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VCE Specialist Mathematics ½
Advanced Trigonometric Functions Exam Skills [3.5]
Homework Solutions

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 02-Pg 16

Section A: Compulsory Questions

Sub-Section [3.5.1]: Simplify the Composition of Inverse Trigonometric Functions

Question 1

a. Simplify $\sin\left(\arcsin\left(\frac{1}{5}\right)\right)$.

$\frac{1}{5}$.

b. Simplify $\sin(\arctan(2))$.

Let $\theta = \arctan(2)$, right triangle with sides, $1, 2, \sqrt{5}$.
Therefore $\sin(\arctan(2)) = \frac{2}{\sqrt{5}}$

c. Simplify $\cos\left(\arcsin\left(\frac{3}{5}\right)\right)$.

Let $\theta = \arcsin\left(\frac{3}{5}\right)$. Right triangle with sides $3, 4, 5$.
Therefore $\cos\left(\arcsin\left(\frac{3}{5}\right)\right) = \frac{4}{5}$

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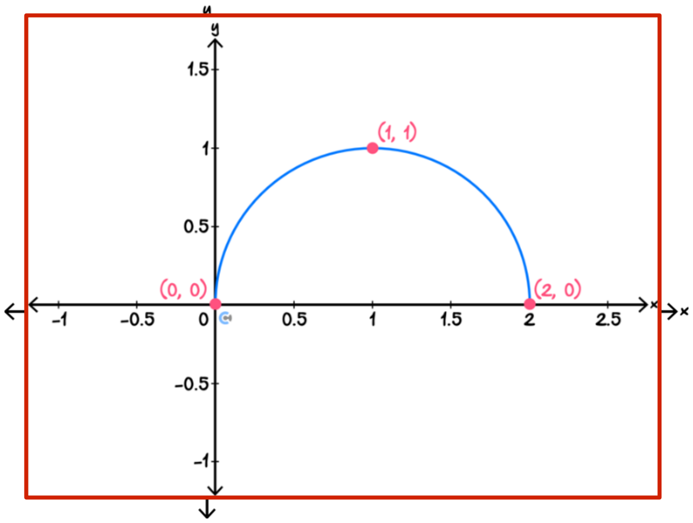
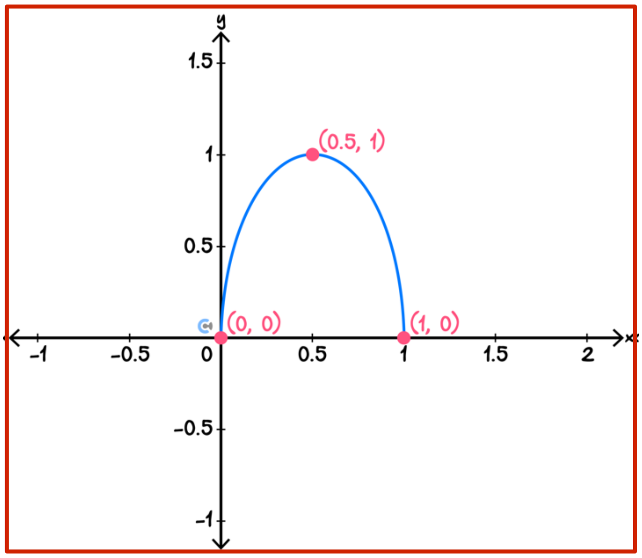
Question 2

a. Simplify and sketch the graph of $f(x) = \cos(\arcsin(x - 1))$.



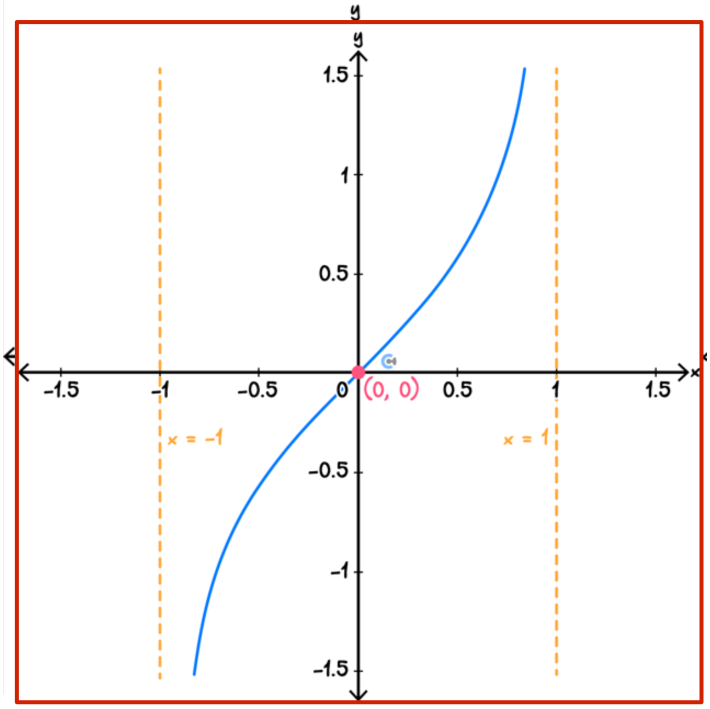
$\sqrt{1-(x-1)^2}$

b. Simplify and sketch the graph of $f(x) = \sin(\arccos(2x + 1))$.



$f(x) = \sqrt{1-(2x-1)^2} = \sqrt{4x-4x^2} = 2\sqrt{x(1-x)}$

c. Simplify and sketch the graph of $\tan(\arcsin(x))$.



Let $\theta = \arcsin(x) \implies \sin(\theta) = x$.
Then $\cos(\theta) = \sqrt{1-x^2}$. Positive because $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.
Thus $\tan(\arcsin(x)) = \tan(\theta) = \frac{x}{\sqrt{1-x^2}}$

Question 3

a. Simplify and state the maximal domain of $f(x) = \tan(\arcsin(2x-1)) + \cos(\arctan(x+2))$.

$$f(x) = \frac{2x-1}{\sqrt{1-(1-2x)^2}} + \frac{1}{\sqrt{1+(2+x)^2}}$$

dom $\tan(\arcsin(2x-1)) = (0,1)$ and dom $\cos(\arctan(x+2)) = \mathbb{R}$
For the maximal domain we require both functions to be defined. Thus $x \in (0,1)$

b. Simplify and state the maximal domain of $f(x) = \sin(\arccos(1 - x^2)) + \cos(\arcsin(x - 1))$.

$f(x) = \sqrt{1-(1-x^2)^2} + \sqrt{1-(1-x)^2}$

dom $\sin(\arccos(1-x^2)) = [-\sqrt{2}, \sqrt{2}]$ and dom $\cos(\arcsin(x-1)) = [0, 2]$
For the maximal domain we require both functions to be defined. Thus $x \in [0, \sqrt{2}]$

c. Simplify and state the maximal domain $f(x) = \tan(\arcsin(2x + 1)). \cos(\arctan(3x))$.

$$f(x) = \frac{1+2x}{\sqrt{1-(1+2x)^2}} \cdot \frac{1}{\sqrt{1+9x^2}} = \frac{1+2x}{2\sqrt{-x(1+x)}\sqrt{1+9x^2}}$$

dom $\tan(\arcsin(2x+1)) = (-1, 0)$ and dom $\cos(\arctan(3x)) = \mathbb{R}$.
For the maximal domain we require both of these functions to be defined. Thus $x \in (-1, 0)$

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Sub-Section [3.5.2]: Simplify $a\cos(x) + b\sin(x)$

Question 4

a. Express $\sin(x) + \cos(x)$ in the form $r\sin(x + \alpha)$.

$$\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$$

b. Express $3\sin(x) + \sqrt{3}\cos(x)$ in the form $r\sin(x + \alpha)$.

$r = \sqrt{12} = 2\sqrt{3}$ and $\alpha = \arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$.

Thus $2\sqrt{3}\sin\left(x + \frac{\pi}{6}\right)$

c. Express $2\cos(x) + \sqrt{2}\sin(x)$ in the form $r\cos(x - \alpha)$.

$r = \sqrt{8} = 2\sqrt{2}$ and $\alpha = \arctan\left(\frac{2}{2}\right) = \frac{\pi}{4}$

Thus $2\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$

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Question 5

a. Solve $\sin(x) - \sqrt{3}\cos(x) = 1$ for $0 \leq x \leq 2\pi$.

Equivalently solve $2\sin\left(x - \frac{\pi}{3}\right) = 1$.

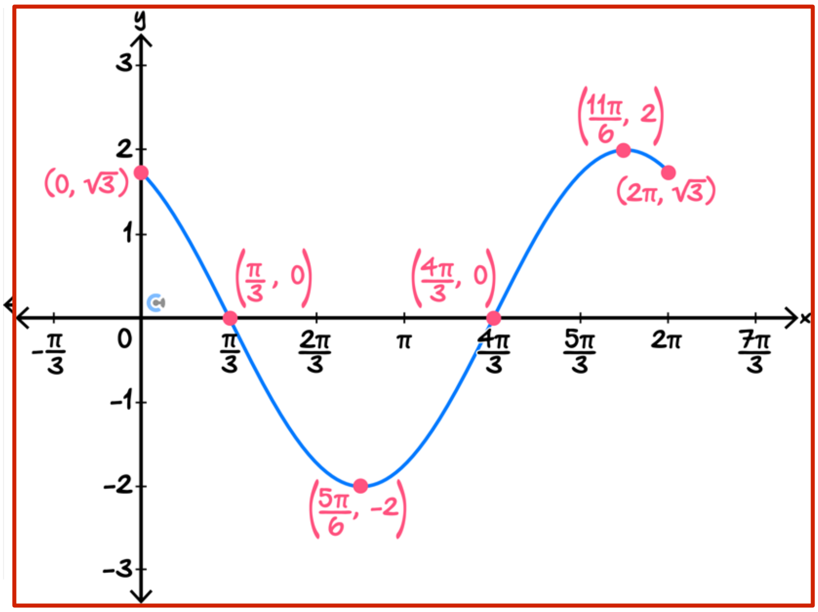
Thus $\sin\left(x - \frac{\pi}{3}\right) = \frac{1}{2} \implies x = \frac{\pi}{2}, \frac{7\pi}{6}$

b. Solve $3\sin(x) - \sqrt{3}\cos(x) = \sqrt{3}$ for $0 \leq x < 2\pi$.

Equivalently solve $2\sqrt{3}\sin\left(x - \frac{\pi}{6}\right) = \sqrt{3}$.

Thus solve $\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2} \implies x = \frac{\pi}{3}, \pi$

c. Sketch the graph of $f(x) = \sqrt{3}\cos(x) - \sin(x)$ for $0 \le x \le 2\pi$. Label all turning points, endpoints and axes intercept with coordinates.



$$f(x) = 2\cos\left(x + \frac{\pi}{6}\right).$$

Question 6

a. Find the maximum and minimum value of $f(x) = 5\sin(x) + 12\cos(x)$.

$r = \sqrt{5^2 + 12^2} = 13$. $f(x) = 13\sin(x + \alpha)$.
Thus max value is 13 and min value is -13

b. Solve $2\sin\left(x-\frac{\pi}{6}\right)+2\sqrt{3}\cos\left(x-\frac{\pi}{6}\right)=2$ for $0\le x\le 2\pi$.

Equivalent to solving $4\sin\left(x+\frac{\pi}{6}\right)=2$.

Thus solve $\sin\left(x+\frac{\pi}{6}\right)=\frac{1}{2}$.

$$x+\frac{\pi}{6}=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6}$$
$$x=0,\frac{2\pi}{3},2\pi$$

c. Show that for $a>0$.

$a\sin(2x)-b\cos^2(x)=\sqrt{4a^2+b^2}\cos(x)\sin(x-\alpha)$, where $\alpha=\arctan\left(\frac{b}{2a}\right)$.

$$\begin{aligned}a\sin(2x)-b\cos^2(x)&=2a\sin(x)\cos(x)-b\cos^2(x)\\&=\cos(x)(2a\sin(x)-b\cos(x))\\&=\cos(x)\left(\sqrt{4a^2+b^2}\sin(x-\alpha)\right),\ \text{where } \alpha=\arctan\left(\frac{b}{2a}\right)\\&=\sqrt{4a^2+b^2}\cos(x)\sin(x-\alpha)\end{aligned}$$

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Sub-Section [3.5.3]: Apply Product-to-Sum and Sum-to-Product Identities to Simplify Trigonometric Expressions

Question 7

a. Express $\sin(4\theta)\cos(2\theta)$ as a sum or difference.

Use the formulas. $\frac{1}{2}\sin(6\theta) + \frac{1}{2}\sin(2\theta)$

b. Express $2\cos(3A)\cos(5A)$ as a sum or difference.

Use the formulas. $\cos(8A) + \cos(2A)$

c. Express $\cos(4A)\sin(2A)$ as a sum or difference.

Use the formulas. $\frac{1}{2}\sin(6A) - \frac{1}{2}\sin(2A)$

d. Express $\sin(2\alpha) + \sin(2\beta)$ as a product.

Use the formulas. $2\sin(\alpha + \beta)\cos(\alpha - \beta)$

e. Express $\cos(2x) + \cos(2y)$ as a product.

Use the formulas. $2\cos(x+y)\cos(x-y)$

f. Express $\sin(x + h) - \cos(x)$ as a product.

$$\sin(x+h) - \sin\left(x + \frac{\pi}{2}\right) = 2\cos\left(x + \frac{2h+\pi}{4}\right)\sin\left(\frac{2h-\pi}{4}\right)$$

Question 8

a. Solve $\sin(3\theta) + \sin(\theta) = 0$ for $0 \leq \theta \leq 2\pi$.

Equivalently solve $2\sin(2\theta)\cos(\theta) = 0$.

$\sin(2\theta) = 0 \implies \theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi.$

$\cos(\theta) = 0 \implies \theta = \frac{\pi}{2}, \frac{3\pi}{2}.$

Thus all solutions are $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi.$

b. Solve $\cos(4x) + \cos(2x) = 0$ for $0 \leq x \leq \pi$.

Equivalently solve $2\cos(3x)\cos(x) = 0$.
$\cos(3x) = 0 \implies x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$
and $\cos(x) = 0 \implies x = \frac{\pi}{2}$.
Thus all solutions are $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

c. Solve $\sin(x) - \sin\left(\frac{3\pi}{4} - x\right) = 0$ for $0 \leq \theta \leq 2\pi$.

Equivalently solve $2\cos\left(\frac{3\pi}{8}\right)\sin\left(x - \frac{3\pi}{8}\right) = 0$.
Thus solve $\sin\left(x - \frac{3\pi}{8}\right) = 0$.
$x = \frac{3\pi}{8}, \frac{11\pi}{8}$

Question 9

a. Express $|a|\sin(x) - |a|\sin(3x)$ as a product and hence find its maximum and minimum values in terms of a.

$2|a|\cos(2x)\sin(x)$, we want to find the max and min values of this.
We do this by recognition. Note that when $x = \frac{\pi}{2}$, we get a min of $-2|a|$.
When $x = \frac{3\pi}{2}$ we get a max of $2|a|$

b. If $\alpha + \beta + \gamma = \pi$, show that $\cos(\alpha) + \cos(\beta) + \cos(\gamma) = 1 + 4\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)\sin\left(\frac{\gamma}{2}\right)$.

We have

$$\begin{aligned}
\cos(\alpha) + \cos(\beta) + \cos(\gamma) &= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + 1 - 2\sin^2\left(\frac{\gamma}{2}\right) \\
&= 2\cos\left(\frac{\pi}{2} - \frac{\gamma}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + 1 - 2\sin^2\left(\frac{\gamma}{2}\right) \\
&= 2\sin\left(\frac{\gamma}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) + 1 - 2\sin^2\left(\frac{\gamma}{2}\right) \\
&= 2\sin\left(\frac{\gamma}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right) - \sin\left(\frac{\gamma}{2}\right)\right] + 1 \\
&= 2\sin\left(\frac{\gamma}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right) - \sin\left(\frac{\pi}{2} - \frac{\alpha+\beta}{2}\right)\right] + 1 \\
&= 2\sin\left(\frac{\gamma}{2}\right)\left[\cos\left(\frac{\alpha-\beta}{2}\right) - \cos\left(\frac{\alpha+\beta}{2}\right)\right] + 1 \\
&= 2\sin\left(\frac{\gamma}{2}\right)\left[-2\sin\left(\frac{\alpha}{2}\right)\sin\left(-\frac{\beta}{2}\right)\right] + 1 \\
&= 2\sin\left(\frac{\gamma}{2}\right)\left[2\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)\right] + 1 \\
&= 1 + 4\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)\sin\left(\frac{\gamma}{2}\right)
\end{aligned}$$

c. Solve the equation $\sin(3x) + \sin(x) - \sin(4x) = 0$ for $x \in [0, 2\pi]$.

LHS is equivalent to:

$$2\sin(2x)\cos(x) - 2\sin(2x)\cos(2x) = 2\sin(2x)(\cos(x) - \cos(2x))$$

Then $\sin(2x) = 0 \implies x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$.

Now consider $\cos(x) - \cos(2x) = -2\sin\left(\frac{3x}{2}\right)\sin\left(-\frac{x}{2}\right) = 2\sin\left(\frac{3x}{2}\right)\sin\left(\frac{x}{2}\right)$

Now $\sin\left(\frac{x}{2}\right) = 0 \implies x = 0, 2\pi$ and $\sin\left(\frac{3x}{2}\right) = 0 \implies x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$.

Thus all solutions to the equation are:

$$x = 0, \frac{\pi}{2}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{3\pi}{2}, 2\pi$$

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Sub-Section: The 'Final Boss'

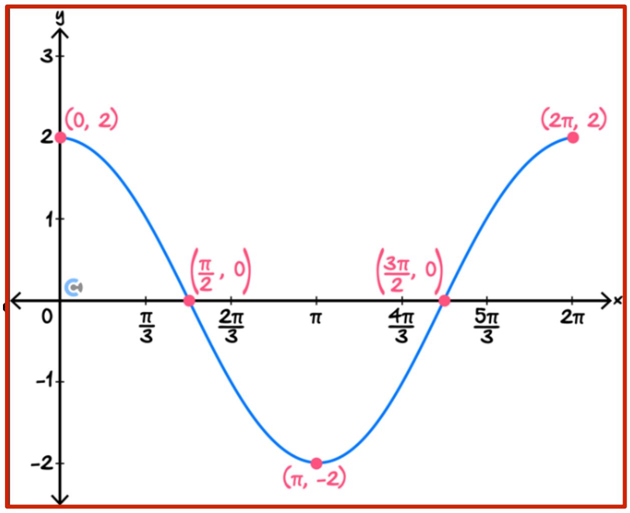
Question 10

a. Solve the equation $\sin(2x) + \sin(4x) = 0, x \in [0, \pi]$.

Equivalent to solving $2\sin(3x)\cos(x) = 0$.
Now $\sin(3x) = 0 \implies x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$.
Also $\cos(x) = 0 \implies x = \frac{\pi}{2}$.
Thus all solutions are $x = 0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi$

b. Consider the function $f: [0, 2\pi] \to \mathbb{R}, f(x) = \sqrt{3}\cos\left(x - \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{3}\right)$.

i. Sketch the graph of f on the axes below. Label all axes intercepts, turning points and endpoints with coordinates.



Note that $f(x) = \sqrt{3}\cos\left(x - \frac{\pi}{6}\right) - \sin\left(x - \frac{\pi}{6}\right) = 2\cos\left(x - \frac{\pi}{6} + \frac{\pi}{6}\right) = 2\cos(x)$

ii. Use your sketch to solve the equation $f(x) = 1$.

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

iii. Hence, find $\{x : f(x) > 1\}$.

$x \in \left[0, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right]$

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