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VCE Specialist Mathematics ½
Transformations I [4.2]
Homework

Admin Info & Homework Outline:

Student Name	
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Section A: Compulsory Questions

Sub-Section [4.2.1]: Using Matrices for Linear Transformations

Question 1

For each of the following, write the transformation matrix for the given mapping.

a. $(x, y) \rightarrow (2x + 3y, 4x + 5y)$

b. $(x, y) \rightarrow (-x + 4y, 6x - 2y)$

c. $(x, y) \rightarrow (7x - 3y, -5x + 8y)$

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Question 2

a. Find the image of the point $(3, -2)$ under the transformation matrix:

$$T = \begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix}$$

b. Find the image of the point $(-1,5)$ under the transformation matrix:

$$T = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$$

c. Find the image of the point $(4,3)$ under the transformation matrix:

$$T = \begin{bmatrix} 5 & -1 \\ 2 & 6 \end{bmatrix}$$

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Question 3

a. A unit square with vertices $(0,0), (1,0), (0,1)$ and $(1,1)$ is transformed into a parallelogram with vertices $(0,0), (4,1), (2,3)$ and $(6,4)$. Find a possible transformation matrix T.

b. Verify that the transformation matrix correctly maps $(1,1)$ to $(6,4)$.

c. Compute the area of the transformed parallelogram.

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Question 4 Tech-Active.

Find the image of the point $(2, -4)$ under the transformation matrix:

$$T = \begin{bmatrix} -3 & 2 \\ 2 & 4 \end{bmatrix}$$

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Sub-Section [4.2.2]: Dilations, Reflections, Shears and Projections

Question 5

a. Write the transformation matrix for a dilation by a factor of $k = 3$ in both the x and y-directions.

b. Write the transformation matrix for a reflection in the x-axis.

c. Write the transformation matrix for a shear in the x-direction with a shear factor of $k = 2$.

d. Write the transformation matrix for a projection onto the x-axis.

e. Use matrices to find the rule for a translation 3 units to the right and 2 units down.

Question 6

a. Apply a reflection in the y-axis to the point $(3, -2)$.

b. Apply a shear in the x-direction with shear factor $k = 2$ to the point $(4,1)$.

c. Apply a projection onto the x-axis to the point $(-2,5)$.

Question 7

a. The point $(4,2)$ is mapped to $(-4,2)$. Find a transformation matrix that achieves this.

b. The point $(3,-1)$ is mapped to $(7,-1)$. Find a transformation matrix that achieves this.

c. The point $(-2,5)$ is mapped to $(-2,0)$. Find a transformation matrix that achieves this.

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Sub-Section [4.2.3]: Inverse Transformations

Question 8

a. Find the inverse transformation matrix for the mapping:

$$(x, y) \rightarrow (2x + 3y, 4x + 5y)$$

b. Find the inverse transformation matrix for the mapping:

$$(x, y) \rightarrow (-x + 4y, 6x - 2y)$$

c. Find the inverse transformation matrix for the mapping:

$$(x, y) \rightarrow (7x - 3y, -5x + 8y)$$

Question 9

a. The transformation matrix:

$$T = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

Maps a point (x, y) to $(10,6)$. Find (x, y).

b. The transformation matrix:

$$T = \begin{bmatrix} 5 & -3 \\ 2 & 7 \end{bmatrix}$$

Maps a point (x, y) to $(4, 11)$. Find (x, y).

c. The transformation matrix:

$$T = \begin{bmatrix} 6 & 2 \\ -3 & 5 \end{bmatrix}$$

Maps a point (x, y) to $(12,7)$. Find (x, y).

Question 10

a. A transformation maps the points:

$$(1,2) \to (5,4), \quad (3,-1) \to (7,6)$$

Find the transformation matrix T.

b. A transformation maps the points:

$$(2,1) \rightarrow (6,5), \qquad (1,3) \rightarrow (4,2)$$

Find the transformation matrix T.

c. A transformation maps the points:

$$(3,2) \rightarrow (9,7), \qquad (0,5) \rightarrow (2,10)$$

Find the transformation matrix T.

Question 11 Tech-Active.

A transformation matrix:

$$T = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$$

Maps a point (x, y) to $(10,6)$. Find (x, y).

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Sub-Section [4.2.4]: Composite Transformations

Question 12

A transformation consists of:

- A reflection in the x-axis.
- A shear in the x-direction with shear factor $k = 3$.

a. Find the composite transformation matrix.

b. Find the image of $(4, 2)$ under this transformation.

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Question 13

A transformation consists of:

- A dilation by a factor of 2 from both the x and y-axes.
- A reflection in the y-axis.

a. Find the composite transformation matrix.

b. Find the image of $(-3, 5)$ under this transformation.

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Question 14

A transformation consists of:

- A reflection in the x-axis.
- A shear in the y-direction with shear factor $k = 2$.
- A projection onto the x-axis.

a. Find the composite transformation matrix.

b. Find the image of $(3, -2)$ under this transformation, followed by a translation 1 unit right and 3 units down.

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Question 15 Tech-Active.

The point $(3, 4)$ is the image of a point (x, y) after the transformations T and then S have been applied in that order. The transformation matrices are:

$$T = \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix} \text{ and } S = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$$

Find the original point (x, y).

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Sub-Section: The 'Final Boss'

Question 16

A company is designing a robotic arm that manipulates objects in a 2D workspace. The workspace is represented by the xy-plane, and transformations are applied using matrices. The arm operates on a unit square with initial vertices at $A(0,0), B(1,0), C(0,1)$, and $D(1,1)$.

a. The robotic arm first applies:

- A shear in the x-direction with shear factor $k = 2$.
- A reflection in the y-axis.

Find the composite transformation matrix T.

b. Apply the transformation matrix T to the original unit square. Find the coordinates of the transformed points A', B', C', D'.

c. Find the area of the transformed shape. Does the transformation preserve area?

d. Inverse transformation.
The robotic arm must reverse the transformation to return the shape to its original position. Find the inverse transformation matrix T^{-1}, if it exists.

e. Alternative transformations.
Instead of the previous transformation, the arm now applies:

- A dilation by a factor of 3 in both the x- and y-directions.
- A projection onto the x-axis.

Find the new composite transformation matrix T'.

f. Calculate the area of the transformed shape under this new transformation. Compare it to the original and explain the result.

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Section B: Supplementary Questions

Sub-Section [4.2.1]: Using Matrices For Linear Transformations

Question 17

For each of the following, write the transformation matrix for the given mapping:

a. $(x, y) \rightarrow (3x - 2y, 5x + 4y)$

b. $(x, y) \rightarrow (6x + y, -x + 3y)$

c. $(x, y) \rightarrow (-2x + 4y, 7x - 5y)$

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Question 18

a. Find the image of the point $(2, -3)$ under the transformation matrix:

$$T = \begin{bmatrix} 5 & 2 \\ -3 & 4 \end{bmatrix}$$

b. Find the image of the point $(-2,4)$ under the transformation matrix:

$$T = \begin{bmatrix} -1 & 3 \\ 2 & 5 \end{bmatrix}$$

c. Find the image of the point $(3,2)$ under the transformation matrix:

$$T = \begin{bmatrix} 4 & -2 \\ 3 & 6 \end{bmatrix}$$

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Question 19

a. A unit square with vertices (0,0), (1,0), (0,1), and (1,1) is transformed to a parallelogram with vertices (0,0), (3,2), (4,1), and (7,3). Find a possible transformation matrix T.

b. Verify that the transformation matrix correctly maps (1,1) to (7,3).

c. Compute the area of the transformed parallelogram.

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Sub-Section [4.2.2]: Dilations, Reflections, Shears, and Projections

Question 20

a. Write the transformation matrix for a dilation by a factor of $k = 4$ in both the x- and y-directions.

b. Write the transformation matrix for a reflection in the y-axis.

c. Write the transformation matrix for a shear in the y-direction with shear factor $k = 3$.

d. Write the transformation matrix for a projection onto the y-axis.

e. Use matrices to find the rule for a translation 5 units left and 4 units up.

Question 21

a. Apply a reflection in the x-axis to the point $(-5,3)$.

b. Apply a shear in the y-direction with shear factor $k = 2$ to the point $(2,3)$.

c. Apply a projection onto the y-axis to the point $(6,-4)$.

Question 22

a. The point $(-3,2)$ is mapped to $(3,2)$. Find a transformation matrix that achieves this.

b. The point $(1,4)$ is mapped to $(1,8)$. Find a transformation matrix that achieves this.

c. The point $(7,-2)$ is mapped to $(7,0)$. Find a transformation matrix that achieves this.

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Sub-Section [4.2.3]: Inverse Transformations

Question 23

a. Find the inverse transformation matrix for the mapping:

$$(x, y) \rightarrow (4x + 5y, 7x + 2y)$$

b. Find the inverse transformation matrix for the mapping:

$$(x, y) \rightarrow (3x - 4y, 6x + 5y)$$

c. Find the inverse transformation matrix for the mapping:

$$(x, y) \rightarrow (2x + 6y, 5x - 3y)$$

Question 24

a. The transformation matrix:

$$T = \begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix}$$

Maps a point (x, y) to $(8,7)$. Find (x, y).

b. The transformation matrix:

$$T = \begin{bmatrix} 5 & -3 \\ 2 & 6 \end{bmatrix}$$

Maps a point (x, y) to $(11, 4)$. Find (x, y).

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Question 25

a. A transformation maps the points:

$$(3,2) \to (7,5), \qquad (1,-1) \to (4,2)$$

Find the transformation matrix T.

b. A transformation maps the points:

$$(2,4) \to (10,3), \qquad (-3,1) \to (5,7)$$

Find the transformation matrix T.

c. A transformation maps the points:

$$(1,3) \rightarrow (8,5), \qquad (2,-2) \rightarrow (4,6)$$

Find the transformation matrix T.

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Sub-Section [4.2.4]: Composite Transformations

Question 26

A transformation consists of:

- A reflection in the x-axis.
- A shear in the y-direction with shear factor $k = 3$.

a. Find the composite transformation matrix.

b. Find the image of $(5, -2)$ under this transformation.

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Question 27

A transformation consists of:

- A shear in the y-direction with shear factor $k = 4$.
- A reflection in the x-axis.

a. Find the composite transformation matrix.

b. Find the image of $(-1,2)$ under this transformation.

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Question 28

A transformation consists of:

- A reflection in the y-axis.
- A shear in the x-direction with shear factor $k = 2$.
- A dilation by a factor of 3 in both directions.

a. Find the composite transformation matrix.

b. Find the image of $(4, -2)$ under this transformation.

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