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VCE Specialist Mathematics ½
Combinatorics I [5.1]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2-Pg 10

Section A: Compulsory Questions

Sub-Section [5.1.1]: Permutations and Combinations (Tech-Free)

Question 1

a. Calculate ${}^5P_2$.

$$
\begin{aligned}
{}^5P_2 &= \frac{5!}{(5-2)!} \\
&= 5 \times 4 \ [1\text{M}] \\
&= 20 \ [1\text{A}]
\end{aligned}
$$

b. Calculate ${}^6C_3$.

$$
\begin{aligned}
{}^6C_3 &= \frac{6!}{3!(6-3)!} \\
&= \frac{6\times5\times4}{3\times2\times1} \ [1\text{M}] \\
&= 20 \ [1\text{A}]
\end{aligned}
$$

c. Calculate $\binom{5}{3}$.

$$
\begin{aligned}
\binom{5}{3} = {}^5C_3 &= \frac{5!}{3!(5-3)!} \\
&= \frac{5\times4}{2\times1} \ [1\text{M}] \\
&= 10 \ [1\text{A}]
\end{aligned}
$$

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Question 2

a. In how many ways can a president, vice president, and secretary be chosen from a committee of 8 people?

Order matters: Use permutation ${}^nP_r$ Setup ${}^8P_3$ **[1M]**
Number of ways $= 8 \times 7 \times 6$ **[1M]**
$= 336$ **[1A]**

b. How many distinct hands of 3 cards can be dealt from a specific set of 10 cards?

Order does not matter: Use combination ${}^nC_r$ Setup ${}^{10}C_3$ **[1M]**
Number of hands $= \frac{10\times9\times8}{3\times2\times1}$ **[1M]**
$= 10 \times 3 \times 4$
$= 120$ **[1A]**

c. How many ways can 4 different books be arranged on a shelf?

Order matters, arranging all items: Use $n!$ Setup $4!$ **[1M]**
Number of ways $= 24$ **[1A]**

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Question 3

Given that, ${}^kP_2 = 30$. Find the value of k.

${}^kP_2 = k(k-1)$ [1M]
$k(k-1) = 30 \implies k^2 - k - 30 = 0$ [1M]
$(k-6)(k+5) = 0$
Since k must be an integer ≥ 2, $k = 6$ [1A]

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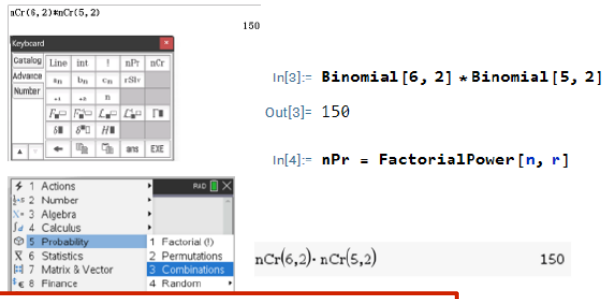
Sub-Section [5.1.2]: Permutations and Combinations with Restrictions/Composite (Tech-Active)

Question 4

a. How many ways can a committee of 4 people be chosen from 6 men and 5 women if there must be exactly 2 men?

Setup: (Choose 2M from 6) × (Choose 2W from 5). ${}^6C_2 \times {}^5C_2$ [1M]
Total ways = 150 [1A]



b. How many ways can 7 people be seated around a circul

In a circular arrangement, fix one person's position Arrange the remaining $7 - 1 = 6$ people relative to the first
Number of ways = 6! [1M]
= 720 [1A]

c. A team of 3 is chosen from 5 boys and a team of 2 is chosen from 4 girls. How many ways can the two separate teams be chosen?

Setup: (Choose 3B from 5) × (Choose 2G from 4). ${}^5C_3 \times {}^4C_2$ [1M]
Total ways = 60 [1A]

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Question 5

a. How many distinct arrangements are there of the letters in the word 'ENGINEERING'?

11 letters: E(3), N(3), G(2), I(2), R(1) Use formula $\frac{n!}{n_1!n_2!\ldots}$ Setup $\frac{11!}{3!3!2!2!}$ **[1M]**
Arrangements $= 277,200$ **[1A]**

b. A student must select 5 books from a reading list of 10 books. However, 2 specific books are mandatory. How many selections are possible?

Choose remaining $5-2=3$ from available $10-2=8$ Use combination ${}^{8}C_{3}$ **[1M]**
Number of selections $= 56$ **[1A]**

c. From a group of 9 students, how many ways can two committees be formed if one committee has 4 students and the other has 3 students, and no student can be on both committees?

Setup: (Choose 4 for C1 from 9) $\times$ (Choose 3 for C2 from remaining 5). ${}^{9}C_{4} \times {}^{5}C_{3}$ **[1M]**
Total ways $= 1260$ **[1A]**

Question 6 (6 marks)

a. How many ways can 4 distinct physics books and 3 distinct chemistry books be arranged on a shelf if all the physics books must be kept together?

Treat physics books as block (P) Arrange (P), C1, C2, C3: 4! Arrange physics internally: 4!
Setup $4! \times 4!$ **[1M]**
Total ways $= 576$ **[1A]**

b. A group of 5 men and 4 women are forming a subcommittee of 4 people. How many ways can this be done if the subcommittee must contain at least 2 women?

Use cases: (2W, 2M) + (3W, 1M) + (4W, 0M) Setup $({}^{4}C_{2}{}^{5}C_{2}) + ({}^{4}C_{3}{}^{5}C_{1}) + ({}^{4}C_{4}{}^{5}C_{0})$ **[1M]**
$= 60 + 20 + 1$
$= 81$ **[1A]**

c. How many ways can a president and vice president be chosen from 10 people, and then a separate committee of 3 chosen from the remaining 8 people?

Setup: (Choose Pres/VP using ${}^{10}P_2$) $\times$ (Choose Committee using ${}^{8}C_3$) **[1M]**
Total ways $= (10 \times 9) \times 56$
$= 90 \times 56$
$= 5040$ **[1A]**

Question 7

Consider arrangements of the letters in the word 'STATISTICS'.

a. How many distinct arrangements are there?

Use formula $\frac{n!}{n_1!n_2!\ldots}$ Setup $\frac{10!}{3!3!2!}$ **[1M]**
Arrangements $= 50400$ **[1A]**

b. How many arrangements begin with 'S' and end with 'T'?

Fix S _____ T Arrange remaining 8 letters: S(2), T(2), I(2), A(1), C(1) Setup $\frac{8!}{2!2!2!}$ **[1M]**
Arrangements $= 5040$ **[1A]**

c. How many arrangements have all the vowels (A, I, I) together?

Treat (AII) as block Arrange (AII), S,T,S,T,C,S,T (8 items: S=3, T=3) Internal arrangements of (AII) $= \frac{3!}{2!} = 3$ Setup $\frac{8!}{3!3!} \times 3$ **[1M]**
Ways $= 1120 \times 3$
$= 3360$ **[1A]**

d. How many arrangements have no two vowels adjacent?

Arrange 7 consonants (S=3, T=3, C=1): $\frac{7!}{3!3!} = 140$ ways **[1M]**
Place 3 vowels (A, I, I) into the 8 gaps created by consonants (_C_C_C_C_C_C_C_)
Choose 3 gaps: ${}^8C_3 = 56$ Place A, I, I in gaps: $\frac{3!}{2!} = 3$ Setup $140 \times 56 \times 3$ **[1M]**
Ways $= 140 \times 56 \times 3$
$= 23520$ **[1A]**

e. How many arrangements have all the 'S' letters together AND all the 'T' letters together?

Treat (SSS) as block S' and (TTT) as block T'
Arrange S', T', I, I, A, C (6 items: I=2) Setup $\frac{6!}{2!}$ **[1M]**
Internal arrangements are 1 **[1M]**
Total ways $= \frac{720}{2} = 360$ **[1A]**

f. How many arrangements have exactly two 'S' letters adjacent?

Total arrangements = 50400
Ways with (SSS) together = 3360 **[1M]**
Ways with no S adjacent: Arrange TTTIIAC ($\frac{7!}{3!2!} = 420$) Place 3 S in 8 gaps (${}^8C_3 = 56$) Ways
$= 420 \times 56 = 23520$ **[1M]**
Exactly two S adjacent = Total - (No S adjacent) - (SSS adjacent)
$= 50400 - 23520 - 3360 = 23520$ **[1A]**

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Sub-Section: The 'Final Boss'

Question 8

A committee of 5 people is to be selected from 6 physicists, 5 chemists, and 4 mathematicians.

a. How many ways can the committee be selected if there are no restrictions?

Choose 5 from 15 Setup ${}^{15}C_5$ **[1M]**
Ways = 3003 **[1A]**

b. How many ways can the committee be selected if it must have exactly 2 physicists and exactly 2 chemists?

Need 2P, 2C, 1M Setup ${}^{6}C_2 \times {}^{5}C_2 \times {}^{4}C_1$ **[1M]**
Ways = $15 \times 10 \times 4$
= 600 **[1A]**

c. How many ways can the committee be selected if it must have at least 3 mathematicians?

Cases: (3M, 2 Others) OR (4M, 1 Other) Others = 11 Setup $({}^{4}C_3 \times {}^{11}C_2) + ({}^{4}C_4 \times {}^{11}C_1)$ **[1M]**
Ways = $(4 \times 55) + (1 \times 11)$
$= 220 + 11$
= 231 **[1A]**

d. How many ways can the committee be selected if two specific physicists, Dr Alpha and Dr Beta, refuse to serve on the committee together?

Use complement: Total ways - Ways with BOTH A and B together
Ways A&B together = Choose A&B (1 way) AND Choose 3 more from remaining 13 $({}^{13}C_3)$
Setup $3003 - {}^{13}C_3$ **[1M]**
Ways = 3003 − 286
= 2717 **[1A]**

e. How many ways can the committee be selected if it must contain exactly 3 people from the same profession?

Case 1 (3P, 2 Others): ${}^{6}C_3 \times {}^{9}C_2 = 20 \times 36 = 720$
Case 2 (3C, 2 Others): ${}^{5}C_3 \times {}^{10}C_2 = 10 \times 45 = 450$
Case 3 (3M, 2 Others): ${}^{4}C_3 \times {}^{11}C_2 = 4 \times 55 = 220$ **[1M for cases] [1M for calcs]**
Total $= 720 + 450 + 220$
$= 1390$ **[1A]**

f. How many ways can the committee be selected if it must contain more physicists than chemists?

Sum cases (P, C, M) where P is greater than C, P+C+M=5: **[1M]**
Case Group 1:
(1, 0, 4): ${}^{6}C_1 {}^{5}C_0 {}^{4}C_4 = 6$
(2, 0, 3): ${}^{6}C_2 {}^{5}C_0 {}^{4}C_3 = 60$
(2, 1, 2): ${}^{6}C_2 {}^{5}C_1 {}^{4}C_2 = 450$ **[1M]**
Case Group 2:
(3, 0, 2): ${}^{6}C_3 {}^{5}C_0 {}^{4}C_2 = 120$
(3, 1, 1): ${}^{6}C_3 {}^{5}C_1 {}^{4}C_1 = 400$
(3, 2, 0): ${}^{6}C_3 {}^{5}C_2 {}^{4}C_0 = 200$ **[1M]**
Case Group 3:
(4, 0, 1): ${}^{6}C_4 {}^{5}C_0 {}^{4}C_1 = 60$
(4, 1, 0): ${}^{6}C_4 {}^{5}C_1 {}^{4}C_0 = 75$
(5, 0, 0): ${}^{6}C_5 {}^{5}C_0 {}^{4}C_0 = 6$ **[1M]**
Total $= 6 + 60 + 450 + 120 + 400 + 200 + 60 + 75 + 6$
$= 1377$ **[1A]**

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