

Website: contoureducation.com.au | Phone: 1800 888 300

Email: hello@contoureducation.com.au

VCE Specialist Mathematics ½ Matrices [4.1]

Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 02 - Pg 19
Supplementary Questions	Pg 20 - Pg 36



Section A: Compulsory Questions



<u>Sub-Section [3.5.1]</u>: Basics of Matrices and Identifying Types of Matrices. Calculate the Transpose and Trace of a Matrix

Qu	nestion 1
a.	What does it mean for a matrix to be a square matrix ? Give an example.
b.	Given the matrix:
	$A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$
	Find $a_{1,1} + a_{1,2}$.
c.	Compute the trace of matrix <i>A</i> .
	Compute the trace of matrix 71.





a. Consider the matrix:

$$B = [7 - 2 5]$$

What type of matrix is B?

b. Compute the **transpose** of the matrix:

$$C = \begin{bmatrix} 1 & 3 \\ -2 & 0 \\ 4 & -5 \end{bmatrix}$$

c. Find the **trace** of:

$$D = \begin{bmatrix} 6 & -1 & 2 \\ 4 & 3 & -5 \\ 0 & 7 & 8 \end{bmatrix}$$

Question 3



a. If a matrix E is such that $E^T = E$, what can we conclude about the matrix's dimensions? Give an example of a 3×3 matrix satisfying this condition.

b. Evaluate the **transpose** of:

$$F = \begin{bmatrix} 2 & -3 & 1 & 0 \\ 4 & 5 & -2 & 6 \\ -1 & 0 & 3 & -4 \end{bmatrix}$$

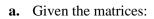
 \mathbf{c} . Explain why the trace of matrix F cannot be found.





<u>Sub-Section [3.5.2]</u>: Perform Matrix Addition, Scalar Multiplication, and Matrix Multiplication

Question 4



$$A = \begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}$$

Evaluate A + B.

b. Given the matrix:

$$C = \begin{bmatrix} -1 & 2 \\ 0 & 4 \end{bmatrix}$$

Find 3*C*.



			_
C.	Given	the	matrices:

$$D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad E = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Evaluate $D \times E$.

Question 5



a. Matrices F and G are given as follows:

$$F = \begin{bmatrix} 4 & -1 & 2 \\ 0 & 3 & -5 \end{bmatrix}, \qquad G = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 0 & 5 \end{bmatrix}$$

Evaluate 2F - 4G.

b. Consider the matrices:

$$H = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -5 \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & -3 \\ 2 & 1 \end{bmatrix}$$

Find H + Q, if possible.

c. Given:

$$J = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ 3 & 4 \end{bmatrix}, \quad k = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$$

Compute $J \times K$.





a. Consider matrices *L* and *M* given by:

$$L = \begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} -1 & 2 \\ 5 & 0 \end{bmatrix}$$

Evaluate L(L + M).

b. Consider the matrices:

$$N = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}, \quad P = \begin{bmatrix} 0 & -1 \\ 2 & 5 \end{bmatrix}$$

Determine whether $N^2 - P^2 = (N + P)(N - P)$.



•	$C_{\alpha n}$	cidor	tha	matrices:
C.	COL	isiuci	uic	maurees.

$$Q = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & -2 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 2 \\ x & 4 \\ 5 & y \end{bmatrix}$$

Find the value of x and y if $QR = \begin{bmatrix} 20 & 6 \\ -6 & 4 \end{bmatrix}$.





<u>Sub-Section [3.5.3]</u>: Calculate the Inverse of a Matrix and Determine Its Determinant

Ouestion	7
Question	,



a. Consider det (A) for:

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

b. Is *A* **invertible**? Justify your answer.

c. If *A* is invertible, find A^{-1} .

Question 8



a. Compute det(*B*) for:

$$B = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix}$$

b. If $BX = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$, find X.

c.	Determine the value of x is the matrix	x	4	does not have an inverse
••	2 00011111110 0110 1110 0110 1110 1110	3	x + 41	





a. Compute det(*D*) for:

$$D = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & 5 \\ -2 & 1 & 6 \end{bmatrix}$$

b. Is *D* invertible? Justify your answer.

c. Find the determinant of D^{-1} given that D is invertible.





<u>Sub-Section [3.5.4]</u>: Apply Matrix Operations to Solve Systems of Linear Equations

Question 10



a. Write the system of equations:

$$\begin{cases} 2x + 3y = 5 \\ 4x - y = 7 \end{cases}$$

in the form AX = C.

b. Compute A^{-1} for:

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$$

c. If AX = C and $A^{-1} = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$, find X given:

$$C = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$





a. Solve for *X* in:

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} X = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

using $X = A^{-1}C$.

b. Determine whether the system:

$$\begin{cases} 4x + y = 2\\ 8x + 2y = 5 \end{cases}$$

Has a unique solution, no solution, or infinitely many solutions.



c.	Determine the value(s) of k for which the system:				
	$ \begin{cases} 4x + 2ky = 5 \\ 2kx + 4y = 5 \end{cases} $				
	Has a unique solution, no solution, or infinitely many solutions.				

Space for Personal Notes

Question 12



a. Write the following system in matrix form and solve using the inverse matrix method.

$$\begin{cases} 2x + 3y = 5 \\ 4x - y = 7 \end{cases}$$

b. Write the following system in matrix form and solve using the inverse matrix method.

$$\begin{cases} x - 2y = 4\\ 3x + 6y = 2 \end{cases}$$

·	



c.	Write the following system in matrix form and solve using the inverse matrix method.
	$\begin{cases} 5x + 2y = 3\\ x - 4y = 6 \end{cases}$
Sp	pace for Personal Notes



Sub-Section: Final Boss

Question 13

A company tracks the sales of two different products, P_1 and P_2 , across two stores, Store A and Store B. The data collected over a week is represented using matrices.

a. The weekly sales data (in units sold) for both products at each store is given as:

$$S = \begin{bmatrix} 40 & 25 \\ 30 & 50 \end{bmatrix}$$

Where the rows correspond to Store A and Store B, and the columns correspond to Product P_1 and Product P_2 .

i.	Identify what type of matrix S is.

ıı.	Compute the trace of S.

111.	. Compute the transpose of S and interpret it in this context.						



i.	Represent the price per unit as a column matrix.
ii.	Compute the total revenue per store using matrix multiplication.
iii.	Compute the overall total revenue.
	the company wants both stores to generate \$500 in weekly revenue, and the sales matrix remains the san termine how the products P_1 and P_2 should be priced.



Section B: Supplementary Questions



<u>Sub-Section [3.5.1]</u>: Basics of Matrices and Identifying Types of Matrices. Calculate the Transpose and Trace of a Matrix

Qu	nestion 14
a.	What does it mean for a matrix to be a column matrix ? Give an example.
b.	Given the matrix:
	$A = \begin{bmatrix} 6 & -2 \\ 1 & 5 \end{bmatrix}$
	$^{\prime\prime}$ = $[1 5]$
	Find $a_{2,1} + a_{2,2}$.
c.	Compute the trace of matrix <i>A</i> .





a. Consider the matrix:

$$B = \begin{bmatrix} -3\\5\\7 \end{bmatrix}$$

What type of matrix is B?

b. Compute the **transpose** of the matrix:

$$C = \begin{bmatrix} -2 & 4\\ 1 & -3\\ 5 & 0 \end{bmatrix}$$

c. Find the **trace** of:

$$D = \begin{bmatrix} 9 & -4 & 3 \\ 2 & 7 & -6 \\ 1 & 8 & -2 \end{bmatrix}$$

Quest	ion	16
Quest	IUII	ıυ



a. If a matrix E is such that $E^T = -E$, what can we conclude about the matrix's structure? Give an example of a 3×3 matrix satisfying this condition.

b. Evaluate the **transpose** of:

 $F = \begin{bmatrix} 1 & -4 & 2 & 3 \\ -3 & 5 & 0 & -1 \\ 7 & 2 & -6 & 0 \end{bmatrix}$

c. Explain why the trace of matrix *F* cannot be found.





<u>Sub-Section [3.5.2]</u>: Perform Matrix Addition, Scalar Multiplication, and Matrix Multiplication

Question 17



a. Given the matrices:

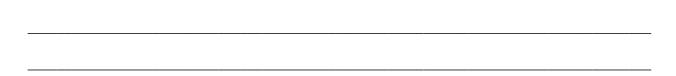
$$A = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 5 \\ 6 & 0 \end{bmatrix}$$

Evaluate A + B.

b. Given the matrix:

$$C = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$$

Find 2*C*.



c. Given the matrices:

$$D = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}, \quad E = \begin{bmatrix} 4 & 1 \\ 7 & -2 \end{bmatrix}$$

Evaluate $D \times E$.

Question 18



a. Matrices F and G are given as follows:

$$F = \begin{bmatrix} 5 & -2 & 1 \\ 3 & 0 & -4 \end{bmatrix}, \qquad G = \begin{bmatrix} -1 & 3 & 2 \\ 4 & -2 & 6 \end{bmatrix}$$

Evaluate 3F - 2G.

b. Consider the matrices:

$$H = \begin{bmatrix} 1 & 4 & -3 \\ 2 & -5 & 7 \end{bmatrix}, \quad Q = \begin{bmatrix} 3 & -1 \\ 6 & 2 \end{bmatrix}$$

Find H + Q, if possible.

c. Given:

$$J = \begin{bmatrix} -1 & 2\\ 4 & 0\\ 3 & -5 \end{bmatrix}, \quad k = \begin{bmatrix} 3 & 1\\ -2 & 4 \end{bmatrix}$$

Compute $J \times K$.





a. Consider matrices *L* and *M* given by:

$$L = \begin{bmatrix} 3 & 2 \\ -4 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$$

Evaluate L(L + M).

b. Consider the matrices:

$$N = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & -1 \\ 2 & 5 \end{bmatrix}$$

Determine whether $N^2 - P^2 = (N + P)(N - P)$.



c. Given the matrices:

$$Q = \begin{bmatrix} 3 & -2 & 5 \\ 4 & 1 & -3 \end{bmatrix}, \quad R = \begin{bmatrix} 2 & 3 \\ x & 4 \\ 7 & y \end{bmatrix}$$

Find the value of x and y if $QR = \begin{bmatrix} 21 & 6 \\ -3 & 13 \end{bmatrix}$.





<u>Sub-Section [3.5.3]</u>: Calculate the Inverse of a Matrix and Determine its Determinant

Question	20
Question	∠ ∪



a. Compute det(A) for:

$$A = \begin{bmatrix} 4 & -2 \\ 3 & 5 \end{bmatrix}$$

b. Is *A* **invertible**? Justify your answer.

c. If *A* is invertible, find A^{-1} .

Question 21



a. Compute det(*B*) for:

$$B = \begin{bmatrix} 6 & -4 \\ 1 & 2 \end{bmatrix}$$

b. If $BX = \begin{bmatrix} 5 & 3 \\ 7 & 1 \end{bmatrix}$, find X.

c. Determine the possible value of x if the matrix, $\begin{bmatrix} x & 5 \\ -2 & x+3 \end{bmatrix}$ is invertible.

Question 22



a. Compute det(*D*) for:

$$D = \begin{bmatrix} 3 & -2 & 5 \\ 1 & 4 & -1 \\ 2 & 0 & 3 \end{bmatrix}$$

b.	Use the fact that $det(AB) = det(A) \cdot det(B)$, for two invertible matrices, A and B, to prove that
	$\det(A^{-1}) = \frac{1}{\det(A)}.$

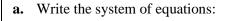
c. Find the determinant of D^{-1} given that, D is invertible.





<u>Sub-Section [3.5.4]</u>: Apply Matrix Operations to Solve Systems of Linear Equations

Question 23



$$\begin{cases} 3x - 2y = 8 \\ 5x + 4y = -6 \end{cases}$$

in the form AX = C.

h.	Compute	A^{-1}	for

$$A = \begin{bmatrix} 2 & 1 \\ -3 & 5 \end{bmatrix}$$

c. If AX = C and $A^{-1} = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, find X given that:

$$C = \begin{bmatrix} -5\\7 \end{bmatrix}$$





a. Solve for *X* in:

$$\begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Using $X = A^{-1}C$.

b. Determine whether the system:

$$\begin{cases} 6x + 2y = 4 \\ 12x + 4y = 10 \end{cases}$$

Has a unique solution, no solution, or infinitely many solutions.



c.	c. Determine the value(s) of k for which the system:						
	$ \begin{cases} 3x + ky = 7 \\ kx + 6y = 5 \end{cases} $						
	Has a unique solution, no solution, or infinitely many solutions.						

Space for Personal Notes		

Question 25



a. Write the following system in matrix form and solve using the inverse matrix method.

$$\begin{cases} 3x + 4y = 7 \\ 5x - y = 9 \end{cases}$$

b. Write the following system in matrix form and solve using the inverse matrix method.

$$\begin{cases} x + 2y = 5 \\ 4x - 3y = 1 \end{cases}$$



с.	Write the following system in matrix form and solve using the inverse matrix method.
	$\begin{cases} 6x + 3y = 4 \\ 2x - 5y = 7 \end{cases}$
Sp	pace for Personal Notes



Website: contoureducation.com.au | Phone: 1800 888 300 | Email: hello@contoureducation.com.au

VCE Specialist Mathematics ½

Free 1-on-1 Consults

What Are 1-on-1 Consults?

- **Who Runs Them?** Experienced Contour tutors (45 + raw scores and 99 + ATARs).
- Who Can Join? Fully enrolled Contour students.
- When Are They? 30-minute 1-on-1 help sessions, after-school weekdays, and all-day weekends.
- What To Do? Join on time, ask questions, re-learn concepts, or extend yourself!
- Price? Completely free!
- One Active Booking Per Subject: Must attend your current consultation before scheduling the next:)

SAVE THE LINK, AND MAKE THE MOST OF THIS (FREE) SERVICE!

6

Booking Link

bit.ly/contour-specialist-consult-2025

