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VCE Specialist Mathematics ½
Transformations I [4.2]
Test Solutions

24.5 Marks. 1 Minute Reading. 25 Minutes Writing

Results:

Test Questions	_____ / 24.5

Section A: Test Questions (24.5 Marks)

Question 1 (4.5 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. A linear transformation is defined by a rule of the form $(x, y) \rightarrow (ax + by, cx + dy)$.	✓	
b. You can find the first column of a linear transformation by finding the coordinates that $(1, 0)$ maps to, and the second column by finding the coordinates that $(0, 1)$ maps to.	✓	
c. The point $(2, -1)$ under the transformation $(x, y) \rightarrow (2x - 3y, -x + 4y)$ has image $(1, -6)$.		✓
d. If a triangle T has area $k\ units^2$, it then undergoes a transformation represented by matrix A, where $\lvert det(A) \rvert = c$, then the area of the transformation triangle T' has an area $\frac{k}{c}\ units^2$.		✓
e. A dilation by a factor 2 from the x-axis is always the same as a dilation by a factor $\frac{1}{2}$ from the y-axis.		✓
f. For a shear parallel to the x-axis, the points further away from the x-axis vertically shift further horizontally than the points closer to the x-axis.	✓	
g. A projection of any point onto the y-axis will reduce the y-coordinate to 0.		✓
h. In order for a transformation to be reversed, the determinant of the transformation matrix must not be 0 and there must be only one pre-image for the image.	✓	
i. If there is a sequence of transformations T_1, T_2, T_3 applied in that order, then the resulting transformation will be given by $T_R = T_1 T_2 T_3$, where each T_i is a 2×2 matrix.		✓

Question 2 (6 marks)

For the following questions, state the composite transformation matrix and find the image of the point $(1, -2)$.

a. Reflection in the y-axis and projection onto x-axis. (2 marks)

$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$ and $(-1,0)$.

b. Shear of factor 4 parallel to the x-axis and reflection in the x-axis. (2 marks)

$\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$ and $(-7, 2)$.

c. Dilation of factor 2 from y-axis and shear of factor 3 parallel to the y-axis. (2 marks)

$\begin{bmatrix} 2 & 0 \\ 6 & 1 \end{bmatrix}$ and $(2,4)$.

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Question 3 (9 marks)

$$\boldsymbol{A} = \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$$

a. Find $det\ \boldsymbol{A}$. (1 mark)

$A = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$

$= \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

(a) Find det A. $= ad - bc$

$\det A = 2(3) - (-1)(-2)$

$= 6 - 2$

$= 4$

b. Find $\boldsymbol{A}^{-1}$. (2 marks)

(b) Find A^{-1}.

$A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$= \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$

The triangle R is transformed into the triangle S by the matrix $\boldsymbol{A}$.

c. Given that the area of triangle S is 72 square units, find the area of triangle R. (2 marks)

The triangle R is transformed to the triangle S by the matrix $\mathbf{A}$.
Given that the area of triangle S is 72 square units,
(c) find the area of triangle R.

Area $R = \frac{1}{4}(72)$

$= 18$ square units

Area $S = (\det A)(\text{Area } R)$

The triangle S has vertices at the points $(0, 4)$, $(8, 16)$ and $(12, 4)$.

d. Find the coordinates of the vertices of R. (4 marks)

The triangle S has vertices at the points $(0,4)$, $(8,16)$ and $(12,4)$.

(d) Find the coordinates of the vertices of R.

Since $AR = S$ $\therefore R = A^{-1}S$

$$= \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 8 & 12 \\ 4 & 16 & 4 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 8 & 56 & 44 \\ 8 & 40 & 20 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 14 & 11 \\ 2 & 10 & 5 \end{pmatrix}$$

$\therefore$ vertices of R: $(2,2)$, $(14,10)$ and $(11,5)$

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Question 4 (5 marks)

Consider the transformation given by $A = \begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$.

a. Find the image of $(1,1)$ under the transformation A. (1 mark)

$(2,7)$

b. Find the inverse matrix A^{-1}. (2 marks)

$\frac{1}{-11}\begin{bmatrix} 5 & -3 \\ -2 & -1 \end{bmatrix}$

It is known that (a, b) under the transformation A was $(11, 33)$.

c. Find the values of a and b. (2 marks)

$a = 4$ and $b = 5$.

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