3.4

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# VCE Specialist Mathematics ½
## Advanced Trigonometric Functions I [3.4]
## Workbook

## Outline:



## Learning Objectives:

- ☐ **SM12 [3.4.1]** - Trigonometric Identities and Solving Exact Values of Reciprocal Functions
- ☐ **SM12 [3.4.2]** - Graph Reciprocal Trigonometric Functions
- ☐ **SM12 [3.4.3]** - Apply compound and Double Angle Formula to Solve Exact Values
- ☐ **SM12 [3.4.4]** - Find Domain, Range and Rule of the Inverse Trigonometric Function
- ☐ **SM12 [3.4.5]** - Graphing the Inverse Trigonometric Functions

## Section A: Reciprocal Trigonometric Functions

### Sub-Section: Introduction of Reciprocal Trigonometric Functions

*What are Reciprocal Trigonometric Functions?*

Definition

**Reciprocal Trigonometric Functions**

- The reciprocal of **sine** is **cosecant**:

$$\mathrm{cosec}(x) = \frac{1}{\sin(x)}$$

- The reciprocal of **cosine** is **secant**:

$$\sec(x) = \frac{1}{\cos(x)}$$

- The reciprocal of **tangent** is **cotangent**:

$$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$$

**Space for Personal Notes**

**Question 1**

Evaluate the following.

**a.** $\sec\left(-\frac{\pi}{3}\right)$

$$= \frac{1}{\cos\left(-\frac{\pi}{3}\right)} = \frac{1}{1/2}$$

$$= 2$$

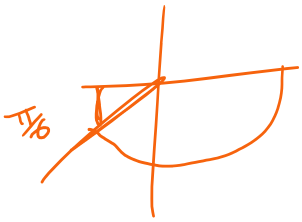
**b.** $\operatorname{cosec}\left(\frac{2\pi}{3}\right)$

$$= \frac{1}{\sin\left(\frac{2\pi}{3}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$


$$= \frac{2\sqrt{3}}{3}$$

**c.** $\cot\left(-\frac{5\pi}{6}\right)$

$$= \frac{1}{\tan\left(-\frac{5\pi}{6}\right)} = \frac{1}{1/\sqrt{3}}$$

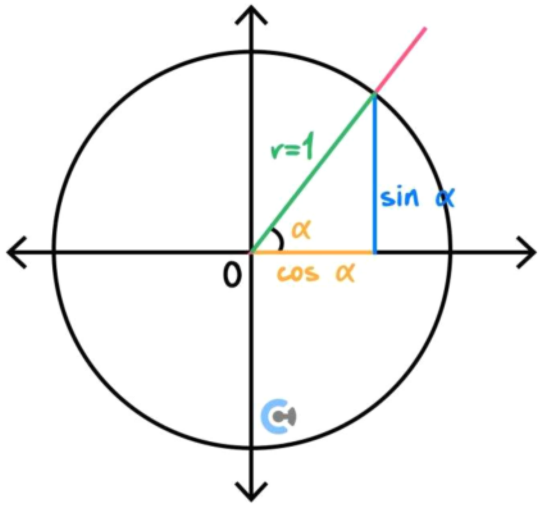



$$= \sqrt{3}$$

**TIP:** Look at their third alphabet!

## Sub-Section: Trigonometric Identities

**REMINDER**




- Trigonometric Identity is given by,

$$\sin^2(\theta) + \cos^2(\theta) = \underline{\quad 1 \quad}$$

***What Happens when we Divide Both Sides by $\sin^2(\theta)$ and $\cos^2(\theta)$?***

**Exploration: Other Trigonometric Identities**

$$\frac{\sin^2(\theta)}{\sin^2\theta} + \frac{\cos^2(\theta)}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

- **Divide each term by** $\cos^2(\theta)$.

$$\tan^2\theta + 1 = \sec^2(\theta)$$

- We can **divide each term by** $\sin^2(\theta)$.

$$1 + \cot^2(\theta) = \operatorname{cosec}^2(\theta)$$

**Trigonometric Identities**

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \operatorname{cosec}^2(\theta)$$

**Question 2**

Given that $\sec(x) = -4$ and $x \in \left[\frac{\pi}{2}, \pi\right]$, find $\operatorname{cosec}(x)$ and $\tan(x)$. Show your working.

$$\frac{1}{\cos(x)} = -4$$

$$\cos(x) = -\frac{1}{4}$$

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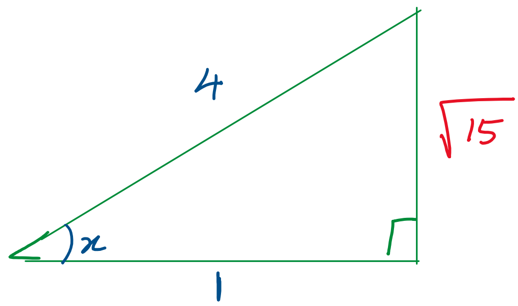



$$\tan(x) = -\frac{\sqrt{15}}{1} = -\sqrt{15}$$

$$\sin(x) = \frac{\sqrt{15}}{4}$$

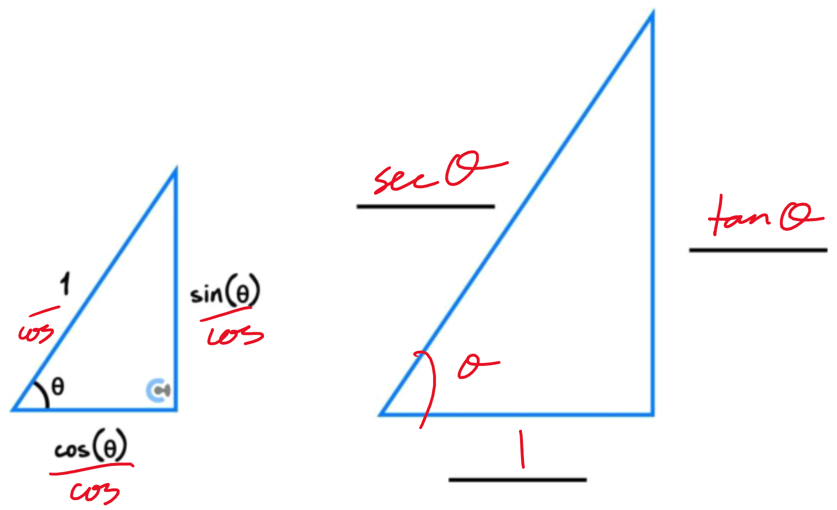
$$\operatorname{cosec}(x) = \frac{4}{\sqrt{15}}$$

## Sub-Section: Further Understanding of Reciprocal Trigonometric Functions

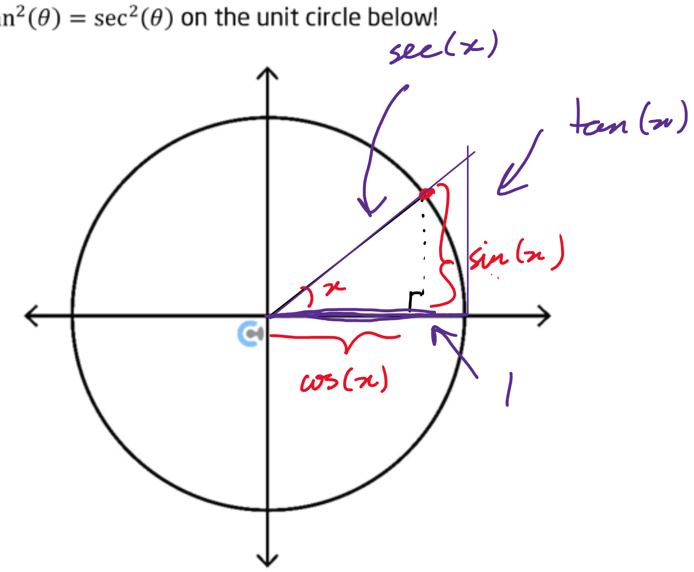
*Let's use the Trigonometric Identities to visualise the Reciprocal Trig Values!*

**Exploration: Visualisation of $1 + \tan^2(\theta) = \sec^2(\theta)$**

- Visualise $1 + \tan^2(\theta) = \sec^2(\theta)$ on the right angle triangle below!



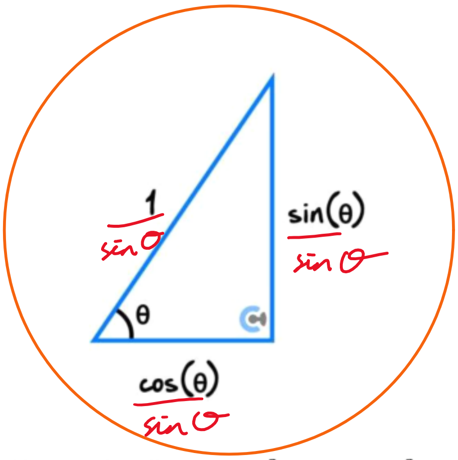

- Hence, visualise $1 + \tan^2(\theta) = \sec^2(\theta)$ on the unit circle below!

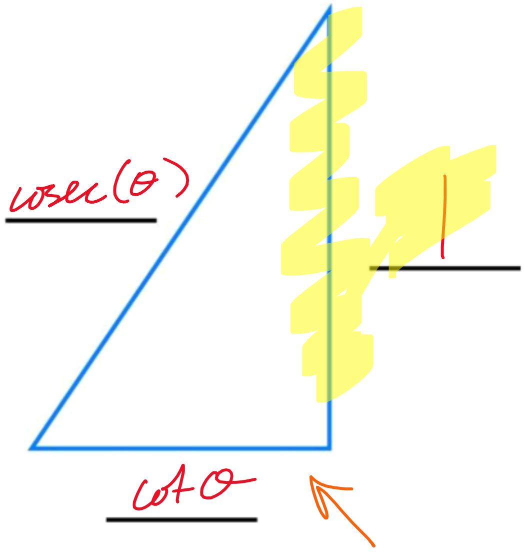


## Try the Next Exploration Yourself!

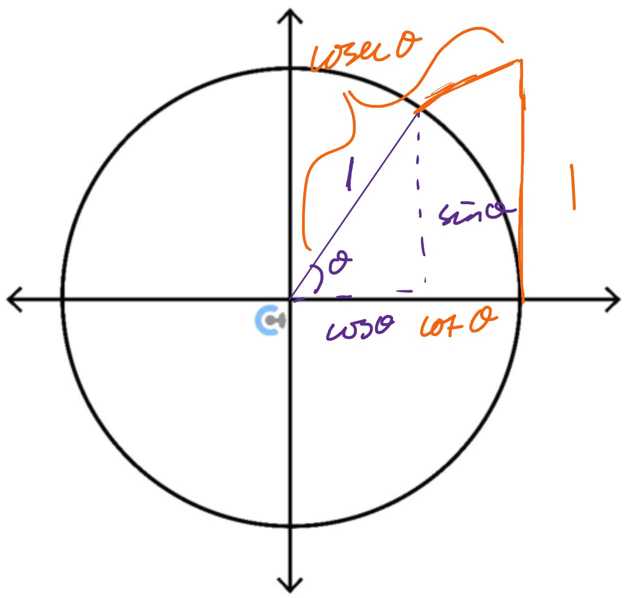
**Exploration: Visualisation of $1 + \cot^2(\theta) = \text{cosec}^2(\theta)$**

- Visualise $1 + \cot^2(\theta) = \text{cosec}^2(\theta)$ on the right angle triangle below!




- Hence, visualise $1 + \cot^2(\theta) = \text{cosec}^2(\theta)$ on the unit circle below!



## Section B: Graphs of Reciprocal Trigonometric Functions

### Sub-Section: Graphs of Reciprocal Functions

**Discussion:** What would the graph of $\frac{1}{f(x)}$ have when the $f(x) = 0$?

$\frac{1}{0}$ Asymptote x-int

**Discussion:** What would the graph of $\frac{1}{f(x)}$ have, when the $f(x)$ is increasing?

$f(x) \uparrow$ $\frac{1}{f(x)} \downarrow$

**Discussion:** What would the graph of $\frac{1}{f(x)}$ have, when the $f(x)$ is decreasing?

$f(x) \downarrow$ $\frac{1}{f(x)} \uparrow$

**Properties of Reciprocal Graphs**

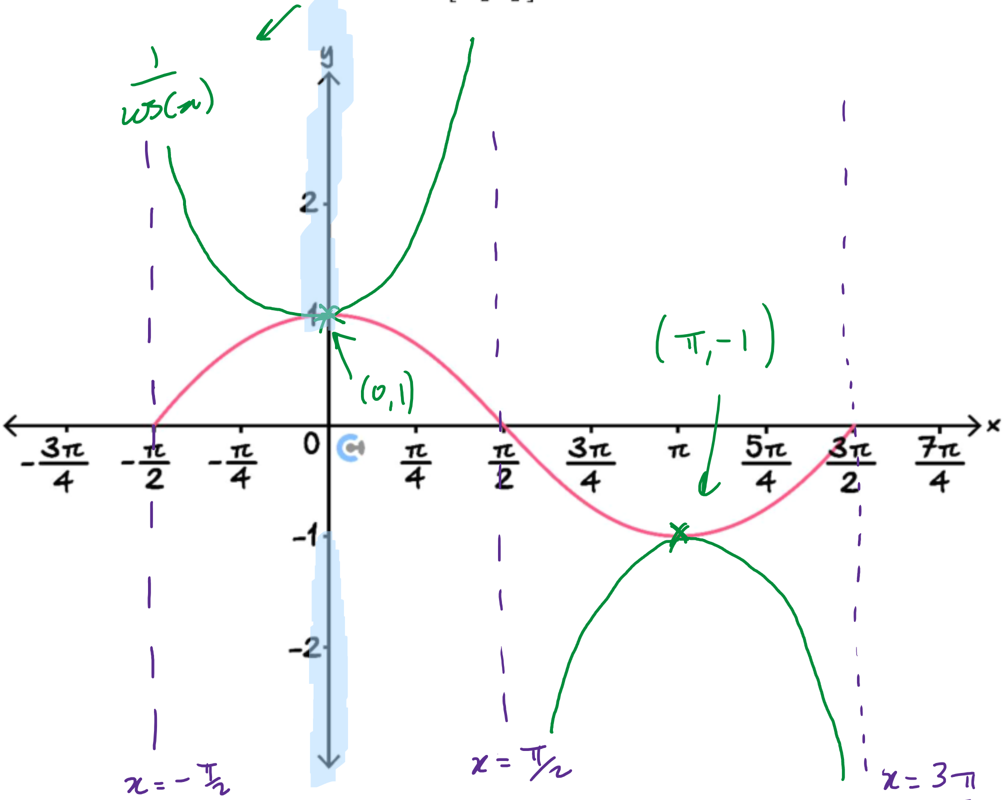
| Feature on $y = f(x)$ | Feature on $y = \frac{1}{f(x)}$ |
|---|---|
| $x$-intercept | Vertical asymptote |
| Positive $y$-values | Positive $y$-values |
| Negative $y$-values | Negative $y$-values |
| Increasing | Decreasing |
| Decreasing | Increasing |
| The graphs intersect only when $f(x) = 1$ or $f(x) = -1$. | |

## Sub-Section: Graphs of Reciprocal Trigonometric Functions

### *Let's Try Sketching the Reciprocal Trigonometric Functions!*

**Question 3 Walkthrough.**

**a.** On the same axes below, sketch $\sec(x)$ for $x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$. Label all your asymptotes and turning points.



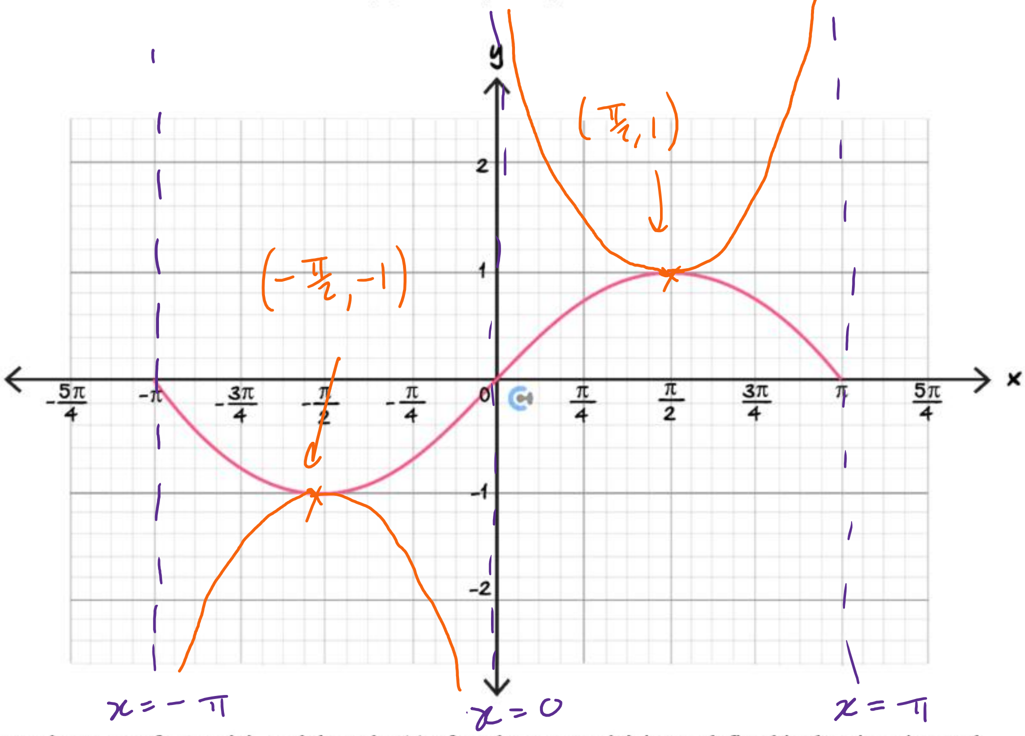

**b.** State the range of $\sec(x)$, and the value(s) of $x$ where $\sec(x)$ is not defined in the given interval.

$\text{ran} = (-\infty, -1] \cup [1, \infty)$

$x = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

**Question 4**

**a.** On the same axes below, sketch $\text{cosec}(x)$ for $x \in [-\pi, \pi]$.




**b.** State the range of $\text{cosec}(x)$, and the value(s) of $x$ where $\text{cosec}(x)$ is not defined in the given interval.

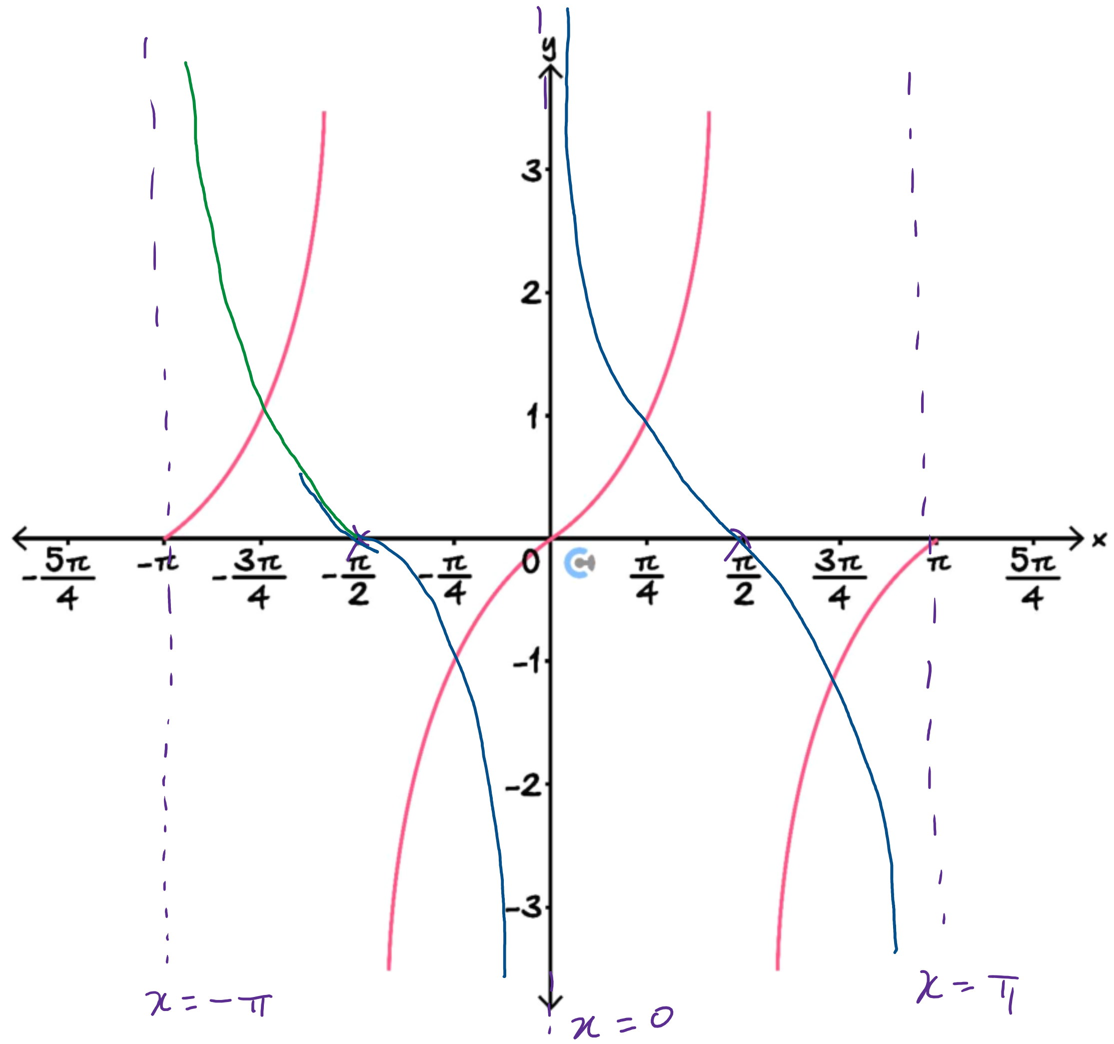
$\text{ran} = (-\infty, -1] \cup [1, \infty)$

$x = -\pi, 0, \pi$

**Question 5 Walkthrough.**

$\frac{1}{\tan}$ $\infty \to \frac{1}{\infty}$

**a.** On the same axes below, sketch $\cot(x)$ for $x \in [-\pi, \pi]$.

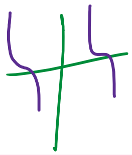



**b.** State the range of $\cot(x)$, and the value(s) of $x$ where $\cot(x)$ is not defined in the given interval.

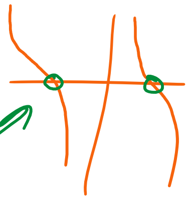
$\text{ran} = \mathbb{R}$

$\boxed{x = -\pi, 0, \pi}$

**Discussion:** Now, what does $\frac{1}{\tan(x)}$ graph look like?



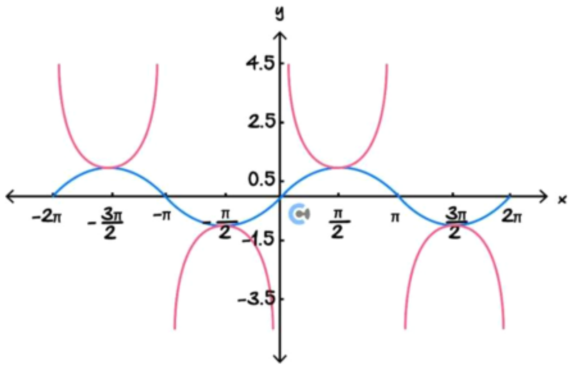
$\cot = \frac{1}{\tan(x)}$ → $\frac{1}{\text{undefined}}$



*In Summary!*

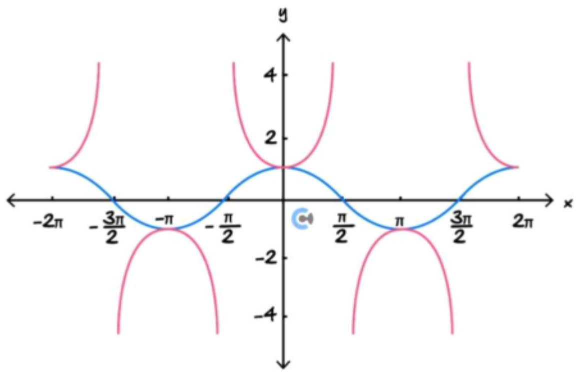
**Graphing Reciprocal Trigonometric Functions**

$$y = \operatorname{cosec}(x)$$



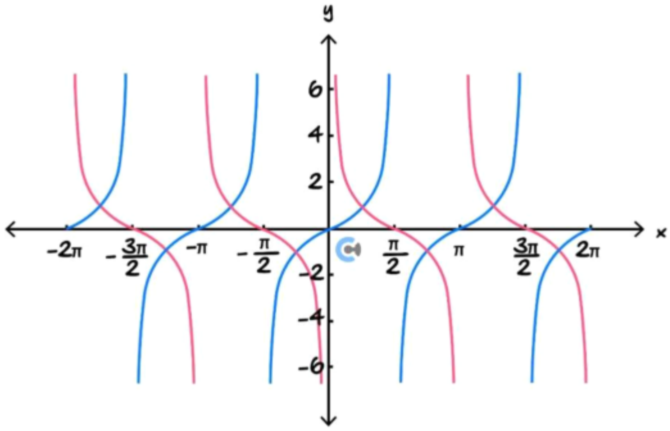

- Maximal Domain: $R\backslash\{x: \sin(x) = 0\}$.
- Range: $(-\infty, -1] \cup [1, \infty)$.

$$y = \sec(x)$$



- Maximal Domain: $R \backslash \{x: \cos(x) = 0\}$.
- Range: $(-\infty, -1] \cup [1, \infty)$.

## $y = \cot(x)$




- Maximal Domain: $R \backslash \{x: \tan(x) = 0\}$.
- Range: $R$.

<u>Discussion:</u> How often do the asymptotes occur for $\operatorname{cosec}$ and $\sec$?

Every $\frac{\pi}{n}$ units

<u>Discussion:</u> How often do the asymptotes occur for $\cot$?

Every $\frac{\pi}{n}$ units

## Sub-Section: Graphs of Complicated Reciprocal Trigonometric Functions

### *Okay, now, about do we Sketch Harder Ones with Transformations?*

**Steps for Sketching Reciprocal Trig Graphs**

- Find **an asymptote**.

$$equate\ \boldsymbol{Angle} = \boldsymbol{0}\ for\ \text{cosec and cot}\ graphs$$

$$equate\ \boldsymbol{Angle} = \frac{\boldsymbol{\pi}}{\boldsymbol{2}}\ for\ \text{sec}\ graphs$$

- Find and mark **all other asymptotes** in the domain.

$$\boldsymbol{Add/Subtract}\ \frac{\boldsymbol{\pi}}{\boldsymbol{n}}\ from\ first\ asymptotes$$

- Plot a point in between the two asymptotes.

$$Midpoint = \boldsymbol{Turning\ Point}\ for\ \text{cosec and sec}\ graphs$$

$$Midpoint = \boldsymbol{Inflection\ Point}\ for\ \text{cot}\ graphs$$

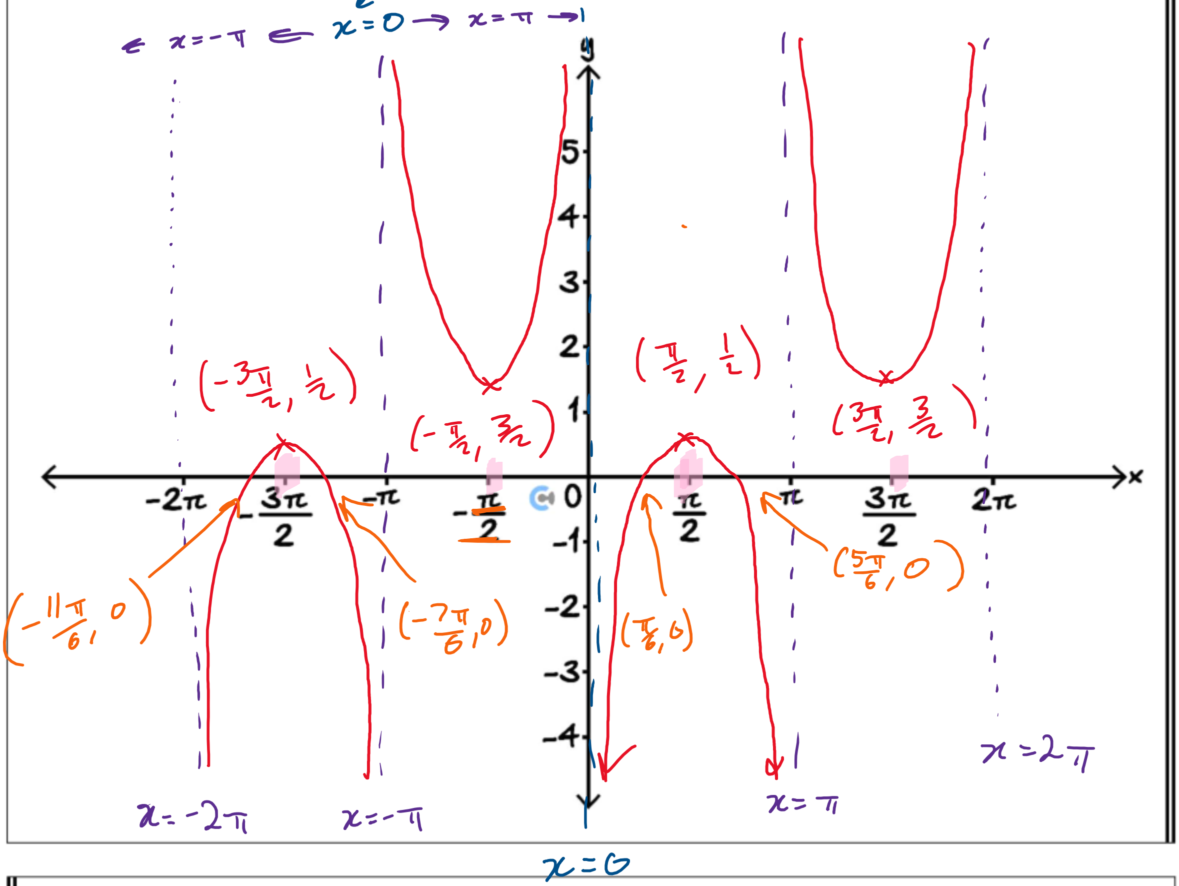
- Solve for axes intercept (if applicable).
- Repeat the shape over the entire domain.
  - For cosec and sec graphs, the "U" shapes **alternate** between asymptotes, while cot graphs look the same between asymptotes.

**NOTE:** Don't forget to label endpoints, and/or axes-intercept(s), turning point(s) and point(s) as required!

**Space for Personal Notes**

**Question 6 Walkthrough.** $\frac{\pi}{2} = \pi$

Sketch the graph of $y = -\frac{1}{2}\operatorname{cosec}(x) + 1$ for $-2\pi \le x \le 2\pi$, labelling all stationary points, axes-intercepts and asymptotes.




**Space for Personal Notes**

$0 = -\frac{1}{2}\operatorname{cosec}(x) + 1$

$\operatorname{cosec}(x) = 2$

$\sin(x) = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \xrightarrow{-\text{period}} -\frac{11\pi}{6}, -\frac{7\pi}{6}$

## Your Turn!

**Question 7**

$\pi(2x-1)=\frac{\pi}{2}$, $2x-1=\frac{1}{2}$, $\boxed{x=\frac{3}{4}}$

Sketch the graph of $y = 2\sec(\pi(2x-1))$ for $-1 \leq x \leq 1$, labelling all stationary points, axes-intercepts and asymptotes.

$\frac{\pi}{n} = \frac{\pi}{2\pi} = \frac{1}{2}$

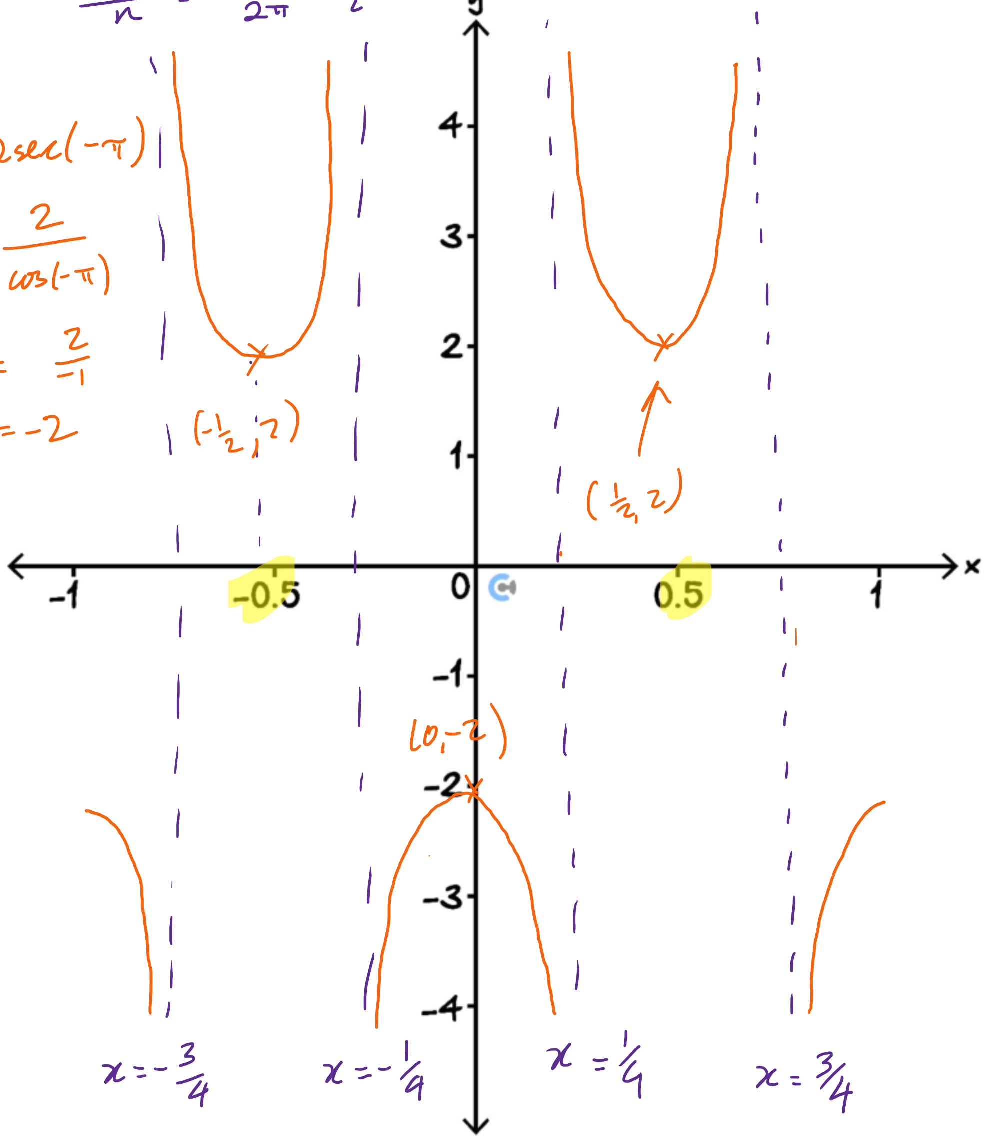
$x=0$,

$y = 2\sec(-\pi)$

$= \frac{2}{\cos(-\pi)}$

$= \frac{2}{-1}$

$= -2$



*Now Cot! Remember they have an Inflection Instead of Turning Points!*

**Question 8**

$x + \frac{\pi}{2} = 0$ , $x = -\frac{\pi}{2}$

Sketch the graph of $y = 1 - \cot\left(x + \frac{\pi}{2}\right)$ for $-\pi \le x \le \pi$, labelling all stationary points, axes-intercepts and asymptotes.

$\frac{\pi}{n} = \pi$

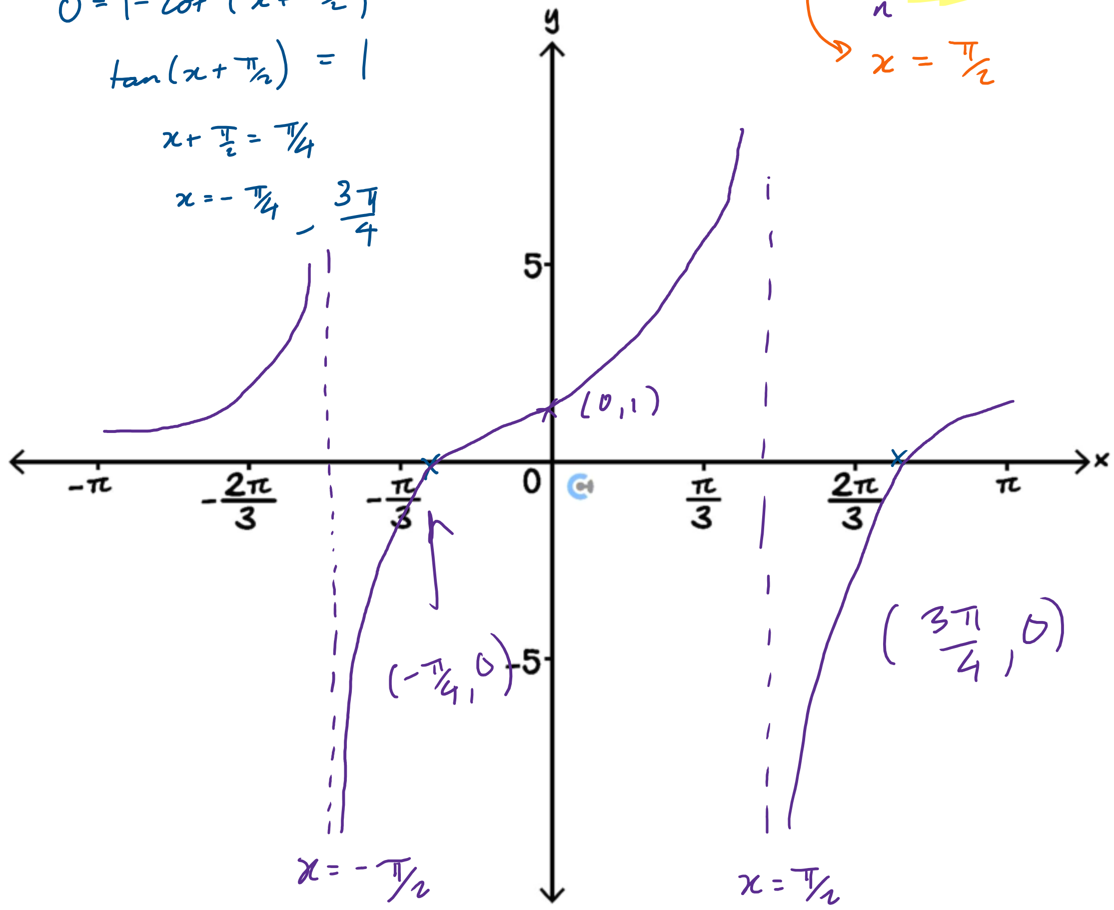
$x = \frac{\pi}{2}$

$0 = 1 - \cot\left(x + \frac{\pi}{2}\right)$

$\tan\left(x + \frac{\pi}{2}\right) = 1$

$x + \frac{\pi}{2} = \frac{\pi}{4}$

$x = -\frac{\pi}{4}$ , $\frac{3\pi}{4}$




**Space for Personal Notes**

## Section C: Compound and Double Angle Formula

### Sub-Section: Compound Angle Formula

*Let's look at the compound angle formula!*

**Compound Angle Formula**

- $\sin$ compound angle formulae.

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

- $\cos$ compound angle formulae.

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

- $\tan$ compound angle formulae.

$$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$$

$$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$$

**Space for Personal Notes**

**Question 9 Walkthrough.**

Using compound angle formula, evaluate $\sin\left(\frac{-\pi}{12}\right)$.

$$\sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$$

$$= \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{2}}{2} \times \frac{1}{2} - \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

**Question 10**

Using compound angle formula, evaluate $\cos\left(\frac{5\pi}{12}\right)$.

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$$

$$= \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$= \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

$$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

## Sub-Section: Double Angle Formula

*What do We Get if $x$ and $y$ Were the Same for the Compound Angle Formula?*

Definition

**Double Angle Formulae**

- $\sin$ double angle formula.

$$\sin(2x) = 2\sin(x)\cos(x)$$

- $\cos$ double angle formula.

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$= 2\cos^2(x) - 1$$

$$= 1 - 2\sin^2(x)$$

- $\tan$ double angle formula.

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

**Question 11 Walkthrough.**

Find $\sin(2t)$, where $\cos(t) = -\frac{1}{8}$.

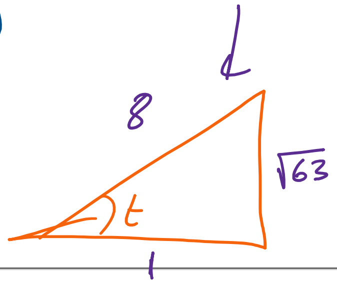
$$\sin(2t) = 2\sin(t)\cos(t)$$

$$= 2\left(\pm\frac{\sqrt{63}}{8}\right)\left(-\frac{1}{8}\right)$$

$$= \pm\frac{\sqrt{63}}{32}$$

$$= \pm\frac{3\sqrt{7}}{32}$$

$\sin(t) = \pm\frac{\sqrt{63}}{8}$



**Question 12**

Find $\cos(2t)$, where $\sin(t) = -\frac{1}{8}$.

$$\cos(2t) = 1 - 2\sin^2(t)$$
$$= 1 - 2\left(-\frac{1}{8}\right)^2$$
$$= 1 - 2 \times \frac{1}{64}$$
$$= 1 - \frac{1}{32}$$

$$\cos(2t) = \pm\frac{31}{32}$$

**Calculator Commands: Expanding Trigonometric Identities**

| Mathematica | TI-Nspire | Casio Classpad |
|---|---|---|
| *"TrigExpand"* | *"texpand"* | *"texpand"* |

**Question 13 Tech-Active.**

Expand $\sin(2x + y)$ in terms of $x$ and $y$.

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$$\text{tExpand}(\sin(2\cdot x+y))$$
$$2\cdot(\cos(x))^2\cdot\sin(y)+2\cdot\sin(x)\cdot\cos(x)\cdot\cos(y)$$

## Section D: Inverse Trigonometric Functions

### Sub-Section: Inversing Trigonometric Functions

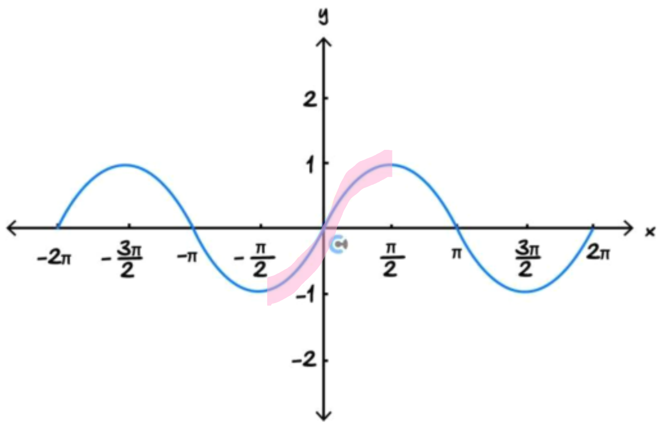
**Discussion**

- What does the original function need to be for the inverse function to exist?

1 : to : 1

**Question 14 Walkthrough.**

Consider the function $\sin(x)$ sketched on the axes below.



**a.** Shade the part of the graph such that the $\sin(x)$ is $1:1$.

**b.** State the domain and range of $\sin(x)$ such that the $\sin^{-1}(x)$ exists.

dom sin $= [-\frac{\pi}{2}, \frac{\pi}{2}]$

ran sin $= [-1, 1]$

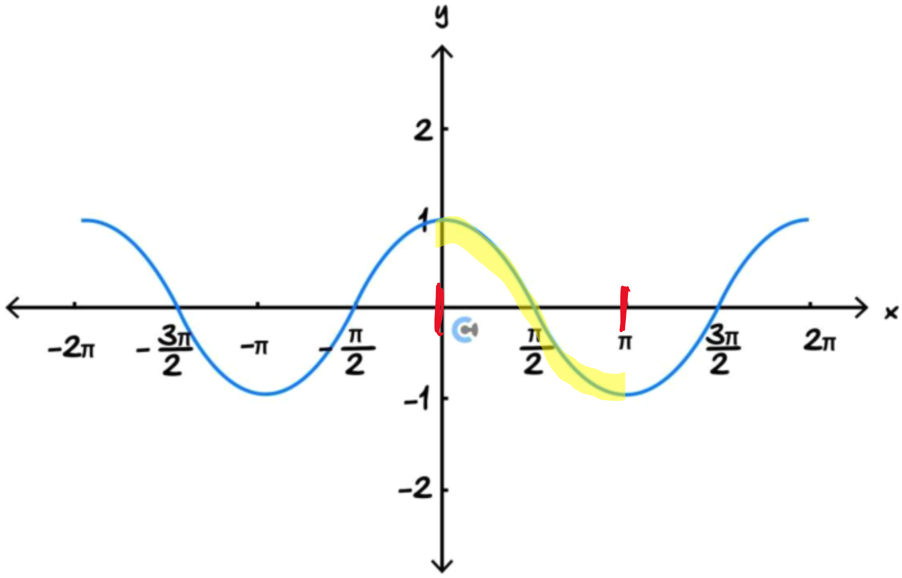
**c.** Hence, state the domain and range of $\sin^{-1}(x)$.

dom $\sin^{-1} = [-1, 1]$

ran $\sin^{-1} = [-\frac{\pi}{2}, \frac{\pi}{2}]$

**Question 15 Walkthrough.**

Consider the function $\cos(x)$ sketched on the axes below.




**a.** Shade the part of the graph such that the $\cos(x)$ is $1:1$.

**b.** State the domain and range of $\cos(x)$, such that the $\cos^{-1}(x)$ exists.

dom $\cos(x) = [0, \pi]$

ran $\cos(x) = [-1, 1]$

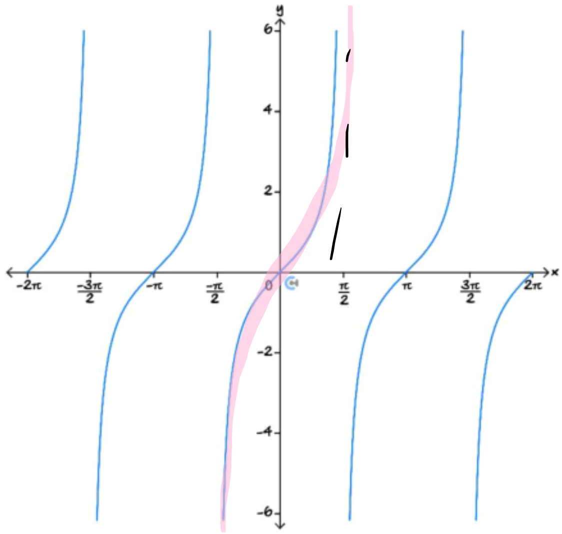
**c.** Hence, state the domain and range of $\cos^{-1}(x)$.

dom $\cos^{-1} = [-1, 1]$

ran $\cos^{-1} = [0, \pi]$

**Question 16 Walkthrough.**

Consider the function $\tan(x)$ sketched on the axes below.




**a.** Shade the part of the graph such that the $\tan(x)$ is $1:1$.

**b.** State the domain and range of $\tan(x)$ such that the $\tan^{-1}(x)$ exists.

dom $\tan(x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

ran $\tan(x) = \mathbb{R}$

**c.** Hence, state the domain and range of $\tan^{-1}(x)$.

dom $\tan^{-1}(x) = \mathbb{R}$

ran $\tan^{-1}(x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

## In Summary!

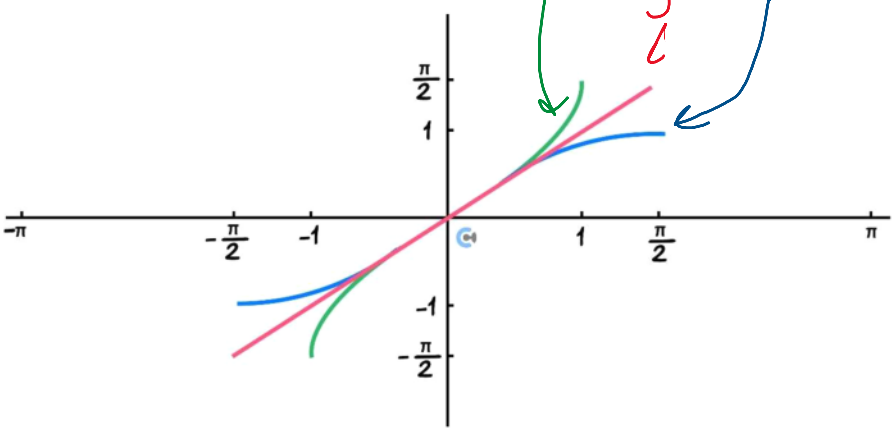
Definition

**Inverse Trig Functions**

- $\sin^{-1}(x)$

inverse orig

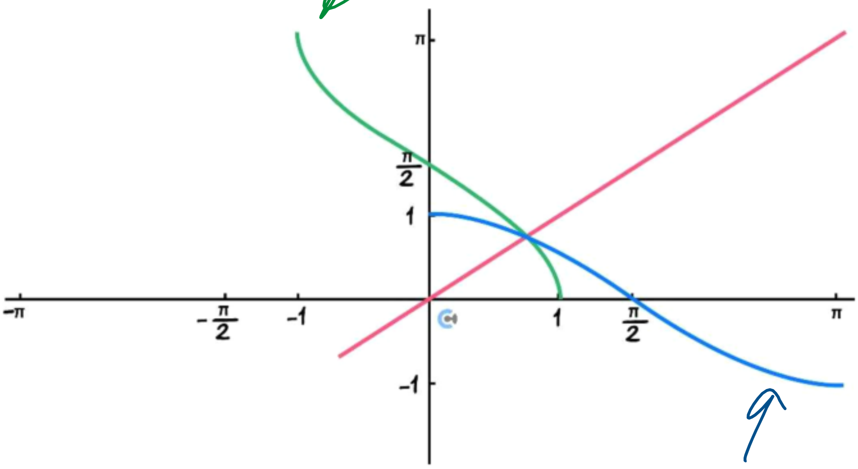
$y=x$




  - The domain of the arcsin function = Range of $\sin = [-1,1]$.
  - The range = Domain of restricted $\sin = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

- $\cos^{-1}(x)$

inv.

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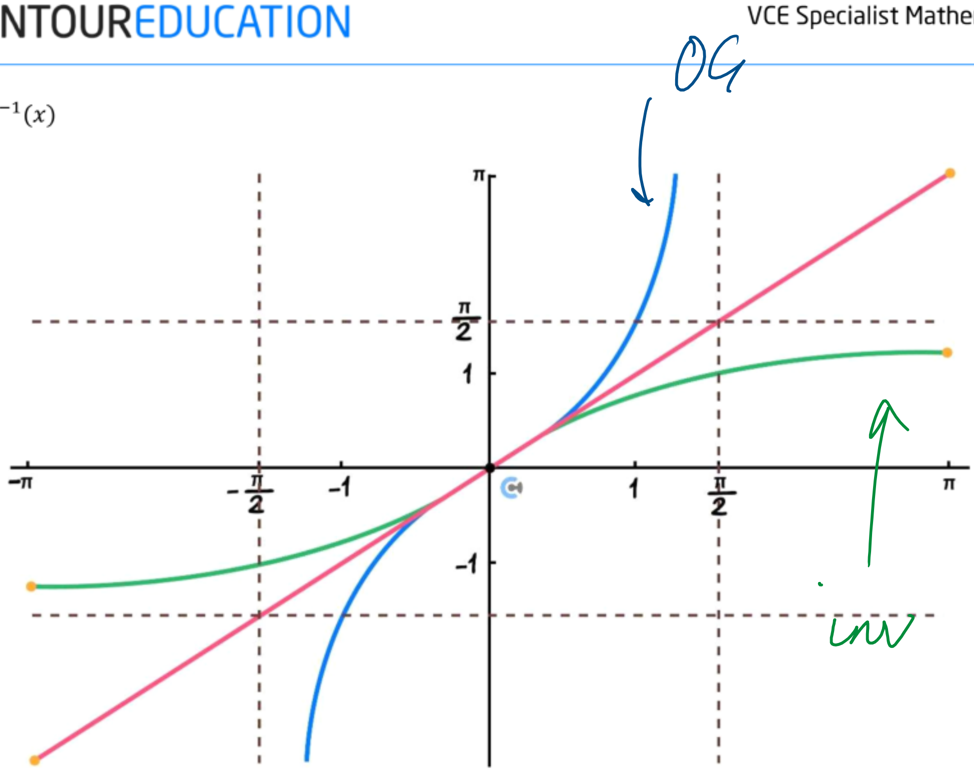
  - The domain of the arccos function = Range of $\cos = [-1,1]$.
  - The range = Domain of restricted $\cos = [0, \pi]$.

➤ $\tan^{-1}(x)$




- The domain of the arctan function = Range of $\tan = R$.
- The range = Domain of restricted $\tan = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

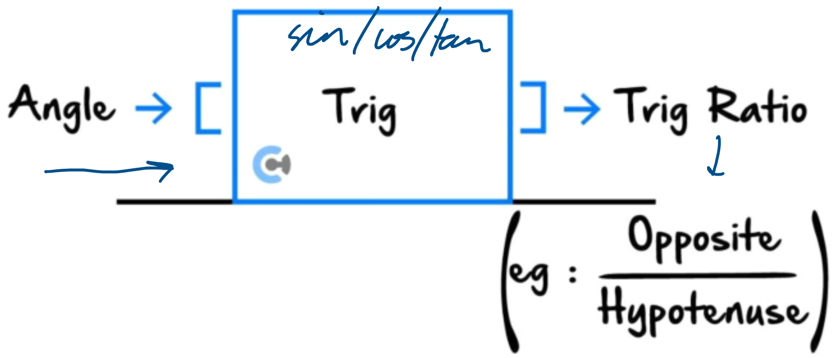
**Space for Personal Notes**

## Sub-Section: Understanding Inverse Trigonometric Functions

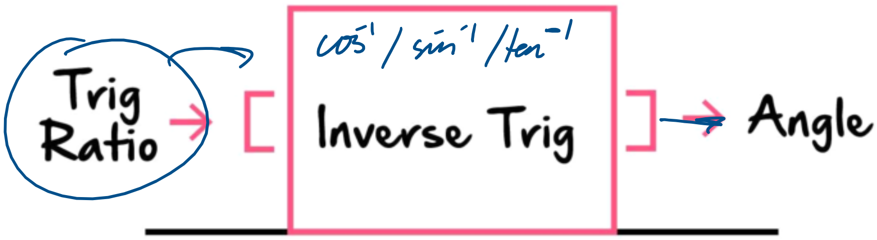
### *What do Inverse Trigonometric Functions Do?*

**Exploration: Understanding Inverse Trig Functions**

- We can consider the normal trigonometric function to be the following:




- Hence, inverse trigonometric functions can be visualised to perform the following:




- In summary, inverse trig functions have:
  - $x$-value: trig ratio $= y$-value original trig functions.
  - $y$-value: Angle $= x$-value of original trig functions.

**Space for Personal Notes**

**Question 17**

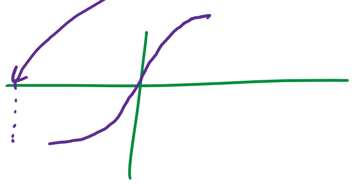
Evaluate the following, or explain why they are undefined.

**a.** $\arcsin\left(-\frac{1}{2}\right)$

$\sin^{-1}$

$= -\frac{\pi}{6}$

$\cancel{-\frac{5\pi}{6}}$



**b.** $\arccos\left(\frac{3}{\sqrt{3}}\right)$

$\cos^{-1}$

$\arccos(\sqrt{3})$

$\sqrt{3} > 1 \rightarrow$ Undefined

**c.** $\arctan\left(\frac{1}{\sqrt{3}}\right)$

$\tan^{-1}$

$\frac{\pi}{6}$, $\cancel{\frac{7\pi}{6}}$

out of domain

**NOTE:** Inverse functions are **angles.**

**NOTE:** Consider the range of the inverse trig functions!

## Sub-Section: Graphs of Inverse Trigonometric Functions

### *How do we Sketch Inverse Trigonometric Functions with Transformations?*

**Steps for Graphing General Arcsin and Arccos**

- Find the implied domain of the function.
  - Restrict inside to be within [−1, 1].
- Find and plot the endpoints of the graph by substituting ends of the domain.
- Find and plot the midpoint of the ends. (It is an inflection point.)
- Find and plot the axis-intercepts if required.
- Using the previously plotted points as a guide, sketch a cubic shape.

**Question 18 Walkthrough.**

Following the steps above, sketch:

$$y = \frac{1}{2}\arccos(1-x) - \frac{\pi}{4}$$

$1 - x \in [-1, 1]$

$-x \in [-2, 0]$ ($\times -1$)

$x \in [0, 2]$

$x = 0$

$y = \frac{1}{2}\arccos(1) - \frac{\pi}{4}$

$= 0 - \frac{\pi}{4}$

$= -\frac{\pi}{4}$

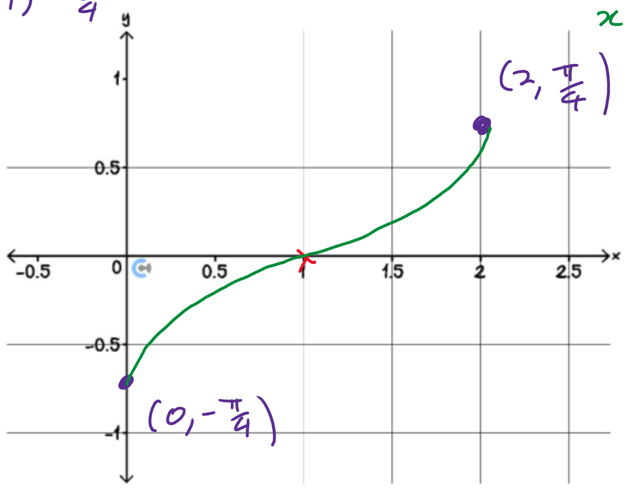
$x = 2$

$y = \frac{1}{2}\arccos(-1) - \frac{\pi}{4}$

$= \frac{\pi}{2} - \frac{\pi}{4}$

$= \frac{\pi}{4}$

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$



**Question 19**

Following the steps above, sketch:

$$y = \frac{\pi}{4} - 2\arcsin x$$

$x \in [-1, 1]$

$x = -1,$

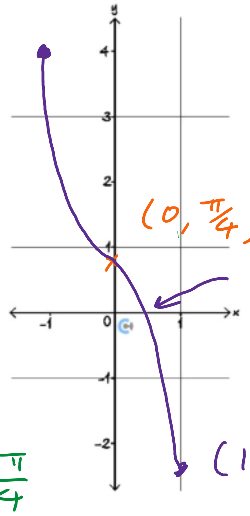
$y = \frac{\pi}{4} - 2\arcsin(-1)$

$= \frac{\pi}{4} - 2\left(-\frac{\pi}{2}\right) = \frac{5\pi}{4}$

$x = 1,$

$y = \frac{\pi}{4} - 2\arcsin(1)$

$= \frac{\pi}{4} - 2\left(\frac{\pi}{2}\right) = -\frac{3\pi}{4}$




## *Now* arctan *Functions!*

**Steps for Graphing General Graphs of arctan**

- Find the <u>horizontal asymptote</u> of the graph and plot them.
  - You can find the asymptotes by finding the **range** of the arctan function.
  - E.g., the range of $\arctan(x) + \pi$ is $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$, so the asymptotes are $y = \frac{\pi}{2}$ and $y = \frac{3\pi}{2}$.
- Inflection point is given by $(h, k)$.
  - The $x$-value can be found by <u>inside $= 0$</u>.
  - The $y$-value can be found by <u>middle of asymptotes</u>.
- Find and plot the <u>axis - intercepts</u> if required.
- Using the previously plotted points and asymptotes as a guide, sketch the function.

**Question 20 Walkthrough.**

Following the steps above, sketch:

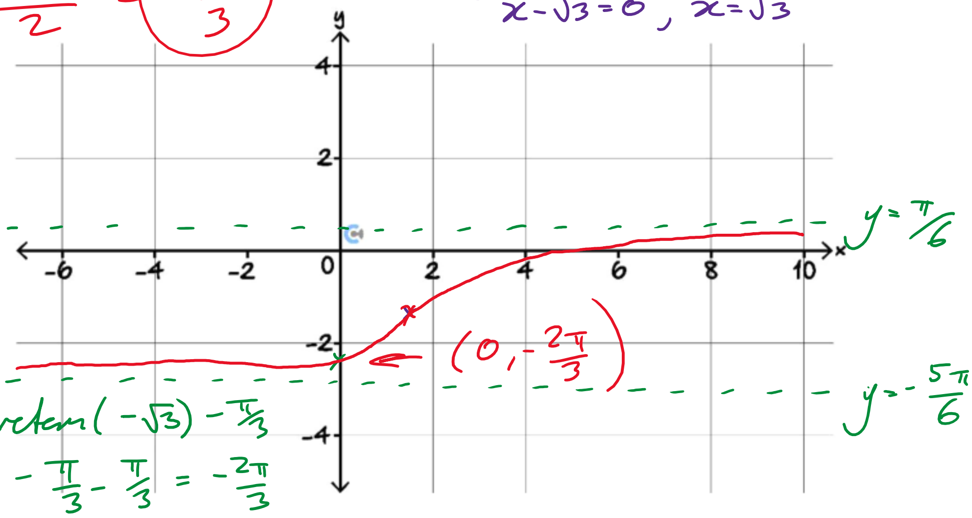
$$y = \arctan(x - \sqrt{3}) - \frac{\pi}{3}$$

$y \in \left(-\frac{\pi}{2} - \frac{\pi}{3}, \frac{\pi}{2} - \frac{\pi}{3}\right)$

$\left(-\frac{5\pi}{6}, \frac{\pi}{6}\right)$

$x - \sqrt{3} = 0, \ x = \sqrt{3}$

$\dfrac{\frac{\pi}{6} - \frac{5\pi}{6}}{2} = y = -\frac{\pi}{3}$




$y = \arctan(-\sqrt{3}) - \frac{\pi}{3}$

$= -\frac{\pi}{3} - \frac{\pi}{3} = -\frac{2\pi}{3}$

**NOTE:** $\tan^{-1}(2) > \tan^{-1}(1) = \frac{\pi}{4}$.

**Question 21** (3 marks)

Consider the function $y = \arctan(2(x-1)) + \frac{\pi}{2}$. Sketch its graph.

$2(x-1) = 0$

$x = 1$

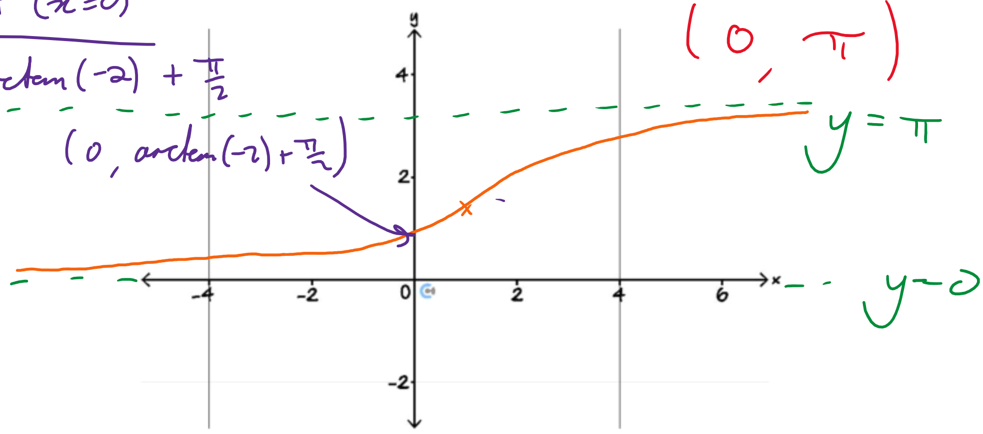
$y \in \left(-\frac{\pi}{2} + \frac{\pi}{2}, \frac{\pi}{2} + \frac{\pi}{2}\right)$

$(0, \pi)$

y-int $(x = 0)$

$y = \arctan(-2) + \frac{\pi}{2}$

$\left(0, \arctan(-2) + \frac{\pi}{2}\right)$



# Contour Check

## Learning Objective: [3.4.1] – Trigonometric Identities and Solving Exact Values of Reciprocal Functions

**Key Takeaways**

$$\sin^2(\theta) + \cos^2(\theta) = \underline{\ 1\ }$$

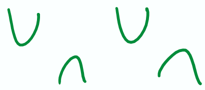
$$1 + \tan^2(\theta) = \underline{\ \sec^2\theta\ }$$

$$1 + \cot^2(\theta) = \underline{\ \operatorname{cosec}^2\theta\ }$$

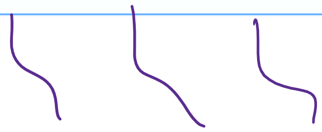
## Learning Objective: [3.4.2] – Graph Reciprocal Trigonometric Functions

**Key Takeaways**

- If the inside of a reciprocal trig function is $nx$, it has asymptotes every $\underline{\ \frac{\pi}{n}\ }$ units.
- cosec and sec ~~have the same~~ [shape] / [flip] after every asymptote.



- cot has the [same shape] / [flips] after every asymptote.



## Learning Objective: [3.4.3] – Apply compound and Double Angle Formula to Solve Exact Values

### Key Takeaways

- To find $\cos\left(\frac{\pi}{8}\right)$, it is more appropriate to use the [double] / [compound] angle formula.
- To find $\sin\left(\frac{7\pi}{12}\right)$, it is more appropriate to use the [double] / [compound] angle formula.

- **Sin double angle formula:**

$$\sin(2x) = 2\sin(x)\cos(x)$$

- **Cos double angle formula:**

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$= 2\cos^2(x) - 1$$
$$= 1 - 2\sin^2(x)$$

- **Tan double angle formula:**

$$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$$

## Learning Objective: [3.4.4] – Find Domain, Range and Rule of the Inverse Trigonometric Function

**Key Takeaways**

- $\cos^{-1}(x)$ and $\sin^{-1}(x)$ have the domain [-1, 1].
- $\tan^{-1}(x)$ has the domain $\mathbb{R}$.
- $\tan^{-1}(x)$ has an inflection point at (0, 0).
- Inverse trig functions output an angle.
- To get the inverse of a trig function, we restrict the function's domain so that it is 1-to-1.

## Learning Objective: [3.4.5] – Graphing the Inverse Trigonometric Functions

**Key Takeaways**

- $\tan^{-1}(x)$ has [horizontal] / [vertical] asymptotes at $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.
- An untransformed $\sin^{-1}$ graph has a point of inflection at (0, 0).