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VCE Specialist Mathematics ½
Matrices [4.1]
Test Solutions

23 Marks. 1 Minute Reading. 18 Minutes Writing.

Results:

Test Questions	________ / 23

Section A: Test Questions (23 Marks)

Question 1 (3 marks)

Tick whether the following statements are **true** or **false**.

Statement	True	False
a. Only square matrices have a chance to be invertible.	✓	
b. A $m \times n$ matrix can be multiplied by a $l \times n$ matrix, $n \neq l$ since they both have the same number of columns.		✓
c. For any two square matrices with the same dimensions, $A + B = B + A$ and $AB = BA$.		✓
d. If a square matrix A is invertible, then there exists another square matrix B such that $AB = BA$.	✓	
e. You can only take the determinant of a square matrix.	✓	
f. If A is invertible, and it is known that $AB = C$, then $B = A^{-1}C$ given that the dimensions of A, B and C allow for these multiplications to exist.	✓	

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Question 2 (3 marks)

If $A = \begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -4 \\ -2 & 8 \end{bmatrix}$, find the matrix X such that $\frac{2}{5}A^T + \frac{3}{2}X = B$.

```
In[1]:= A = {{3, 2}, {-1, 1}}
Out[1]= {{3, 2}, {-1, 1}}

In[2]:= B = {{0, -4}, {-2, 8}}
Out[2]= {{0, -4}, {-2, 8}}

In[4]:= 2/3 * (B - 2/5 * Transpose[A]) // MatrixForm
Out[4]//MatrixForm=
```

$$\begin{pmatrix} -\frac{4}{5} & -\frac{12}{5} \\ -\frac{28}{15} & \frac{76}{15} \end{pmatrix}$$

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Question 3 (3 marks)

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A^T = I$, then find the value of α, $\alpha \in [0,\ \pi]$.

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

To find the value of α, equate the corresponding terms, we get

$$2\cos\alpha = 1$$

$$\cos\alpha = \cos\frac{\pi}{3}$$

$$\alpha = \frac{\pi}{3}$$

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Question 4 (2 marks)

$C = \begin{bmatrix} 1 & 6 \\ -2 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} -5 & 10 \\ -4 & 8 \end{bmatrix}$.

For which matrix, C or D, does an inverse matrix **not** exist? Why?

For matrix D because the determinant is $-40 - -(40) = 0$ so it does not have an inverse.

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Question 5 (3 marks)

Consider

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 4 \\ -1 & -1 & 3 \end{bmatrix}$$

Find $det(A)$.

In[11]:= $\text{Det}\left[\begin{pmatrix} 1 & 3 & 2 \\ 0 & 5 & 4 \\ -1 & -1 & 3 \end{pmatrix}\right]$

Out[11]= 17

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Question 6 (5 marks)

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for any $n \in N$.

Hint: Use an induction proof.

$P(n): \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix} \quad \forall n \in N$

$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

Base case: $n = 1$

$LHS = A^1 = A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1+2(1) & -4(1) \\ (1) & 1-2(1) \end{bmatrix}$

$= RHS$

$\therefore$ Base Case True

Induction Step:

Let $k \in N$ be arbitrary but fixed, suppose $P(k)$ true, i.e. $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$

Then $A^{k+1} = A^k A$

$= A^k \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 3(1+2k)-4k & -12k-4(1-2k) \\ 1+2k-k & -4k-1+2k \end{bmatrix}$

$= \begin{bmatrix} 2k+3 & -4k-4 \\ k+1 & -2k-1 \end{bmatrix}$

$= \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix}$

$= RHS$ of $P(k+1)$

$\therefore P(n)$ true for all $n \in N$

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Question 7 (4 marks)

Solve the following systems of linear equations using matrices.

a. $x + 2y = 2$
$4x - 2y = 5$ (2 marks)

```
In[13]:= A = {{1, 2}, {4, -2}};
In[14]:= X = (x
              y);
In[15]:= b = {{2}, {5}};
In[16]:= Inverse[A] // MatrixForm
Out[16]//MatrixForm=
```

$$\begin{pmatrix} \frac{1}{5} & \frac{1}{5} \\ \frac{2}{5} & -\frac{1}{10} \end{pmatrix}$$

```
In[17]:= Inverse[A].b // MatrixForm
Out[17]//MatrixForm=
```

$$\begin{pmatrix} \frac{7}{5} \\ \frac{3}{10} \end{pmatrix}$$

```
In[18]:= Solve[x + 2 y == 2 && 4 x - 2 y == 5]
```

Out[18]= $\left\{\left\{x \to \frac{7}{5},\ y \to \frac{3}{10}\right\}\right\}$

b. $2x + 2y = 2$
$3x + 3y = 3$ (2 marks)

$Det = 0$. We see that the equations are scalar multiples of each other. Therefore, no unique solution.

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