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VCE Specialist Mathematics ½
Matrices [4.1]
Workbook

Outline:

Learning Objectives:

- ☐ **SM12 [4.1.1]** – Basics of Matrices and Identifying Types of Matrices. Calculate the Transpose and Trace of a Matrix
- ☐ **SM12 [4.1.2]** – Perform Matrix Addition, Scalar Multiplication, and Matrix Multiplication
- ☐ **SM12 [4.1.3]** – Calculate the Inverse of a Matrix and Determine its Determinant
- ☐ **SM12 [4.1.4]** – Apply Matrix Operations to Solve Systems of Linear Equations

Section A: Introduction to Matrices

Sub-Section: Understanding Matrices

Context

- Why matrices?
  - Consider the system of n many linear equations.

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$\vdots$$

$$a_nx + b_ny + c_nz = d_n$$

  - Imagine solving them!
  - That would be very tedious.

What if there was a way to solve it as one equation?

$$A \cdot Var = D$$

  - We can do that using matrices!

Space for Personal Notes

Let's have a look at what a matrix is!

Matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- A matrix is an ________ of numbers.
- It has ________ and ____________.
- Dimension (similar to size) of a matrix is given by rows × columns.

$$Dimension = Rows \times Columns$$

- Each number is called an ______________.

Discussion: For a 2×2 matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ how can we represent elements in the 1^{st} row, 2^{nd} column?

Labelling Elements

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$a_{i,j} = i^{\text{th}}\ row, \qquad j^{\text{th}} column$$

- Elements of matrix A is given by ________________ with row and column number.

Question 1

Consider the following matrix.

$$A = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 3 & 0 \end{bmatrix}$$

a. State the dimension of the matrix.

b. $a_{1,1} + a_{2,1}$

c. $a_{2,1} + a_{2,3}$

TIP: Remember the order ARC! A matrix with R row and C column.

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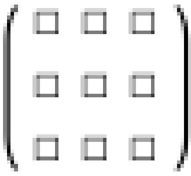
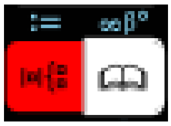
Calculator Commands: Defining Matrices on Technology

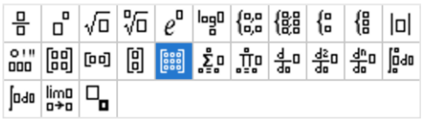
Mathematica

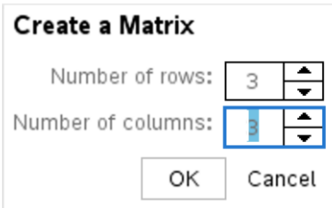


- We use round brackets.
- Control Comma: Makes new columns.
- Control Enter: Makes new rows.

TI-Nspire

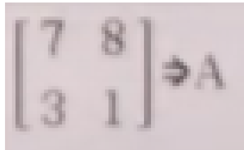






- Press the button **next to** the book.

Casio Classpad



- Under **maths 2.**
- Press it multiple times to change the dimension.
- Use the arrow to save the matrix.

Question 2 Tech-Active.

On your technology, save the following matrix.

$$A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & 2 & -5 \end{bmatrix}$$

Sub-Section: Types of Matrices

Let's have a look at different types of matrices!

Row and Column Matrix

- Row Matrix
  - A matrix that has only one row.

$$\begin{bmatrix}1\end{bmatrix}$$
$$\begin{bmatrix}1 & 2\end{bmatrix}$$
$$\begin{bmatrix}1 & 2 & 3\end{bmatrix}$$
$$\begin{bmatrix}1 & 2 & 3 & 4\end{bmatrix}$$
$$\begin{bmatrix}1 & 2 & 3 & 4 & 5\end{bmatrix}$$

- Column Matrix
  - A matrix that has only one column.

$$\begin{bmatrix}1\end{bmatrix} \begin{bmatrix}1\\2\end{bmatrix} \begin{bmatrix}1\\2\\3\end{bmatrix} \begin{bmatrix}1\\2\\3\\4\end{bmatrix} \begin{bmatrix}1\\2\\3\\4\\5\end{bmatrix}$$

Square Matrix

$$\begin{bmatrix}a & b\\c & d\end{bmatrix}, \begin{bmatrix}a & b & c\\d & e & f\\g & h & i\end{bmatrix}$$

- Matrices that have the _________ number of rows and columns.

$$Dimension = n \times n$$

Zero Matrix

$$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

- Matrix with all the elements equaling to _____________.

Sub-Section: Transpose and Trace

What does it mean to transpose a matrix?

Transpose

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

- Simply swap the row and the column.
  - Eg: 1^{st} row becomes the 1^{st} column.
- You may also see the transpose written as A'.

Question 3 Walkthrough.

Consider the matrix below.

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & 5 \end{bmatrix}$$

Find A^T.

Question 4

For each of the following, find A^T.

a. $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \end{bmatrix}$

b. $A = \begin{bmatrix} 7 \\ 6 \\ 5 \end{bmatrix}$

c. $A = \begin{bmatrix} 5 & 1 \\ 0 & 7 \\ 9 & 8 \end{bmatrix}$

Let's now look at the trace of a square matrix!

Definition

Trace

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$tr(A) = a + d$$

- Find the sum of all the _______________________________.
- We can only find the trace of a ______________ matrix.

Question 5

For each of the following square matrix, calculate $tr(A)$.

a. $A = \begin{bmatrix} 1 & 1 & 3 \\ \frac{1}{2} & -7 & 0 \\ e & \pi & 1 \end{bmatrix}$

b. $A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

c. $A = [\pi]$

Section B: Operations of Matrices

Sub-Section: Addition and Subtraction

Discussion: Could we add two matrices of different size?

Addition and Subtraction of Matrices

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{bmatrix}$$

- Add the elements in the ______________ position of each matrix.
- We can only add/subtract matrices of the ____________________________.

Question 6

Let $X = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, Y = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

Find $X - Y$.

Sub-Section: Scalar Multiplication

Discussion: What will happen to the matrix if we multiply it by two?

Scalar Multiplication

$$k \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

- Multiply all the elements by k.

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Question 7

Let $X = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, $Y = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$, $A = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 \\ -2 & 4 \end{bmatrix}$, find:

a. $2X - 3Y$

b. $5A + \frac{1}{2}B$

c. $4Y + X + B$

Space for Personal Notes

Sub-Section: Matrix Multiplication

How do we multiply two matrices?

Exploration: Matrix Multiplication

- Let's take a look at two matrices A and B.

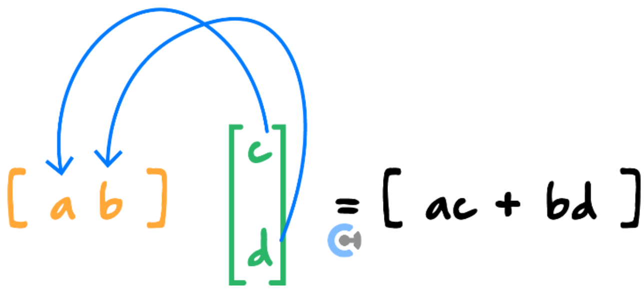
$$A = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

- What would the multiplication look like?

$$A \times B = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} =$$

- We simply take the row of the 1^{st} and column of the 2^{nd} matrix and multiply them.



- Let's take a look at two matrices A and B.

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

- What would the multiplication look like?

$$A \times B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$

- What does the multiplication of two matrices always give us?

<u>Discussion:</u> For $A \times B$, what does the number of columns for A has to be the same as?

<u>Discussion:</u> What happens if we multiply 2×3 and 3×1 matrix?

<u>Matrix Multiplication</u>

$$A \times B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \times 3 + (-1) \times 1 \\ 1 \times 3 + 2 \times 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

Number of Columns of 1^{st} Matrix = Number of Rows of 2^{nd}

- The answer will always be a matrix.

Space for Personal Notes

Question 8 Walkthrough.

For $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 2 \\ 0 & 3 \end{bmatrix}$, find AB.

NOTE: $m \times n$ matrix multiplied with $n \times k$ gives us $m \times k$ matrix.

Space for Personal Notes

Question 9

Find AB for the following set of matrices.

a. $A = \begin{bmatrix} 2 & 1 \\ 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

b. $A = \begin{bmatrix} 0 & 2 & 1 \\ 5 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$

<u>Discussion:</u> Is $A \times B$ the same as $B \times A$?

Sub-Section: Identity Matrix

What matrix if you multiply doesn't change the other matrix? Similar to 1 in numbers?

Question 10

Consider $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Find $A \times B$.

Space for Personal Notes

We call B in the previous question an identity matrix!

Identity Matrix

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Identity matrix I_n has a size $n \times n$.
- Its diagonal elements are equal to ________.
- Identity matrices are always a ________________________.

$$AI = IA = A$$

Discussion: What would $tr(I_n)$ always equal to?

Properties of Matrix Multiplication

$$A(B + C) = AB + AC$$

$$(A + B)C = AC + BC$$

$$A(BC) = (AB)C$$

$$AI = IA = A$$

$$A0 = 0A = 0$$

Section C: Inverse Matrices and Determinants

Sub-Section: Inverse Matrix

Inverse of a 2×2 Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- Inverse only exists for a square matrix.
- Matrix that has an inverse is called ______________________.

Question 11 Walkthrough.

Consider $A = \begin{bmatrix} 4 & 1 \\ 2 & -3 \end{bmatrix}$

Find A^{-1}.

Space for Personal Notes

Question 12

Consider $A = \begin{bmatrix} 3 & -3 \\ -2 & 1 \end{bmatrix}$

a. Find A^{-1}.

b. Find AA^{-1}.

Discussion: For the previous question, what do you notice when you multiply a matrix by its inverse?

Multiplication of Inverse Matrices

$$AA^{-1} = I_n = A^{-1}A$$

- The order does not matter.

Sub-Section: Determinant

Discussion: How can we tell if a square matrix does not have an inverse?

- Consider for $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Determinant of a 2×2 Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$det(A) = ad - bc$$

- If determinant equals to ____________, then A does not have an inverse.
- A is not ________________________.
- Notation for the determinant also includes using straight lines | | around the matrix.

$$det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Space for Personal Notes

Question 13

Calculate the determinant of the following matrices and state whether it is invertible.

a. $A = \begin{bmatrix} 3 & 4 \\ -2 & 1 \end{bmatrix}$

b. $A = \begin{bmatrix} 2 & 8 \\ 1 & 4 \end{bmatrix}$

How about 3×3 matrix?

Definition

Determinant of 3×3 Matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\det(A) = a \times \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \times \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \times \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

- For the first term, simply cover the first column and row and copy the 4 numbers.

$$\det(A) = a \times (ei - fh) - b \times (di - fg) + c \times (dh - eg)$$

Question 14 Walkthrough.

Calculate the determinant of the following matrix.

Find the $A = \begin{bmatrix} 1 & 3 & 4 \\ -3 & 2 & 5 \\ 0 & 3 & -1 \end{bmatrix}$

Space for Personal Notes

Question 15

Calculate the determinant of the following matrices and state whether it is invertible.

a. $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 3 \\ 1 & 5 & 1 \end{bmatrix}$

b. $A = \begin{bmatrix} 0 & 2 & 5 \\ 7 & 1 & 3 \\ -1 & 2 & 4 \end{bmatrix}$

Calculator Commands: Finding Determinant on Technology

- **Mathematica**
  - Control comma/enter to make cells.

    Det[$\begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix}$]

- **TI-Nspire**
  - **(Menu) > Matrix and Vector > Determinant:** $\boldsymbol{det(A)}$
  - Where A is a matrix, you enter in the brackets or a previously defined matrix.

- **Casio Classpad**
  - **(Interactive) > Matrix > Calculation > det:** $\boldsymbol{det(A)}$
  - Where A is a matrix you enter in the bracket, or a previously defined matrix.

Question 16 Tech-Active.

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 3 \\ 1 & 5 & 1 \end{bmatrix}$$

Find the determinant of A.

Space for Personal Notes

Section D: Systems of Linear Equations

Recall the context at the start of the class.

REMINDERS

- Why matrices?
  - Consider the system of n many linear equations.

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$\vdots$$

$$a_nx + b_ny + c_nz = d_n$$

  - Imagine solving them!
  - That would be very tedious.

What if there was a way to solve it as one equation?

$$A \cdot Var = D$$

  - We can do that using matrices!

Space for Personal Notes

Question 17

Consider the two matrices.

$$A = \begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix} \text{ and } B = \begin{bmatrix} x \\ y \end{bmatrix}$$

a. Find $A \times B$.

b. Hence, solve $A \times B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

NOTE: We can use matrices to represent simultaneous equations.

Discussion: Hence, how can we represent $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ using matrices?

Space for Personal Notes

System of Linear Equation Using Matrices

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

- We can extend the idea to more than 2 equations.

Discussion: Previously we used matrices to only represent the equations. Is there any way to solve the equation using matrices?

Exploration: Solving System of Linear Equation using Matrices

- Consider the system of linear equations.

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

- We can simplify the equation:

$$AX = C$$

- Let's try to solve for our variables by solving our matrix X.
- What happens when you multiply both sides by A^{-1}.

$$A^{-1}AX = A^{-1}C$$

Definition

Solving System of Linear Equation Using Matrices

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$AX = C$$

$$X = A^{-1}C$$

Question 18 Walkthrough.

Convert the following system of linear equations into matrix equation and solve for x and y.

$$-x + 6y = 14$$

$$-5x + 4y = -8$$

Question 19

Convert the following system of linear equations into matrix equation and solve for x and y.

a. $3x + 5y = 21$ and $6x - 2y = 6$.

b. $$\begin{aligned} -x + y &= 2 \\ x + y &= 3 \end{aligned}$$

Discussion: Okay seems like the inverse matrix is important. What if the matrix is not invertible?

Definition

Determinants to Determine Number of Solutions

$$AX = C$$

$$X = A^{-1}C$$

$$det(A) = 0 \Rightarrow \text{No or infinite solution}$$

- If the determinant equal to 0, the equation cannot be solved as there is no inverse of A.

$$det(A) \neq 0 \Rightarrow \text{Unique Solution}$$

- If the determinant is non-zero, equation can be solved as there is an inverse of A.

Question 20

Consider the system of linear equations. Determine whether the system of linear equations has a unique solution or not by using determinants.

a. $2x + y = 4$ and $-4x - 2y = 10$.

b. $2x + y + z = 1,\ -x - y + z = 10$ and $x + z = 4$.

Contour Check

Learning Objective: [4.1.1] - Basics of matrices and identifying types of matrices. Calculate the transpose and trace of a matrix

Key Takeaways

- **Matrices**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

  - A matrix is an ________ of numbers.
  - It has ________ and ____________.
  - Dimension (similar to size) of a matrix is given by rows $\times$ columns.

$$Dimension = Rows \times Columns$$

  - Each number is called an ______________.

- **Row and Column Matrix**
  - **Row Matrix**
    - A matrix that has only one row.
  - **Column Matrix**
    - A matrix that has only one column.

- **Square Matrix:** Has dimension ________.

- **Zero Matrix:** All elements are equal to _______.

Identity Matrix

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Identity matrix I_n has a size $n \times n$.
- Its diagonal elements are equal to ________.
- Identity matrices are always a ________________________.

$$AI = IA = A$$

Transpose

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$A^T = \_\_\_\_\_\_\_\_\_\_\_\_$$

Trace

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$tr(A) = a + d$$

- Find the sum of all the ___________________________________.
- We can only find the trace of a _______________ matrix.

Learning Objective: [4.1.2] – Perform matrix addition, scalar multiplication, and matrix multiplication

Key Takeaways

- **Addition and Subtraction of Matrices**

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

  - We can only add/subtract matrices of the ____________________.
  - Add the elements in the __________ position of each matrix.

- **Scalar Multiplication**

$$k \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \_\_\_\_\_\_\_\_\_\_\_\_$$

  - Multiply all the elements by k.

- **Matrix Multiplication**

$$A \times B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \times 3 + (-1) \times 1 \\ 1 \times 3 + 2 \times 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

Number of ________________ of 1^{st} Matrix
$=$ Number of ____________ of 2^{nd}

  - The answer will always be a matrix.

- Properties of Matrix Multiplication

$$A(B+C) = \_\_\_\_\_\_\_\_\_\_$$

$$(A+B)C = \_\_\_\_\_\_\_\_\_\_$$

$$A(BC) = \_\_\_\_\_\_\_\_\_\_$$

$$AI = IA = \_\_\_\_\_\_\_\_\_\_$$

$$A0 = 0A = \_\_\_\_\_\_\_\_\_\_$$

Learning Objective: [4.1.3] – Calculate the inverse of a matrix and determine its determinant

Key Takeaways

- Inverse of a 2×2 Matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

  - Inverse only exists for a square matrix.
  - Matrix that has an inverse is called ____________________.

- Multiplication of Inverse Matrices

$$AA^{-1} = \_\_\_\_\_\_\_\_\_\_ = A^{-1}A$$

  - The order does not matter.

- **Determinant of a 2×2 Matrix**

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$det(A) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

  - If determinant equals to __________, then A does not have an inverse.
  - A is not ____________________.

- **Determinant of 3×3 Matrix**

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$det(A) = a \times \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \times \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \times \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Learning Objective: [4.1.4] – Apply matrix operations to solve systems of linear equations

Key Takeaways

- Solving System of Linear Equation using Matrices

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$AX = C$$

$$X = \_\_\_\_\_\_\_\_\_\_$$

- Determinants to Determine Number of Solutions

$$AX = C$$

$$X = A^{-1}C$$

$$det(A) = 0 \Rightarrow \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

  - If the determinant equal to 0, the equation cannot be solved as there is no inverse of A.

$$det(A) \neq 0 \Rightarrow \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

  - If the determinant is non-zero, equation can be solved as there is an inverse of A.

VCE Specialist Mathematics ½

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