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VCE Specialist Mathematics ½

Matrices [4.1]

Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
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Section A: Compulsory Questions

Sub-Section [3.5.1]: Basics of Matrices and Identifying Types of Matrices. Calculate the Transpose and Trace of a Matrix

Question 1

a. What does it mean for a matrix to be a **square matrix**? Give an example.

b. Given the matrix:

$$A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$$

Find $a_{1,1} + a_{1,2}$.

c. Compute the **trace** of matrix A.

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Question 2

a. Consider the matrix:

$$B = [7 \quad -2 \quad 5]$$

What type of matrix is B?

b. Compute the **transpose** of the matrix:

$$C = \begin{bmatrix} 1 & 3 \\ -2 & 0 \\ 4 & -5 \end{bmatrix}$$

c. Find the **trace** of:

$$D = \begin{bmatrix} 6 & -1 & 2 \\ 4 & 3 & -5 \\ 0 & 7 & 8 \end{bmatrix}$$

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Question 3

a. If a matrix E is such that $E^T = E$, what can we conclude about the matrix's dimensions? Give an example of a 3×3 matrix satisfying this condition.

b. Evaluate the **transpose** of:

$$F = \begin{bmatrix} 2 & -3 & 1 & 0 \\ 4 & 5 & -2 & 6 \\ -1 & 0 & 3 & -4 \end{bmatrix}$$

c. Explain why the trace of matrix F cannot be found.

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Sub-Section [3.5.2]: Perform Matrix Addition, Scalar Multiplication, and Matrix Multiplication

Question 4

a. Given the matrices:

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}$$

Evaluate $A + B$.

b. Given the matrix:

$$C = \begin{bmatrix} -1 & 2 \\ 0 & 4 \end{bmatrix}$$

Find $3C$.

c. Given the matrices:

$$D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad E = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Evaluate $D \times E$.

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Question 5

a. Matrices F and G are given as follows:

$$F = \begin{bmatrix} 4 & -1 & 2 \\ 0 & 3 & -5 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & 5 \end{bmatrix}$$

Evaluate $2F - 4G$.

b. Consider the matrices:

$$H = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -5 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix}$$

Find $H + Q$, if possible.

c. Given:

$$J = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 3 & 4 \end{bmatrix}, \quad k = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$$

Compute $J \times K$.

Question 6

a. Consider matrices L and M given by:

$$L = \begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} -1 & 2 \\ 5 & 0 \end{bmatrix}$$

Evaluate $L(L + M)$.

b. Consider the matrices:

$$N = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}, \quad P = \begin{bmatrix} 0 & -1 \\ 2 & 5 \end{bmatrix}$$

Determine whether $N^2 - P^2 = (N + P)(N - P)$.

c. Consider the matrices:

$$Q = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & -2 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 2 \\ x & 4 \\ 5 & y \end{bmatrix}$$

Find the value of x and y if $QR = \begin{bmatrix} 20 & 6 \\ -6 & 4 \end{bmatrix}$.

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Sub-Section [3.5.3]: Calculate the Inverse of a Matrix and Determine Its Determinant

Question 7

a. Consider det (A) for:

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

b. Is A **invertible**? Justify your answer.

c. If A is invertible, find A^{-1}.

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Question 8

a. Compute $\det(B)$ for:

$$B = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix}$$

b. If $BX = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$, find X.

c. Determine the value of x is the matrix $\begin{bmatrix} x & 4 \\ 3 & x+4 \end{bmatrix}$ does not have an inverse.

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Question 9

a. Compute $\det(D)$ for:

$$D = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & 5 \\ -2 & 1 & 6 \end{bmatrix}$$

b. Is D invertible? Justify your answer.

c. Find the determinant of D^{-1} given that D is invertible.

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Sub-Section [3.5.4]: Apply Matrix Operations to Solve Systems of Linear Equations

Question 10

a. Write the system of equations:

$$\begin{cases} 2x + 3y = 5 \\ 4x - y = 7 \end{cases}$$

in the form $AX = C$.

b. Compute A^{-1} for:

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$$

c. If $AX = C$ and $A^{-1} = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$, find X given:

$$C = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

Question 11

a. Solve for X in:

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

using $X = A^{-1}C$.

b. Determine whether the system:

$$\begin{cases} 4x + y = 2 \\ 8x + 2y = 5 \end{cases}$$

Has a unique solution, no solution, or infinitely many solutions.

c. Determine the value(s) of k for which the system:

$$\begin{cases} 4x + 2ky = 5 \\ 2kx + 4y = 5 \end{cases}$$

Has a unique solution, no solution, or infinitely many solutions.

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Question 12

a. Write the following system in matrix form and solve using the inverse matrix method.

$$\begin{cases} 2x + 3y = 5 \\ 4x - y = 7 \end{cases}$$

b. Write the following system in matrix form and solve using the inverse matrix method.

$$\begin{cases} x - 2y = 4 \\ 3x + 6y = 2 \end{cases}$$

c. Write the following system in matrix form and solve using the inverse matrix method.

$$\begin{cases} 5x + 2y = 3 \\ x - 4y = 6 \end{cases}$$

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Sub-Section: Final Boss

Question 13

A company tracks the sales of two different products, P_1 and P_2, across two stores, Store A and Store B. The data collected over a week is represented using matrices.

a. The weekly sales data (in units sold) for both products at each store is given as:

$$S = \begin{bmatrix} 40 & 25 \\ 30 & 50 \end{bmatrix}$$

Where the rows correspond to Store A and Store B, and the columns correspond to Product P_1 and Product P_2.

i. Identify what type of matrix S is.

ii. Compute the trace of S.

iii. Compute the transpose of S and interpret it in this context.

b. Each unit of Product P_1 generates \$5 in revenue, while each unit of Product P_2 generates \$8.

i. Represent the price per unit as a column matrix.

ii. Compute the total revenue per store using matrix multiplication.

iii. Compute the overall total revenue.

c. If the company wants both stores to generate \$500 in weekly revenue, and the sales matrix remains the same, determine how the products P_1 and P_2 should be priced.

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Section B: Supplementary Questions

Sub-Section [3.5.1]: Basics of Matrices and Identifying Types of Matrices. Calculate the Transpose and Trace of a Matrix

Question 14

a. What does it mean for a matrix to be a **column matrix**? Give an example.

b. Given the matrix:

$$A = \begin{bmatrix} 6 & -2 \\ 1 & 5 \end{bmatrix}$$

Find $a_{2,1} + a_{2,2}$.

c. Compute the **trace** of matrix A.

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Question 15

a. Consider the matrix:

$$B = \begin{bmatrix} -3 \\ 5 \\ 7 \end{bmatrix}$$

What type of matrix is B?

b. Compute the **transpose** of the matrix:

$$C = \begin{bmatrix} -2 & 4 \\ 1 & -3 \\ 5 & 0 \end{bmatrix}$$

c. Find the **trace** of:

$$D = \begin{bmatrix} 9 & -4 & 3 \\ 2 & 7 & -6 \\ 1 & 8 & -2 \end{bmatrix}$$

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Question 16

a. If a matrix E is such that $E^T = -E$, what can we conclude about the matrix's structure? Give an example of a 3×3 matrix satisfying this condition.

b. Evaluate the **transpose** of:

$$F = \begin{bmatrix} 1 & -4 & 2 & 3 \\ -3 & 5 & 0 & -1 \\ 7 & 2 & -6 & 0 \end{bmatrix}$$

c. Explain why the trace of matrix F cannot be found.

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Sub-Section [3.5.2]: Perform Matrix Addition, Scalar Multiplication, and Matrix Multiplication

Question 17

a. Given the matrices:

$$A = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 5 \\ 6 & 0 \end{bmatrix}$$

Evaluate $A + B$.

b. Given the matrix:

$$C = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$$

Find $2C$.

c. Given the matrices:

$$D = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}, \quad E = \begin{bmatrix} 4 & 1 \\ 7 & -2 \end{bmatrix}$$

Evaluate $D \times E$.

Question 18

a. Matrices F and G are given as follows:

$$F = \begin{bmatrix} 5 & -2 & 1 \\ 3 & 0 & -4 \end{bmatrix}, \quad G = \begin{bmatrix} -1 & 3 & 2 \\ 4 & -2 & 6 \end{bmatrix}$$

Evaluate $3F - 2G$.

b. Consider the matrices:

$$H = \begin{bmatrix} 1 & 4 & -3 \\ 2 & -5 & 7 \end{bmatrix}, \quad Q = \begin{bmatrix} 3 & -1 \\ 6 & 2 \end{bmatrix}$$

Find $H + Q$, if possible.

c. Given:

$$J = \begin{bmatrix} -1 & 2 \\ 4 & 0 \\ 3 & -5 \end{bmatrix}, \quad k = \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$$

Compute $J \times K$.

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Question 19

a. Consider matrices L and M given by:

$$L = \begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$$

Evaluate $L(L + M)$.

b. Consider the matrices:

$$N = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & -1 \\ 2 & 5 \end{bmatrix}$$

Determine whether $N^2 - P^2 = (N + P)(N - P)$.

c. Given the matrices:

$$Q = \begin{bmatrix} 3 & -2 & 5 \\ 4 & 1 & -3 \end{bmatrix}, \quad R = \begin{bmatrix} 2 & 3 \\ x & 4 \\ 7 & y \end{bmatrix}$$

Find the value of x and y if $QR = \begin{bmatrix} 21 & 6 \\ -3 & 13 \end{bmatrix}$.

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Sub-Section [3.5.3]: Calculate the Inverse of a Matrix and Determine its Determinant

Question 20

a. Compute $\det(A)$ for:

$$A = \begin{bmatrix} 4 & -2 \\ 3 & 5 \end{bmatrix}$$

b. Is A **invertible**? Justify your answer.

c. If A is invertible, find A^{-1}.

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Question 21

a. Compute $\det(B)$ for:

$$B = \begin{bmatrix} 6 & -4 \\ 1 & 2 \end{bmatrix}$$

b. If $BX = \begin{bmatrix} 5 & 3 \\ 7 & 1 \end{bmatrix}$, find X.

c. Determine the possible value of x if the matrix, $\begin{bmatrix} x & 5 \\ -2 & x+3 \end{bmatrix}$ is invertible.

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Question 22

a. Compute $\det(D)$ for:

$$D = \begin{bmatrix} 3 & -2 & 5 \\ 1 & 4 & -1 \\ 2 & 0 & 3 \end{bmatrix}$$

b. Use the fact that $\det(AB) = \det(A) \cdot \det(B)$, for two invertible matrices, A and B, to prove that $\det(A^{-1}) = \frac{1}{\det(A)}$.

c. Find the determinant of D^{-1} given that, D is invertible.

Sub-Section [3.5.4]: Apply Matrix Operations to Solve Systems of Linear Equations

Question 23

a. Write the system of equations:

$$\begin{cases} 3x - 2y = 8 \\ 5x + 4y = -6 \end{cases}$$

in the form $AX = C$.

b. Compute A^{-1} for:

$$A = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$$

c. If $AX = C$ and $A^{-1} = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, find X given that:

$$C = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$$

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Question 24

a. Solve for X in:

$$\begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Using $X = A^{-1}C$.

b. Determine whether the system:

$$\begin{cases} 6x + 2y = 4 \\ 12x + 4y = 10 \end{cases}$$

Has a unique solution, no solution, or infinitely many solutions.

c. Determine the value(s) of k for which the system:

$$\begin{cases} 3x + ky = 7 \\ kx + 6y = 5 \end{cases}$$

Has a unique solution, no solution, or infinitely many solutions.

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Question 25

a. Write the following system in matrix form and solve using the inverse matrix method.

$$\begin{cases} 3x + 4y = 7 \\ 5x - y = 9 \end{cases}$$

b. Write the following system in matrix form and solve using the inverse matrix method.

$$\begin{cases} x + 2y = 5 \\ 4x - 3y = 1 \end{cases}$$

c. Write the following system in matrix form and solve using the inverse matrix method.

$$\begin{cases} 6x + 3y = 4 \\ 2x - 5y = 7 \end{cases}$$

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