the positions between?

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Question 17

A student committee has members from three year levels: Year 10, Year 11, and Year 12. There are 7 students from Year 10, 2 from Year 11, and 2 from Year 12.

Five members are selected to form a subcommittee. How many different selections include at least one member from each year level?

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Question 18

Consider the digits of the number, 1345789.

a. How many different 7-digit numbers can be made using all seven digits, if all the odd digits are placed together and no digits are repeated?

b. How many 4-digit numbers can be made using these digits if the number is even, lies between 3000 and 5000, and no digits are repeated?

c. How many numbers less than 1000 can be made using some or all of the digits, if the number is a multiple of 5 and digits can be repeated?

Question 19

Nine cards are labelled: 1, 2, 2, 3, 3, 4, 6, 6, 6

a. All nine cards are arranged in a line to make a 9-digit number.

i. How many such numbers can be formed if all the even digits appear together?

ii. How many such 9-digit numbers can be formed if the first and last digits are both odd?

b. Three of the nine cards are chosen and arranged to make a 3-digit number.

i. How many such numbers are possible if there are no repeated digits?

ii. How many such 3-digit numbers can be made if the number is between 200 and 300?

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