Website: contoureducation.com.au | **Phone:** 1800 888 300
Email: hello@contoureducation.com.au

VCE Specialist Mathematics ½
Advanced Trigonometric Functions Exam Skills [3.5]
Workbook

Outline:

Learning Objectives:

- ☐ **SM12 [3.5.1]** -Simplify the Composition of Inverse Trigonometric Functions
- ☐ **SM12 [3.5.2]** -Simplify $a\cos(x) + b\sin(x)$
- ☐ **SM12 [3.5.3]** -Apply Product to Sum and Sum to Product Identities to Simplify Trigonometric Expressions

Section A: Recap

If you were here last week, skip to section B - warmup test.

Reciprocal Trigonometric Functions

- The reciprocal of **sine** is **cosecant:**

$$\mathbf{cosec}(x) = \frac{1}{\sin(x)}$$

- The reciprocal of **cosine** is **secant:**

$$\sec(x) = \frac{1}{\cos(x)}$$

- The reciprocal of **tangent** is **cotangent:**

$$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$$

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Question 1

Evaluate the following.

a. $\sec\left(-\frac{\pi}{3}\right)$

b. $\operatorname{cosec}\left(\frac{2\pi}{3}\right)$

c. $\cot\left(-\frac{5\pi}{6}\right)$

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Trigonometric Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \text{cosec}^2(\theta)$$

Question 2

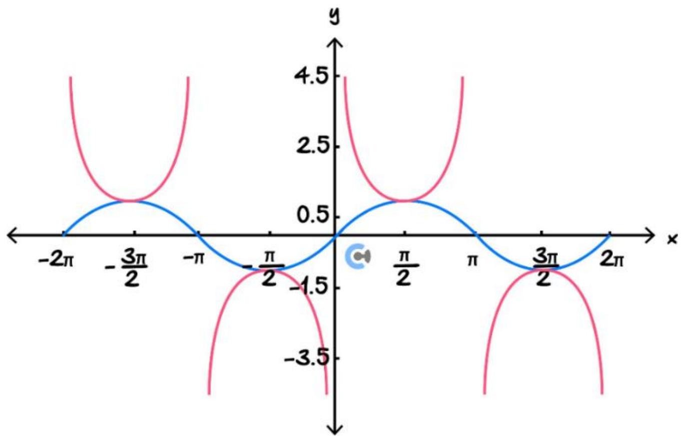
Given that $\sec(x) = -4$ and $x \in \left[\frac{\pi}{2}, \pi\right]$, find $\text{cosec}(x)$ and $\tan(x)$. Show your working.

Properties of Reciprocal Graphs

Feature on $y = f(x)$	Feature on $y = \frac{1}{f(x)}$
x-intercept	Vertical asymptote
Positive y-values	Positive y-values
Negative y-values	Negative y-values
Increasing	Decreasing
Decreasing	Increasing
The graphs intersect only when $f(x) = 1$ or $f(x) = -1$.	

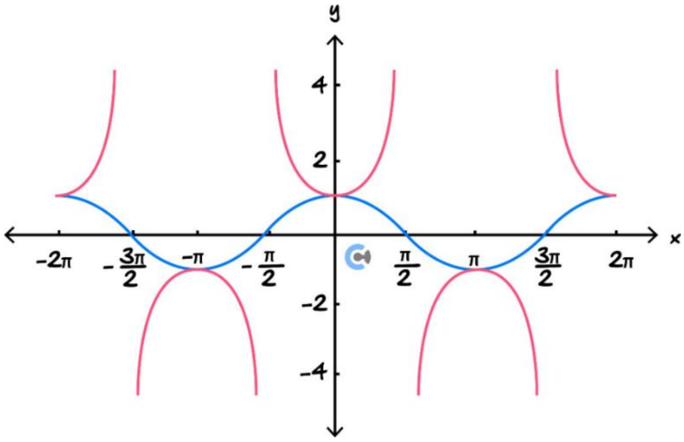
Graphing Reciprocal Trigonometric Functions

$y = \text{cosec}(x)$



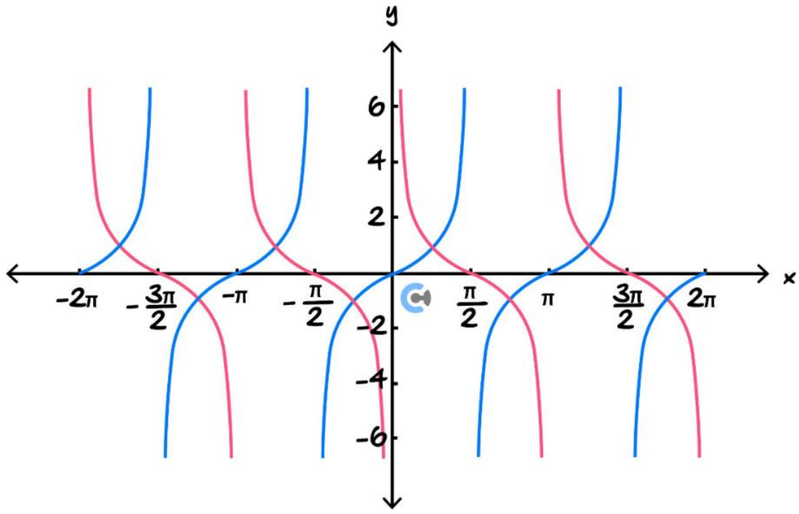
- Maximal Domain: $R\backslash\{x: \sin(x) = 0\}$.
- Range: $(-\infty, -1] \cup [1, \infty)$.

$y = \sec(x)$



- Maximal Domain: $R \backslash \{x: \cos(x) = 0\}$.
- Range: $(-\infty, -1] \cup [1, \infty)$.

$y = \cot(x)$



- Maximal Domain: $R \backslash \{x: \tan(x) = 0\}$.
- Range: R.

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Steps for Sketching Reciprocal Trig Graphs

- Find **an asymptote.**

$$equate\ \boldsymbol{Angle} = \mathbf{0}\ for\ \mathrm{cosec}\ and\ \mathrm{cot}\ graphs$$

$$equate\ \boldsymbol{Angle} = \frac{\boldsymbol{\pi}}{\mathbf{2}}\ for\ \mathrm{sec}\ graphs$$

- Find and mark **all other asymptotes** in the domain.

$$\boldsymbol{Add/Subtract}\ \frac{\boldsymbol{\pi}}{\boldsymbol{n}}\ from\ first\ asymptotes$$

- Plot a point in between the two asymptotes.

$$Midpoint = \boldsymbol{Turning\ Point}\ for\ \mathrm{cosec}\ and\ \mathrm{sec}\ graphs$$

$$Midpoint = \boldsymbol{Inflection\ Point}\ for\ \mathrm{cot}\ graphs$$

- Solve for axes intercept (if applicable).
- Repeat the shape over the entire domain.
  - For cosec and sec graphs, the "U" shapes **alternate** between asymptotes, while cot graphs look the same between asymptotes.

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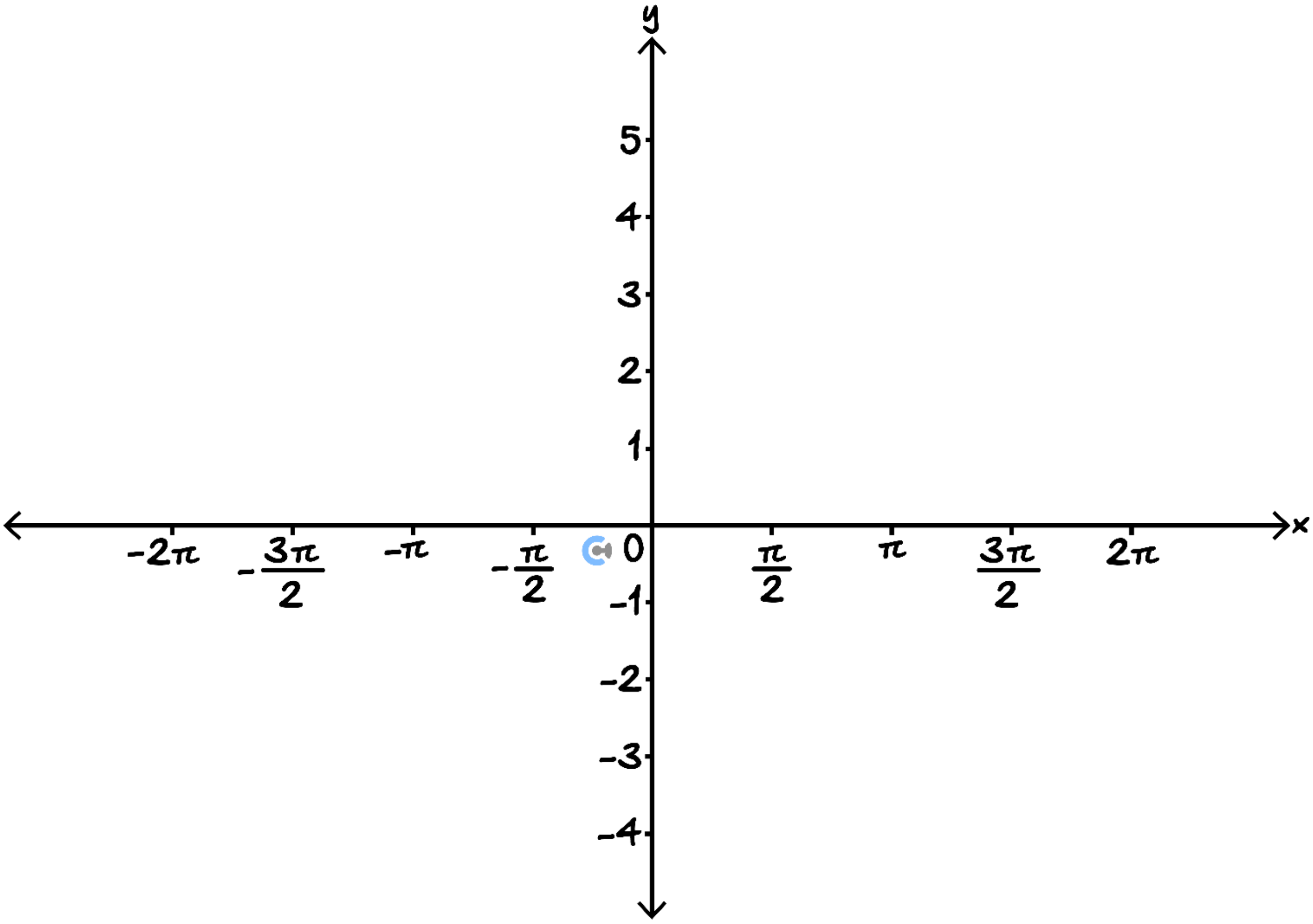
Question 3

Sketch the graph of $y = -\frac{1}{2}\operatorname{cosec}(x) + 1$ for $-2\pi \leq x \leq 2\pi$, labelling all stationary points, axes-intercepts and asymptotes.



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Definition

Compound Angle Formula

- sin compound angle formulae.

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$$

- cos compound angle formulae.

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$$

- tan compound angle formulae.

$$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$$

$$\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$$

Question 4

Using the compound angle formula, evaluate $\cos\left(\frac{5\pi}{12}\right)$.

Definition

Double Angle Formulae

- sin double angle formula.

$$\sin(2x) = 2\sin(x)\cos(x)$$

- cos double angle formula.

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$= 2\cos^2(x) - 1$$
$$= 1 - 2\sin^2(x)$$

- tan double angle formula.

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

Question 5

Find $\cos(2t)$, where $\sin(t) = -\frac{1}{8}$.

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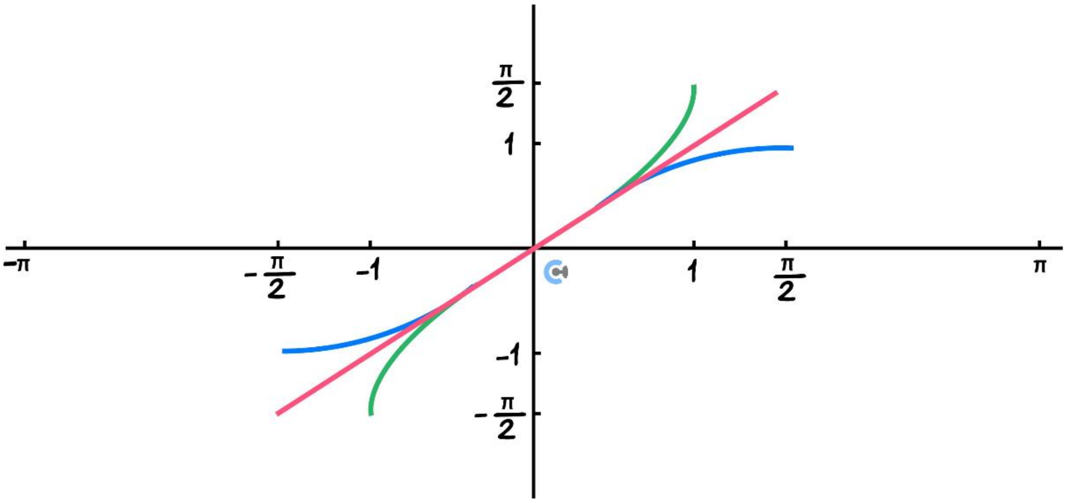
Definition

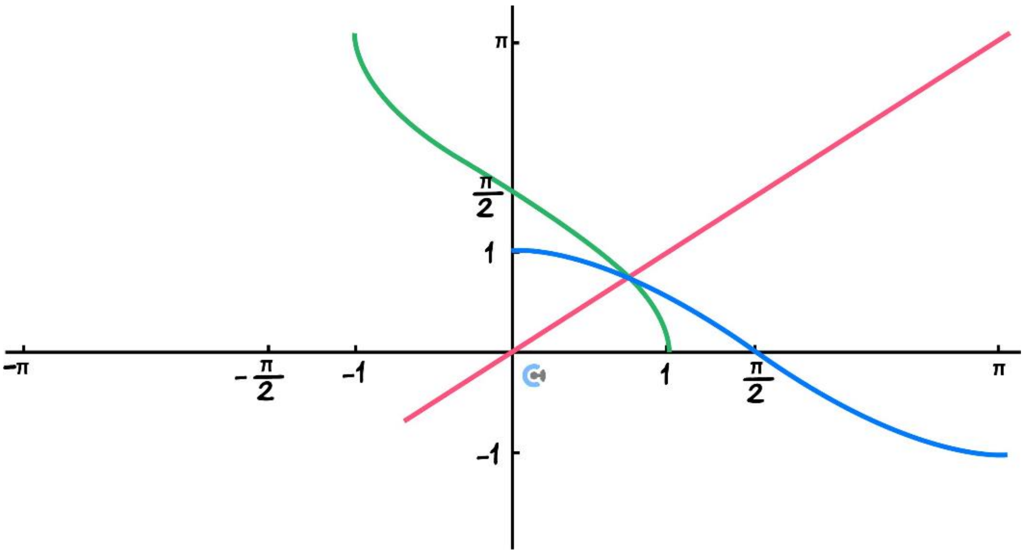
Inverse Trig Functions

- $\sin^{-1}(x)$



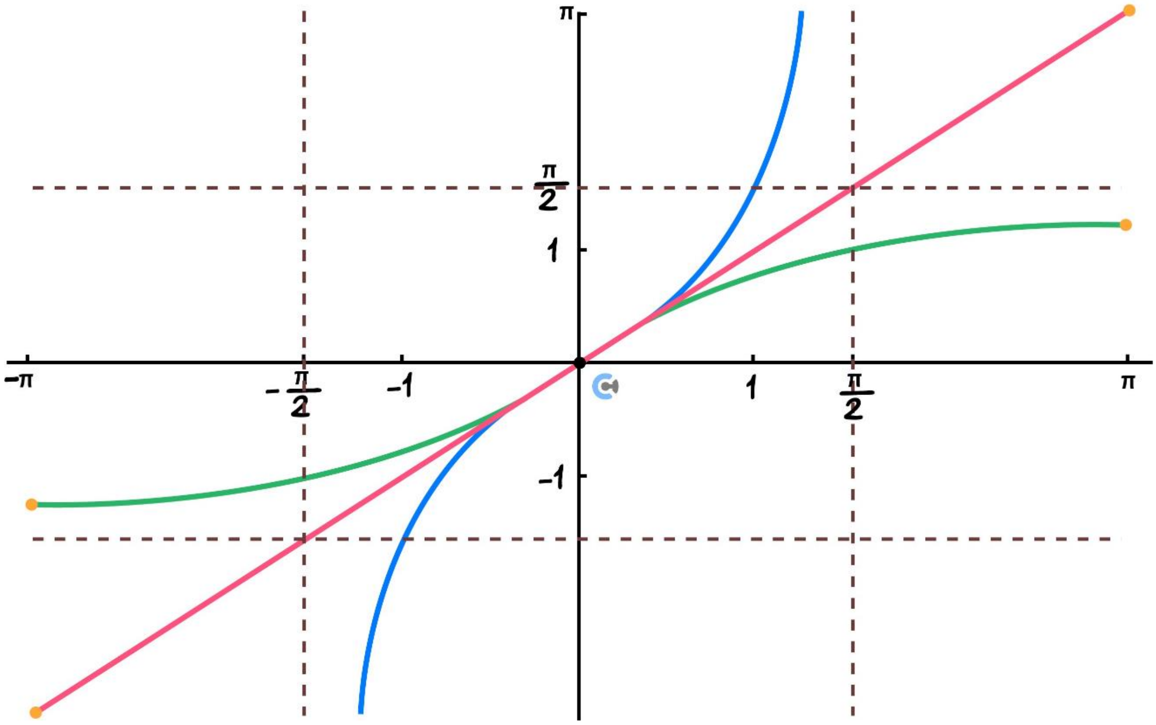
  - The domain of the $\arcsin$ function = Range of $\sin = [-1,1]$.
  - The range = Domain of restricted $\sin = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

- $\cos^{-1}(x)$



  - The domain of the $\arccos$ function = Range of $\cos = [-1,1]$.
  - The range = Domain of restricted $\cos = [0, \pi]$.

- $\tan^{-1}(x)$



- The domain of the $\arctan$ function $=$ Range of $\tan = R$.
- The range $=$ Domain of restricted $\tan = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Steps for Graphing General Arcsin and Arccos

1. Find the implied domain of the function.
   - Restrict **inside** to be within $[-1, 1]$.
2. Find and plot the endpoints of the graph by substituting the ends of the domain.
3. Find and plot the midpoint of the ends. (It is an inflection point.)
4. Find and plot the axes intercepts if required.
5. Using the previously plotted points as a guide, sketch a “cubic-like” shape.

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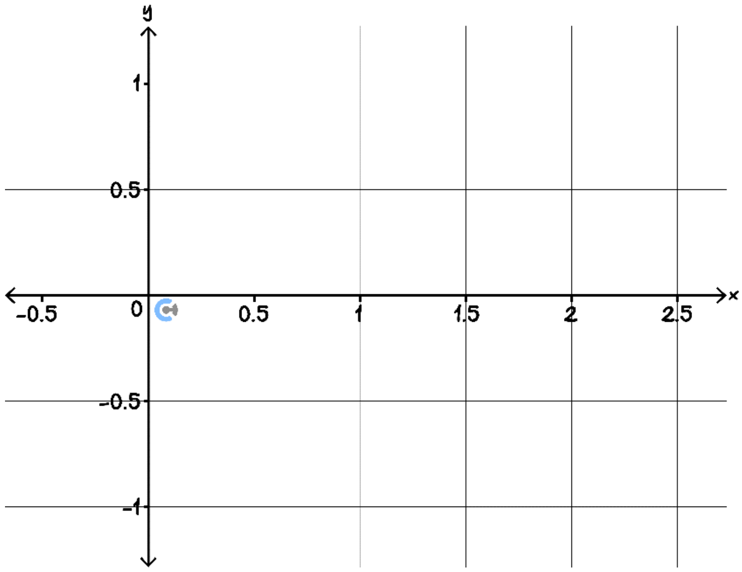
Question 6

Following the steps above, sketch:

$$y = \frac{1}{2}\arccos(1 - x) - \frac{\pi}{4}$$



Steps for Graphing General Graphs of arctan

1. Find the horizontal asymptotes of the graph and plot them.
   - You can find the asymptotes by finding the **range** of the arctan function.
   - E.g., the range of $\arctan(x) + \pi$ is $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, so the asymptotes are $y = \frac{\pi}{2}$ and $y = \frac{3\pi}{2}$.
2. Inflection point is given by (h, k).
   - The x-value can be found by making inside $= 0$.
   - The y-value can be found by averaging the asymptotic values (midpoint).
3. Find and plot the axes intercepts if required.
4. Using the previously plotted points and asymptotes as a guide, sketch the function.

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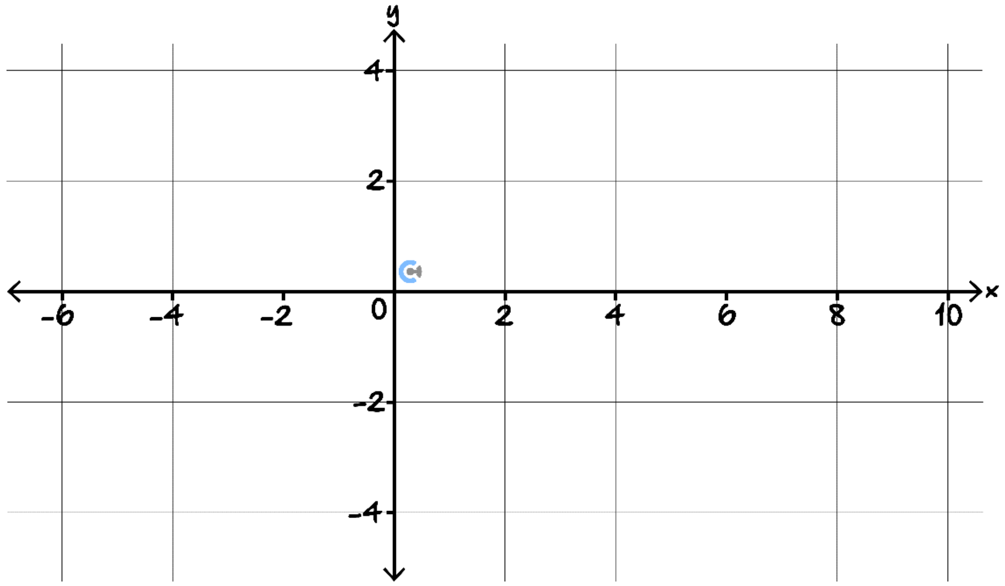
Question 7

Following the steps above, sketch:

$$y = \arctan\left(x - \sqrt{3}\right) - \frac{\pi}{3}$$



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Section B: Warmup Test (15 Marks)

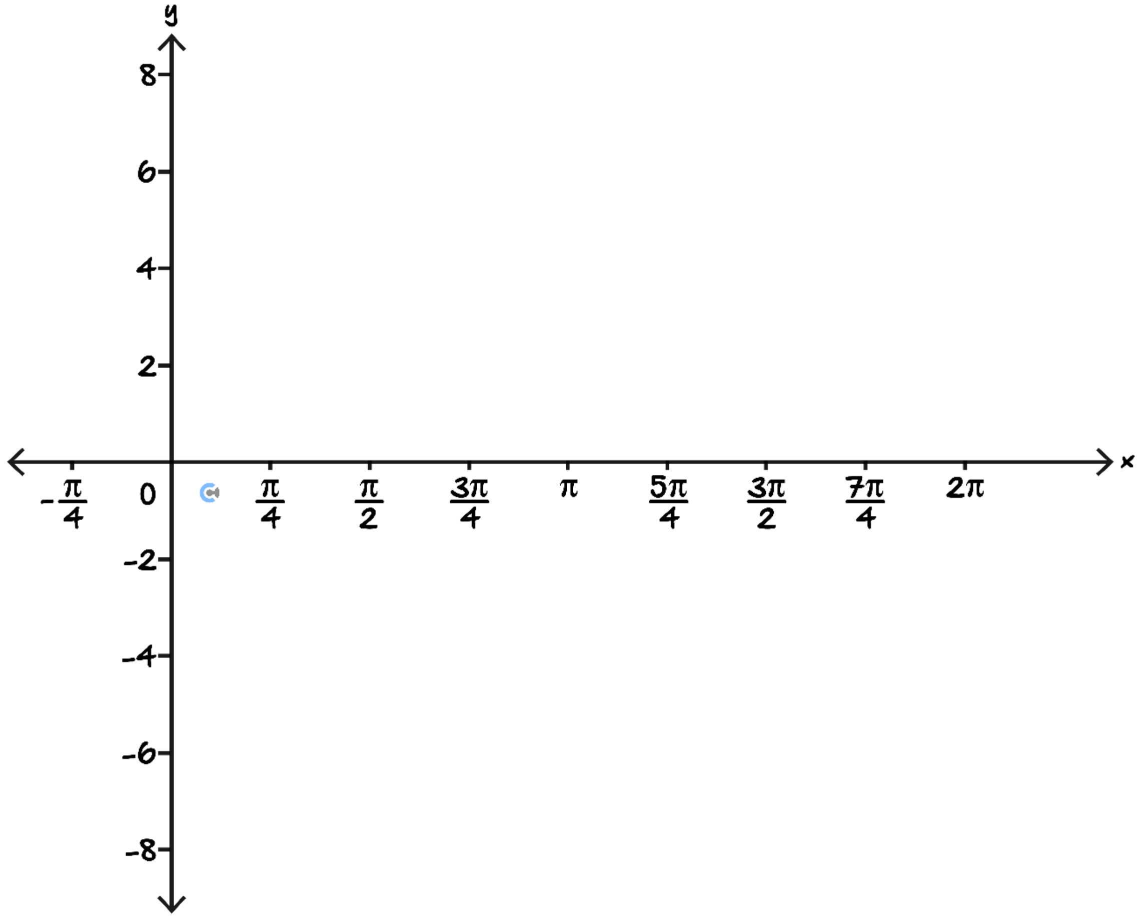
Question 8 (8 marks)

Consider the function $f(x) = \sec\left(2x - \frac{\pi}{4}\right) - 2$ for $x \in [0, 2\pi]$.

a. Find the equation(s) of any asymptotes of the function. (2 marks)

b. Find all solutions to the equation $f(x) = 0$ for $x \in [0, 2\pi]$. (3 marks)

c. Hence, sketch the graph of f on the axes below, labelling all axes intercepts, turning points and endpoints with their coordinates, and all asymptotes with their equations. (3 marks)



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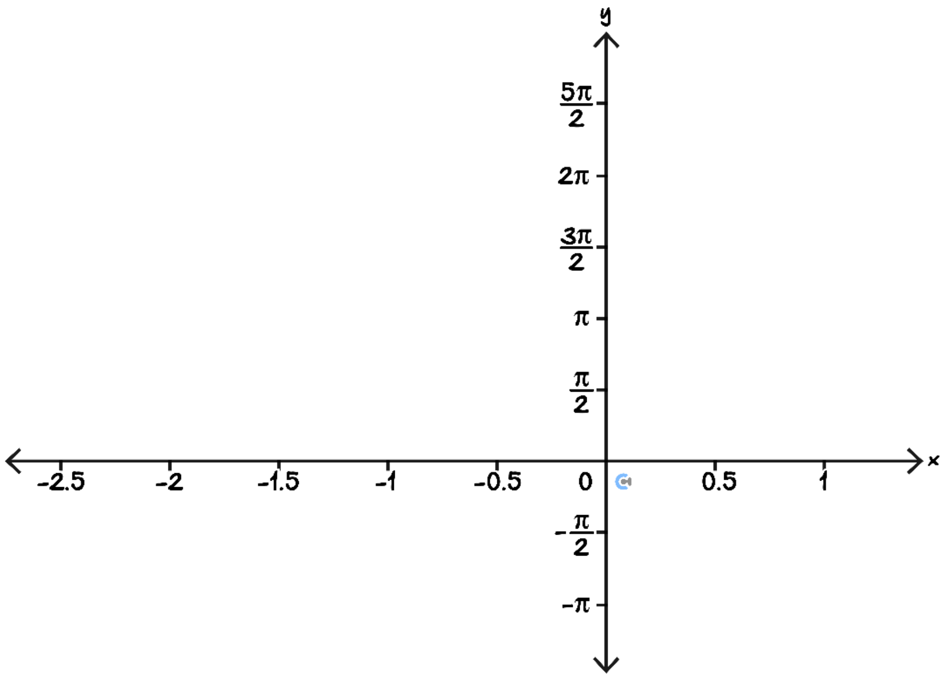
Question 9 (3 marks)

Prove the identity

$$\frac{\sin(x)}{1+\cos(x)} = \tan\left(\frac{x}{2}\right)$$

Question 10 (4 marks)

Sketch the graph of $y = 2\arccos\left(x+\frac{1}{2}\right) - \frac{\pi}{2}$ over its maximal domain, labelling endpoints and axes intercepts with their coordinates.



Section C: Circular Functions Exam Skills

Sub-Section: Simplifying Composite Inverse Trigonometric Functions

Active Recall

- The range of $\sin^{-1}(x)$ is ____________.
- The range of $\cos^{-1}(x)$ is ___________.

Question 11 Walkthrough.

Simplify the function $f(x) = \cos(\arcsin(x))$.

NOTE: Inverse trig functions are essentially an angle. Let them equal theta!

ALSO NOTE: Reject by using the range restriction of inverse trigonometric functions.

Your turn!

Question 12

Simplify the function $f(x) = \sin(\arccos(x - 2))$.

NOTE: To check this on technology, simply use *texpand/trigexpand*.

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Question 13 Tech Active.

Simplify the function $f(x) = \cos(\arcsin(x - 4))$.

$$\cos\left(\sin^{-1}(x-4)\right) \qquad \sqrt{-x^2+8\cdot x-15}$$

In[142]:= Cos[ArcSin[x - 4]]

Out[142]= $\sqrt{1 - (4 - x)^2}$

$$\cos(\sin^{-1}(x-4))$$

$$\sqrt{-x^2+8\cdot x-15}$$

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Sub-Section: Simplifying $a\cos(x) + b\sin(x)$

Exploration: Simplifying $a\cos(x) + b\sin(x)$

- Take a look at this expression!

$$\sqrt{3}\sin(x) + 1\cos(x)$$

- How could we simplify this into one trigonometric function?
- Let's first find the Pythagoras of the coefficients of $\sin$ and $\cos$.

We will call this our ______________

$$r = \sqrt{\left(\sqrt{3}\right)^2 + (1)^2} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

- Now factorise the radius out from the previous equation.

$$\sqrt{3}\sin(x) + 1\cos(x) = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

- What do you notice about the coefficient of $\sin$ and $\cos$ now?

They are ______________________ of $\sin$ and $\cos$!

- Let's replace $\frac{\sqrt{3}}{2}$ as $\cos\left(\frac{\pi}{3}\right)$ and $\frac{1}{2}$ as $\sin\left(\frac{\pi}{3}\right)$

NOTE: It is important that we use the same angle. We will see why!

$$\sqrt{3}\sin(x) + 1\cos(x) = 2(\text{\_\_\_\_\_\_}\sin(x) + \text{\_\_\_\_\_\_}\cos(x))$$

- What formula can we use to simplify this now? **Compound angle formula!**

$$\sqrt{3}\sin(x) + 1\cos(x) = 2\ \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

- Can you see why using the same angle in the above step is important?

Discussion: Was this the only way to simplify?

Simplifying the sum of trig functions:

Definition

- Step 1: Find the radius by taking the two coefficients:

$$r = \sqrt{a^2 + b^2}$$

- Step 2: Factor the expression by the radius.
- Step 3: Replace the coefficients with $\cos(\alpha)$ and $\sin(\alpha)$. Remember to use the same angles.
- Step 4: Use the compound angle formula to express in terms of a single trigonometric function.

$$a\cos(x) + b\sin(x) = r\cos(x - \alpha)$$

$$a\cos(x) - b\sin(x) = r\cos(x + \alpha)$$

$$a\sin(x) + b\cos(x) = r\sin(x + \alpha)$$

$$a\sin(x) - b\cos(x) = r\sin(x - \alpha)$$

$$\text{where } r = \sqrt{a^2 + b^2} \text{ and } \alpha = \tan^{-1}\left(\frac{b}{a}\right)$$

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Question 14 Walkthrough.

Simplify the expression $\sin(x) + \cos(x)$.

Active Recall: Simplifying the sum of trig functions:

- Step 1: Find the radius by taking the two coefficients:

$$r = \sqrt{a^2 + b^2}$$

- Step 2: Factor the expression by the________________.
- Step 3: Replace the coefficients with $\cos(a)$ and $\sin(a)$. Remember to use the same angles.
- Step 4: Use ____________________ formula to express in terms of a single trigonometric function.

$$a\cos(x) + b\sin(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$a\cos(x) - b\sin(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$a\sin(x) + b\cos(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$a\sin(x) - b\cos(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\text{where } r = \sqrt{a^2 + b^2} \text{ and } \alpha = \tan^{-1}\left(\frac{b}{a}\right)$$

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Question 15

Simplify the expression $\sqrt{3}\sin(x) - \cos(x)$.

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Sub-Section: Sums and Products of $\cos(x)$ and $\sin(x)$

Definition

Product to Sum Identities

$$2\cos(x)\cos(y) = \cos(x-y) + \cos(x+y)$$

$$2\sin(x)\sin(y) = \cos(x-y) - \cos(x+y)$$

$$2\sin(x)\cos(y) = \sin(x+y) + \sin(x-y)$$

How does it work? Let's prove the first one!

Exploration: Proof of Product to Sum Identity

$$\cos(x-y) + \cos(x+y) = 2\cos(x)\cos(y)$$

- Consider that $\cos(a-b) = \cos(\text{a})\cos(\text{b}) + \sin(\text{a})\sin(\text{b})$
  and $\cos(a+b) = \cos(\text{a})\cos(\text{b}) - \sin(\text{a})\sin(\text{b})$: Compound angle formula

$$\cos(x-y) + \cos(x+y)$$

$= $ ____

$= $ ____________________________

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Question 16 Walkthrough.

Express $2\sin(3x)\cos(x)$ as a sum or difference:

Active Recall: Product to Sum Identities

$$2\cos(x)\cos(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$2\sin(x)\sin(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$2\sin(x)\cos(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Question 17

Express $4\cos(3x)\cos(x)$ as a sum or difference:

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Now backwards!

Sum to Product Identities

$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin(x) - \sin(y) = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$$

How does it work? Let's try to prove the first one using product to sum identities.

Exploration Proof of Sum to Product Identity

$$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \cos(x) + \cos(y)$$

- Consider that $2\cos(a)\cos(b) = \cos(a-b) + \cos(a+b)$: Product to Sum Identity

$$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = cos(\_\_\_\_\_ - \_\_\_\_\_) + cos(\_\_\_\_\_ + \_\_\_\_\_)$$

$$= cos(\_\_\_\_) + cos(\_\_\_\_)$$

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Question 18 Walkthrough.

Express $\sin(x+\alpha) - \sin(x)$ as a product:

Active Recall: Sum to product identities:

$$\cos(x) + \cos(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\cos(x) - \cos(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\sin(x) + \sin(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\sin(x) - \sin(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Question 19

Express $\cos(3x) - \cos(x)$ as a product:

Section D: Exam 1 Questions (21 Marks)

INSTRUCTION: 21 Marks. 25 Minutes Writing.

Question 20 (4 marks)

Let $f: [0, 2\pi] \to R, f(x) = \cos(4x) + \cos(2x)$.

a. Express f as the product of two trigonometric functions. (1 mark)

b. Hence, solve the equation $f(x) = 0$ for $x \in [0, 2\pi]$. (3 marks)

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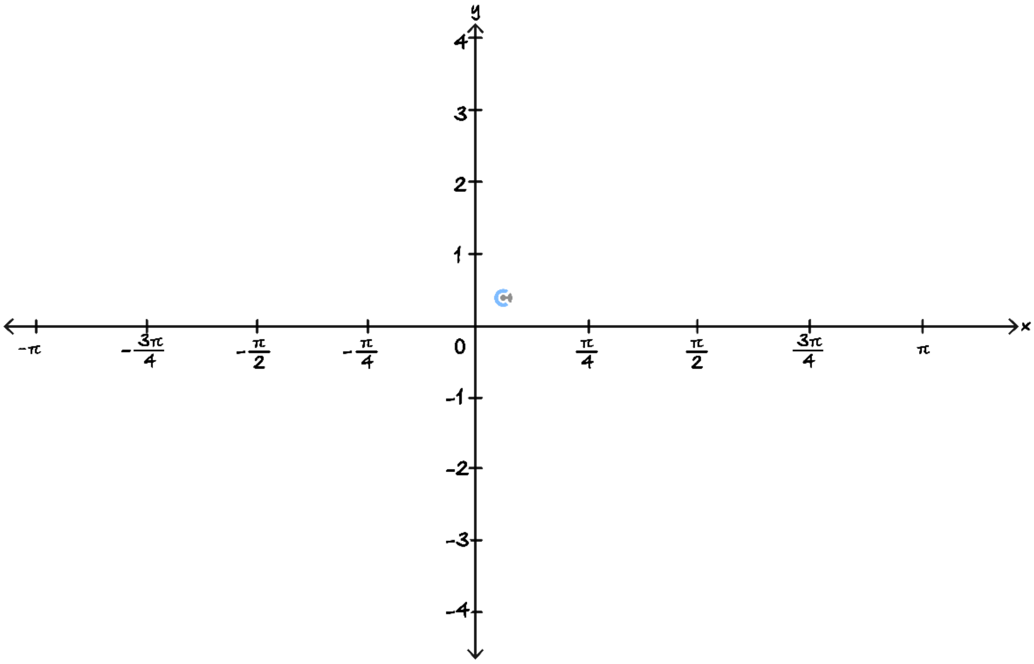
Question 21 (3 marks)

Sketch the graph of $f(x) = \text{cosec}(2x)$ for $x \in [-\pi, \pi]$ on the axes below. Label any asymptotes with their equations and label any turning points with their coordinates.



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Question 22 (3 marks)

If $\cos(x) = 3\sin(x)$, find the value of $\sin(2x)$ where $x \in \left(0, \frac{\pi}{2}\right)$.

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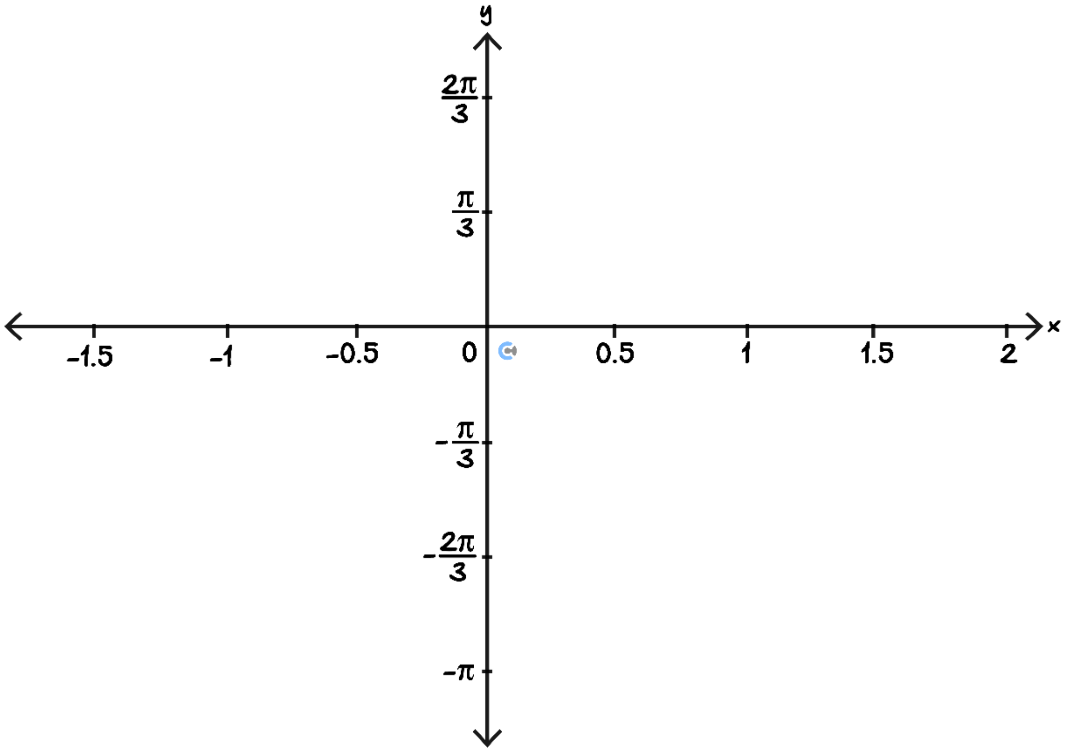
Question 23 (5 marks)

Consider the function $f: [-1, 1] \to R, f(x) = \arcsin(x) - \frac{\pi}{2}$.

a. Sketch the graph of $y = f(x)$ on the axes below, labelling the axes intercepts and endpoints with their coordinates. (3 marks)



b. Find the rule and domain of the function $g(x) = \cos(f(x))$. (2 marks)

Question 24 (2 marks)

Show that $\tan\left(\frac{5\pi}{12}\right) = 2 + \sqrt{3}$.

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Question 25 (4 marks)

Consider the function f with rule $f(x) = \arccos(x^2 - 1)$.

a. State the domain of f. (2 marks)

b. Find the range of f. (2 marks)

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Section E: Tech Active Exam Skills

Calculator Commands: Finding asymptote of reciprocal trigonometric functions.

TI	Casio	Mathematica
solve(denominator $trig(..) = 0, x)$ | domain restriction	solve(denominator $trig(..) = 0, x)$ | domain restriction	Solve[denominator $trig[] == 0$ && domain restriction, x]
| is under control equal.	| is under maths 3.	

Question 26 Tech-Active.

Find the asymptotes of the function $f(x) = 2\sec\left(2x + \frac{\pi}{4}\right) - 3$ for $0 \le x \le 2\pi$.

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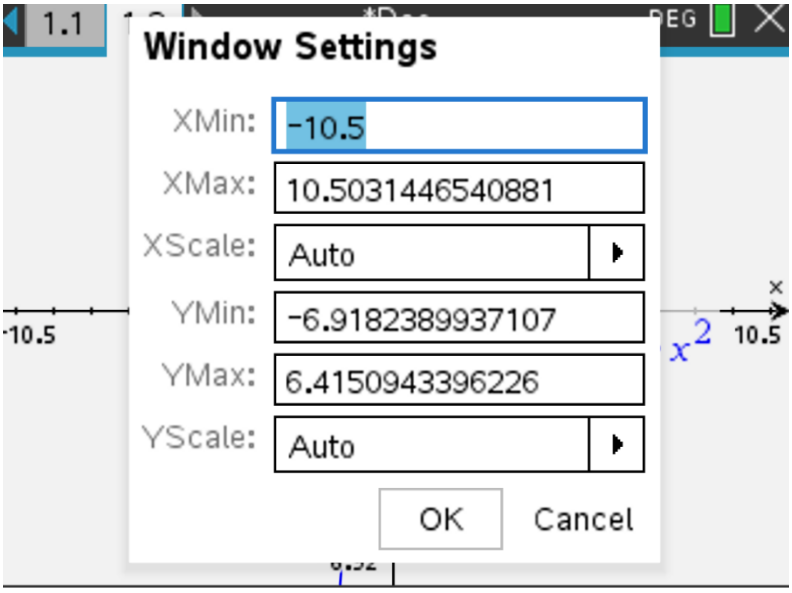
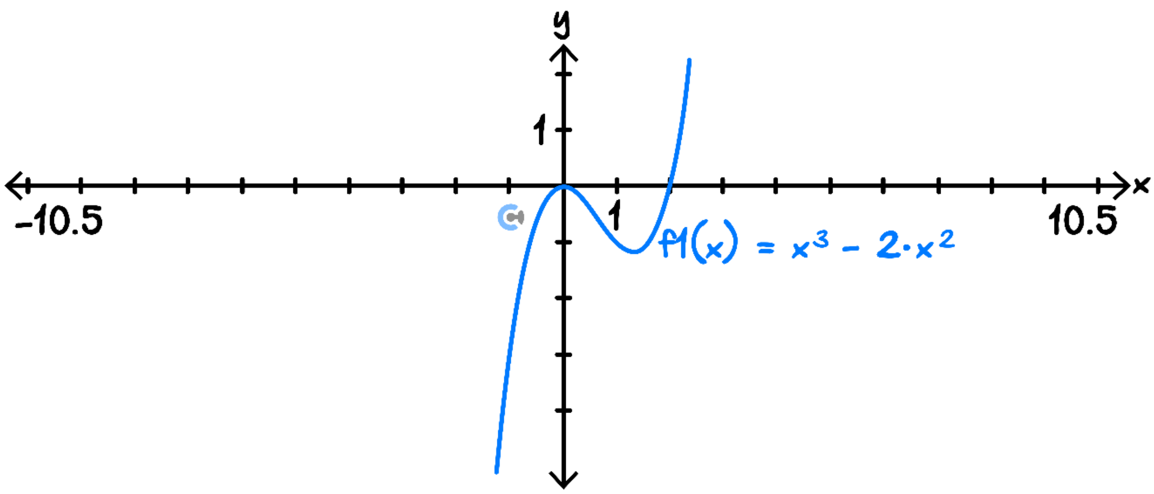
<u>Calculator Commands:</u> Graphing

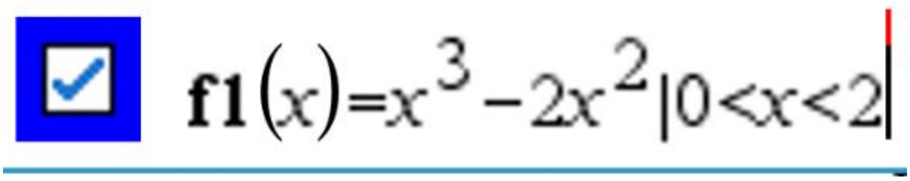
- Open a graph page and plot your function.
- Zoom settings: Menu→ 4 (window/zoom)→ 1 enter your x and y-ranges.



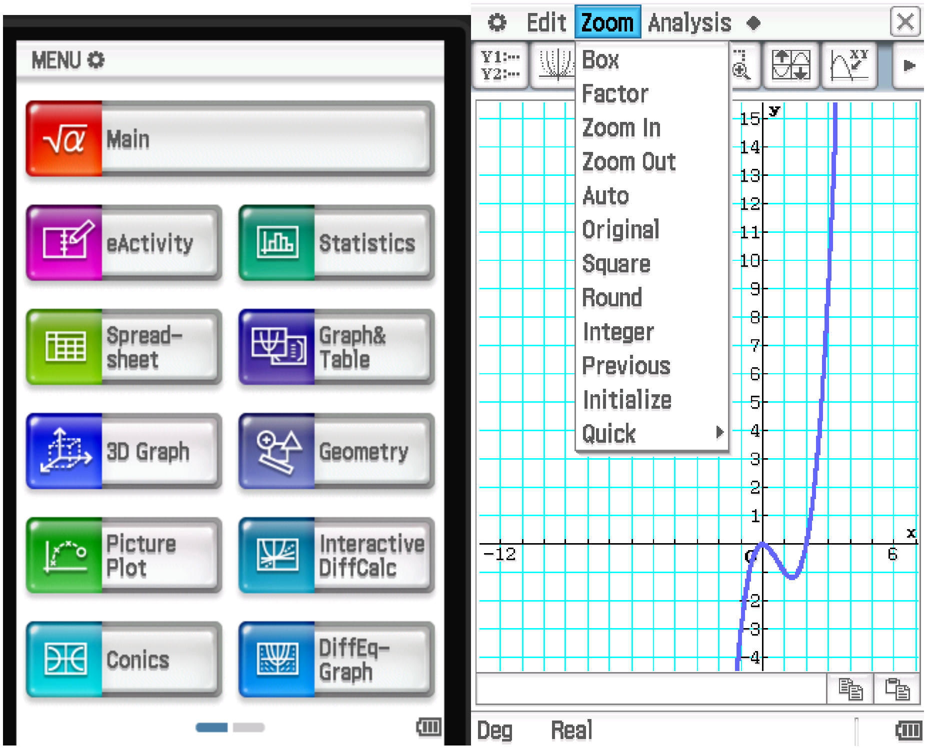
- Can also click the axis numbers on the graph and alter them directly.



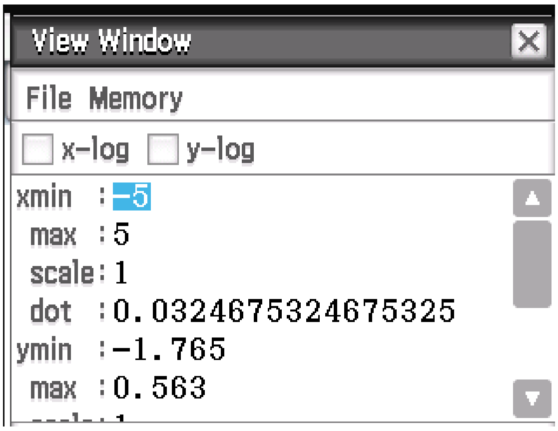
- Menu→ 6 (Analyse) to find min/max x and y-intercepts.
- Restrict domain to $0 < x < 2$ use the bar can get it from $\text{ctrl}+=$



- **Casio:** Click Graph & Table, and enter the function.



- Analysis→ G-Solve to find intercepts.
- Use this button  to set the view window.



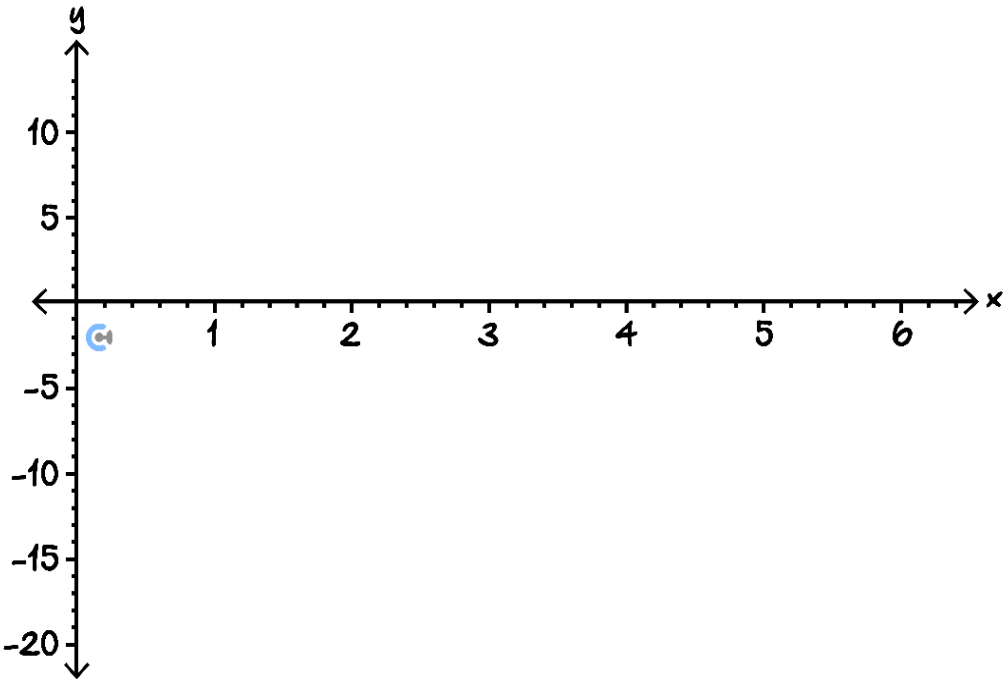
- Use | to restrict domain → find it in Math 3.

$$y1=x^3-2\cdot x^2 \mid 0<x<2$$

- **Mathematica:** $\text{Plot}[\text{function}, \{x, x\text{min}, x\text{max}\}, \text{PlotRange} \rightarrow \{y\text{min}, y\text{max}\}]$
  - PlotRange is optional but can be used to make the scale appropriate for the question.

Question 27 Tech-Active.

Sketch the graph of $y = 2\sec\left(2x + \frac{\pi}{4}\right) - 3$ for $0 \leq x \leq 2\pi$.



Calculator Commands: Simplifying compound angle/double angle and composition.

Mathematica	TI-Nspire	Casio Classpad
"TrigExpand"	"texpand"	"texpand"
	Menu→ $3 \rightarrow B \rightarrow 1$	Action→Transformation→tExpand
	Or Type t and menu 33	

Question 28 Tech-Active.

Simplify $\cos\left(\sin^{-1}(2x-2)\right)+1$.

Calculator Commands: Finding domain and rage

- **TI:** domain$(f(x), x)$, $f\text{Min}$ and $f\text{Max}$

Define $f(x)=\sqrt{9-x^2}$	*Done*
$\text{domain}(f(x),x)$	$-3\le x\le 3$
$\text{fMin}(f(x),x)$	$x=-3$ or $x=3$
$\text{fMax}(f(x),x)$	$x=0$
$f(3)$	0
$f(0)$	3

- **TI UDF:**

Analyse a Function: Find intercepts, critical points and their nature, maximal domain, asymptote.	`analyse(<function>, <variable>)`
	
	`analysed(<function>, <variable>, <lower bound>, <upper bound>)`

$$\textit{analysed}\left(\frac{x^4-2\cdot x^3-3\cdot x^2+3\cdot x+1}{-3\cdot x^3-6\cdot x^2-x+1},x,-5,5\right)$$

- Start Point: $\begin{bmatrix} -5 & \frac{262}{77} \end{bmatrix}$
- End Point: $\begin{bmatrix} 5 & \frac{-316}{529} \end{bmatrix}$
- Maximal Domain:
  $x\neq-1.68469$ and
  $x\neq-0.629579$ and
  $x\neq0.314273$ and
  $-5\le x\le 5$

- **Casio:** Graph the function and use G-Solve to find min and max values for the range.
- **Mathematica:** FunctionDomain[$f[x], x$]. FunctionRange[$f[x], x, y$].

```
In[127]:= f[x_] := √(9 - x^2)

In[128]:= FunctionDomain[f[x], x]
Out[128]= -3 ≤ x ≤ 3

In[129]:= FunctionRange[f[x], x, y]
Out[129]= 0 ≤ y ≤ 3
```

- **Mathematica UDF:** FInfo[$f[x], x$] – gives domain and range all together.

FInfo[f[x], {x, x min, x max}, y]
Returns useful information about a function, including derivative, domain, range, period, horizontal intercepts, vertical intercepts, stationary points, inflexion points, left and rig sided asymptotes, oblique asymptotes, and vertical asymptotes.

$$\text{FInfo}\left[\frac{x^2-1}{x\,(x^2-3)}, \{x, -\text{Infinity}, \text{Infinity}\}, y\right]$$

The function is $\frac{x^2-1}{x\,(x^2-3)}$

The derivative is $-\frac{x^4+3}{x^2\,(x^2-3)^2}$

Domain: $x < -\sqrt{3} \lor -\sqrt{3} < x < 0 \lor 0 < x < \sqrt{3} \lor x > \sqrt{3}$

Range: $y \in \mathbb{R}$

Period: 0

Horizontal Intercepts: {-1, 1}

Vertical Intercepts: None

Stationary points: {}

Inflexion points: {{-0.871..., -0.123...}, {0.871..., 0.123...}}

Left sided asymtote: y=0

Right sided asymtote: y=0

Oblique asymtote: {0}

Vertical asymtote: $\{x=0, x=-\sqrt{3}, x=\sqrt{3}\}$

Question 29 Tech-Active.

Find the domain and range of $\cos\left(\sin^{-1}(2x-2)\right)+1$.

Space for Personal Notes

Section F: Exam 2 Questions (16 Marks)

INSTRUCTION: 16 Marks. 4 Minutes Reading. 20 Minutes Writing.

Question 30 (1 mark)

The implied domain of $y = \arcsin\left(\frac{x-a}{b}\right)$ where $b > 0$ is:

A. $[-1,1]$

B. $[b, -b]$

C. $[a - b, a + b]$

D. $[a + b, a - b]$

Question 31 (1 mark)

Let $f(x) = \frac{1}{\operatorname{cosec}(2x)+2}$.

The number of asymptotes that the graph of f has in the interval $[-\pi, \pi]$ is:

A. 2

B. 3

C. 4

D. 5

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Question 32 (1 mark)

The range of the function $f(x) = \sin(\arccos(2x+1)) - 1$ is:

A. $[-1,0]$

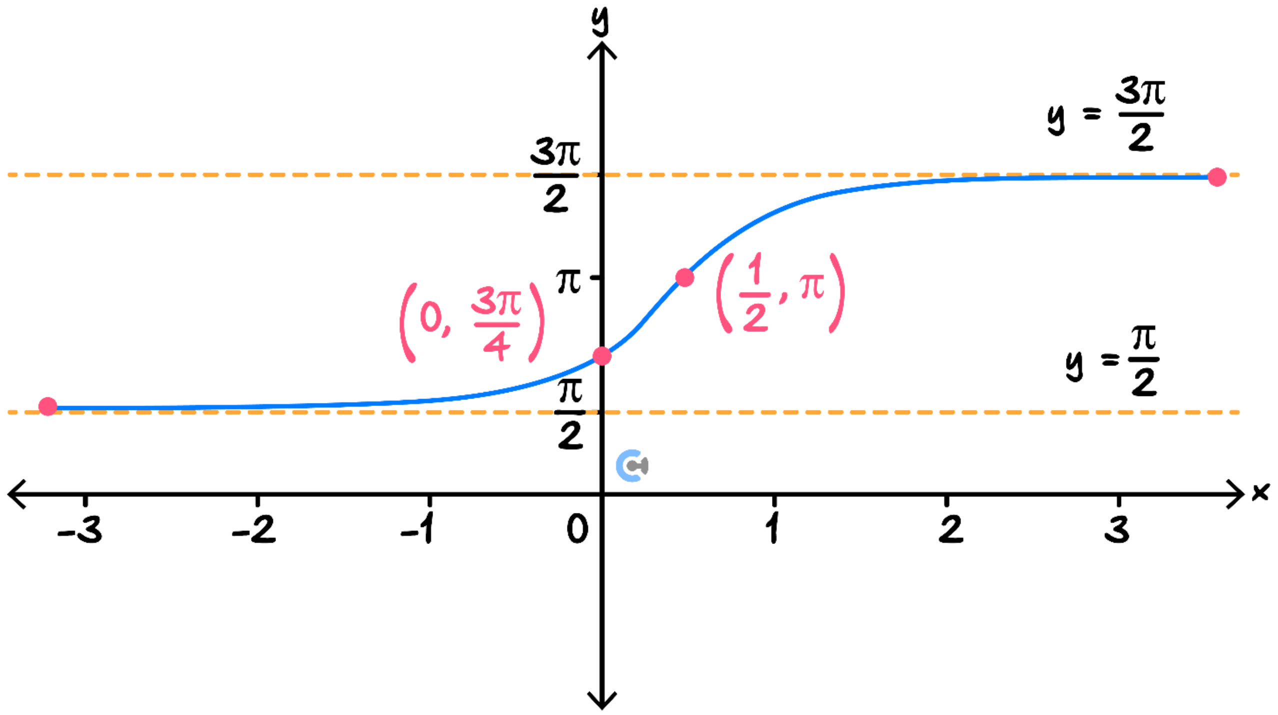
B. $\left[\frac{2}{3}, 1\right]$

C. $[0,1]$

D. $[1,2]$

Question 33 (1 mark)

Part of the graph of $y = f(x)$ is shown below.



The rule for f could be:

A. $\cot(2x+1) + \pi$

B. $\arctan(2x-1) + \pi$

C. $\arctan(2x) + \frac{\pi}{2}$

D. $\arcsin(2x) + \pi$

Question 34 (1 mark)

If $\sin(x+y) = a$ and $\sin(x-y) = b$, then $\sin(x)\cos(y)$ is equal to:

A. ab

B. $\sqrt{a^2+b^2}$

C. $\sqrt{a^2-b^2}$

D. $\frac{a+b}{2}$

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Question 31 (11 marks)

a.

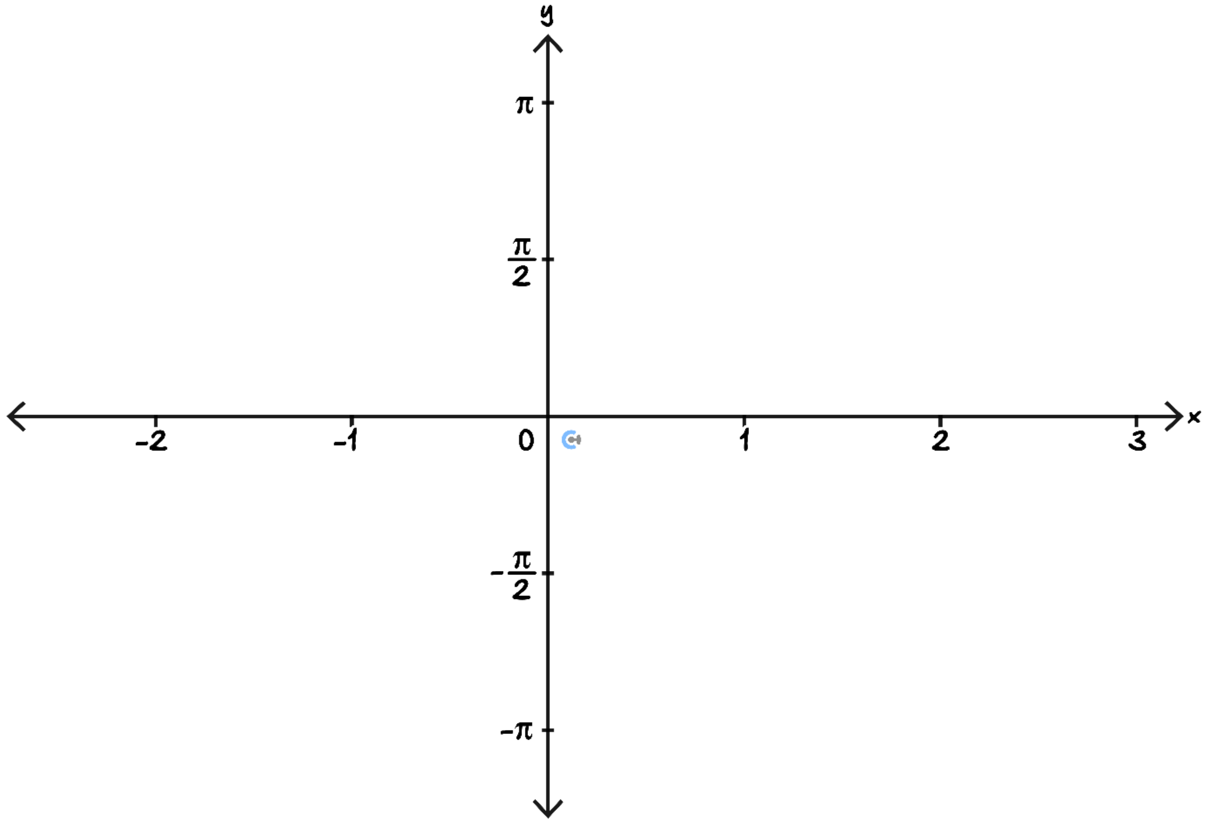
i. Use an appropriate double-angle formula with $t = \tan\left(\frac{\pi}{8}\right)$ to deduce a quadratic equation of the form $t^2 + bt + c = 0$, where b and c are real values. (2 marks)

ii. Hence, show that $\tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1$. (1 mark)

b. Consider the function $f: D \to R, f(x) = -2\arcsin(x^2 - 1)$.

i. Determine the maximal domain D and the range of f. (2 marks)

ii. Sketch the graph of $y = f(x)$ on the axes below, labelling any endpoints and the y-intercept with their coordinates. (3 marks)



c. Let $x = \sec(t)$ and $y = \operatorname{cosec}(t)$, where $t \in \left(0, \frac{\pi}{2}\right)$.

Use trigonometric identities to find y as a function of x. (3 marks)

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Contour Check

- [ ] **Learning Objective**: [3.5.1] - Simplify the composition of inverse trigonometric functions

Key Takeaways

- [ ] The range of $\sin^{-1}(x)$ is ____________.
- [ ] The range of $\cos^{-1}(x)$ is ___________.
- [ ] Simplify the composition by treating the inside function as an angle θ.

Learning Objective: [3.5.2] - Simplify $a\cos(x) + b\sin(x)$

Key Takeaways

- Step 1: Find the radius by taking the two coefficients

$$r = \sqrt{a^2 + b^2}$$

- Step 2: Factor the expression by the ____________
- Step 3: Replace the coefficients with $\cos(a)$ and $\sin(a)$. Remember to use the same angles.
- Step 4: Use _______________ formula to express in terms of a single trigonometric function.

$$a\cos(x) + b\sin(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$a\cos(x) - b\sin(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$a\sin(x) + b\cos(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$a\sin(x) - b\cos(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\text{where } r = \sqrt{a^2 + b^2} \text{ and } \alpha = \tan^{-1}\left(\frac{b}{a}\right)$$

- **Learning Objective**: [3.5.3] - Apply Product to Sum and Sum to Product Identities to simplify trigonometric expressions

Key Takeaways

- Product to Sum Identities

$$2\cos(x)\cos(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$2\sin(x)\sin(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$2\sin(x)\cos(y) = \_\_\_\_\_\_\_\_\_\_\_\_$$

- Sum to product identities:

$$\cos(x) + \cos(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\cos(x) - \cos(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\sin(x) + \sin(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

$$\sin(x) - \sin(y) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

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