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VCE Specialist Mathematics ½
Transformations II [4.3]
Test Solutions

21 Marks. 1 Minute Reading. 20 Minutes Writing.

Results:

Test Questions	____________ / 21

Section A: Test Questions (21 Marks)

Question 1 (3 marks)

State whether the statement is **true** or **false**.

Statement	**True**	**False**
a. To transform a function, we simply substitute in the x and y in terms of x' and y'.	✓	
b. Rotations and reflections preserve the length of shapes, but dilations and shears do not.	✓	
c. The rotation of θ clockwise around the origin is given by the following matrix: $$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$	It's clockwise	✓
d. A rotation by $60°$ anti-clockwise about the point $(1, 2)$ is the same as translating one unit down, two units to the left, rotating $60°$ about the origin, and then translating one unit upward, and finally translating two units to the right.	✓	
e. To reflect around a point (a, b), we first translate a units right and b units up. We go the other way around.		✓
f. To reflect a point about the line $y = -2x + 1$, first you need to translate the point one unit down, and then reflect it about the line $y = -2x$.		✓

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Question 2 (3 marks)

Find the equation of the line $y = \frac{1}{2}x - 1$ after it undergoes a shear of factor -2 parallel to the y-axis.

$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad \begin{bmatrix} x' = x \\ y' = y - 2 \cdot x \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} x' \\ y' \end{bmatrix}$ $\quad \begin{bmatrix} x = x' \\ y = 2 \cdot x' + y' \end{bmatrix}$

$\text{solve}\left(2 \cdot x + y = \frac{x}{2} - 1, y\right)$ $\quad y = \frac{-3 \cdot x}{2} - 1$

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Question 3 (4 marks)

Find the matrix corresponding to each of the following linear transformations, and hence find the image of the point (1,2) after undergoing each of the transformations.

a. Rotation by 60° anticlockwise. (2 marks)

$$\begin{bmatrix} \cos\left(\frac{\pi}{3}\right) & -\sin\left(\frac{\pi}{3}\right) \\ \sin\left(\frac{\pi}{3}\right) & \cos\left(\frac{\pi}{3}\right) \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} - \sqrt{3} \\ \frac{\sqrt{3}}{2} + 1 \end{bmatrix}$$

b. Reflection in the line $y = \frac{1}{\sqrt{3}}x$. (2 marks)

$$\begin{bmatrix} \cos\left(\frac{\pi}{3}\right) & \sin\left(\frac{\pi}{3}\right) \\ \sin\left(\frac{\pi}{3}\right) & -\cos\left(\frac{\pi}{3}\right) \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} \sqrt{3} + \frac{1}{2} \\ \frac{\sqrt{3}}{2} - 1 \end{bmatrix}$$

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Question 4 (5 marks)

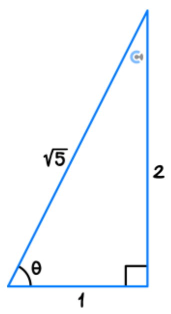
a. Find the matrix that will reflect the point (x, y) in the line through the origin at an angle of $30°$ to the positive direction of the x-axis. (2 marks)

We simply let $\theta = 30°$, and so the required reflection matrix is:

$$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix} = \begin{bmatrix} \cos(60°) & \sin(60°) \\ \sin(60°) & -\cos(60°) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$$

b. Find the matrix that will reflect the point (x, y) in the line $y = 2x$. (3 marks)

Since $\tan(\theta) = 2 = \frac{2}{1}$, we draw a right-angled triangle with opposite and adjacent lengths 2 and 1 respectively.



Pythagoras' theorem gives the hypotenuse as $\sqrt{5}$. Therefore

$$\cos(\theta) = \frac{1}{\sqrt{5}} \text{ and } \sin(\theta) = \frac{2}{\sqrt{5}}.$$

We then use the double angle formulas to show that:

$$\cos(2\theta) = 2\cos^2(\theta) - 1 = 2\left(\frac{1}{\sqrt{5}}\right)^2 - 1 = \frac{2}{5} - 1 = -\frac{3}{5}$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta) = 2 \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{4}{5}$$

Therefore, the required reflection matrix is:

$$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix} = \begin{bmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

Question 5 (6 marks)

Find the equation of the graph of $y = 3x + 1$ under a reflection in the line $y = \frac{1}{\sqrt{3}}x$.

Solve $\left(\frac{\sqrt{3}.x}{2} - \frac{y}{2} = 3.\left(\frac{x}{2} + \frac{\sqrt{3}y}{3}\right) + 1, y\right)$

$$y = \frac{(3.\sqrt{3} - 1).((\sqrt{3} - 3).x - 2)}{26}$$

$$\begin{bmatrix} \cos\left(\frac{\pi}{3}\right) & \sin\left(\frac{\pi}{3}\right) \\ \sin\left(\frac{\pi}{3}\right) & -\cos\left(\frac{\pi}{3}\right) \end{bmatrix} \qquad \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix} \qquad \begin{bmatrix} \frac{x}{2} + \frac{\sqrt{3}.y}{2} = x' \\ \frac{\sqrt{3}.x}{2} - \frac{y}{2} = y' \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix}^{-1} . \begin{bmatrix} x' \\ y' \end{bmatrix} \qquad \begin{bmatrix} x = \frac{x'}{2} + \frac{\sqrt{3}.y'}{2} \\ y = \frac{\sqrt{3}.x'}{2} - \frac{y'}{2} \end{bmatrix}$$

Solve $\left(\frac{\sqrt{3}.x}{2} - \frac{y}{2} = 3.\left(\frac{x}{2} + \frac{\sqrt{3}y}{3}\right) + 1, y\right)$

$$y = \frac{(3\sqrt{3} - 1)((\sqrt{3} - 3)x - 2)}{26}$$

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