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VCE Specialist Mathematics ½
Advanced Trigonometric Functions [3.4]
Homework Solutions

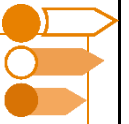
Admin Info & Homework Outline:



Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2- Pg 18
Supplementary Questions	Pg 19- Pg 34

Section A: Compulsory Questions

Sub-Section [3.4.1]: Trigonometric Identities and Solving Exact Values of Reciprocal Functions



Question 1



Evaluate the following:

a. $\operatorname{cosec}\left(\frac{\pi}{4}\right)$

$$\sqrt{2}$$

b. $\sec\left(\frac{\pi}{6}\right)$

$$\frac{2}{\sqrt{3}}$$

c. $\cot\left(\frac{3\pi}{4}\right)$

$$-1$$

Question 2



Evaluate the following:

a. $\operatorname{cosec}\left(\frac{15\pi}{4}\right)$

$$-\sqrt{2}$$

b. $\sec\left(-\frac{7\pi}{6}\right)$

$$-\frac{2}{\sqrt{3}}$$

c. $\cot\left(\frac{7\pi}{3}\right)$

$$\frac{1}{\sqrt{3}}$$

Question 3



- a. If $\cos(x) = \frac{2}{3}$ and x is not in the first quadrant, find in simplest surd form, the value of:

$$\frac{\cos(x) - 2 \cot(x)}{\tan(x) - 3 \sin(x)}$$

4th quadrant angle. Then $\sin(x) = -\frac{\sqrt{5}}{3}$ and $\tan(x) = -\frac{\sqrt{5}}{2}$ and $\cot(x) = -\frac{2}{\sqrt{5}}$. So

$$\begin{aligned} \frac{\cos(x) - 2 \cot(x)}{\tan(x) - 3 \sin(x)} &= \frac{\frac{2}{3} + \frac{4}{\sqrt{5}}}{-\frac{\sqrt{5}}{2} + \sqrt{5}} \\ &= \frac{2}{3} \times \frac{2}{\sqrt{5}} + \frac{4}{\sqrt{5}} \times \frac{2}{\sqrt{5}} \\ &= \frac{4}{3\sqrt{5}} + \frac{8}{5} \\ &= \frac{4}{15}(6 + \sqrt{5}) \end{aligned}$$

b. Prove the trigonometric identity. Only use the Pythagorean identity.

$$(1 - \tan(x))^2 + (1 + \tan(x))^2 = 2 \sec^2(x)$$

$$\begin{aligned} (1 - \tan(x))^2 + (1 + \tan(x))^2 &= 1 + \tan^2(x) - 2 \tan(x) + 1 + \tan^2(x) + 2 \tan(x) \\ &= 2 + 2 \tan^2(x) \\ &= \frac{2 \sin^2(x)}{\cos^2(x)} + \frac{2 \cos^2(x)(\sin^2(x) + \cos^2(x))}{\cos^2(x)} \\ &= \frac{2 \sin^2 + 2 \cos^2(x)}{\cos^2(x)} \\ &= \frac{2}{\cos^2(x)} \\ &= 2 \sec^2(x) \end{aligned}$$

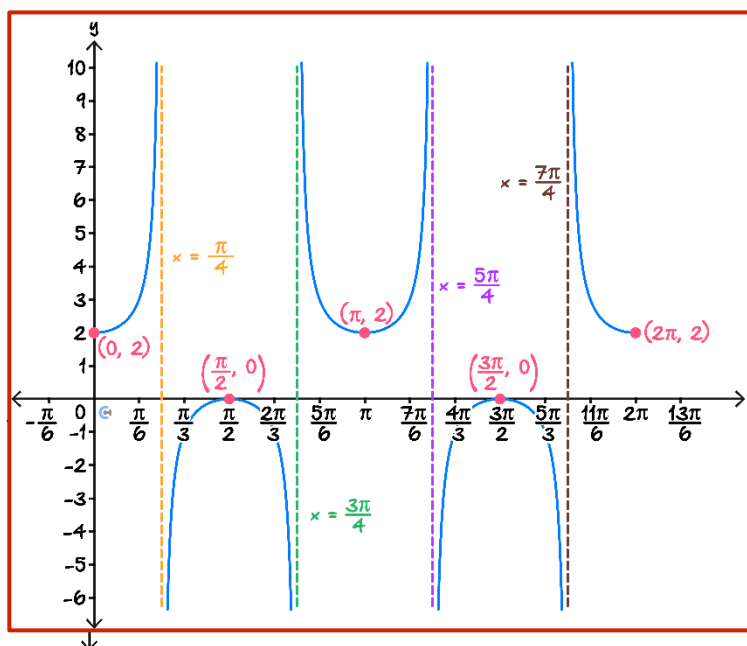
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Sub-Section [3.4.2]: Graph Reciprocal Trigonometric Functions

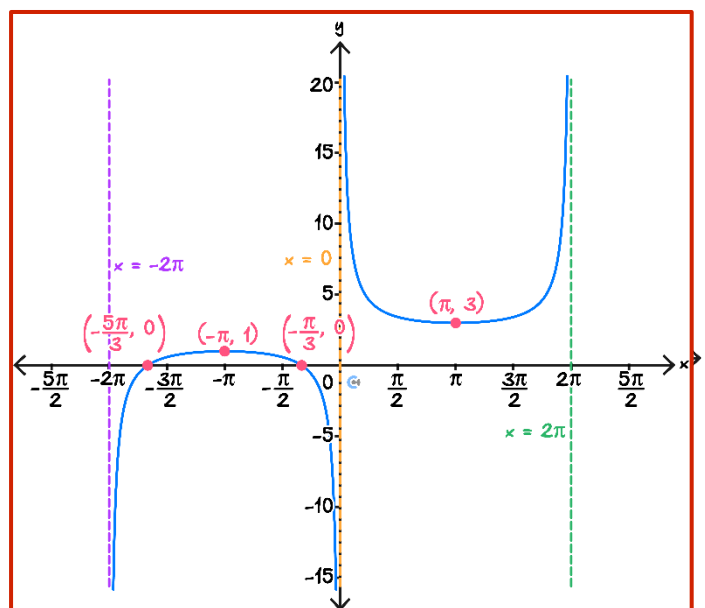
Question 4

Sketch the graphs of the following functions on the axes below. Label all axes intercepts, turning points and asymptotes.

a. $f(x) = \sec(2x) + 1$, for $x \in [0, 2\pi]$.



b. $f(x) = \operatorname{cosec}\left(\frac{x}{2} + 2\right)$, for $x \in [-2\pi, 2\pi]$.

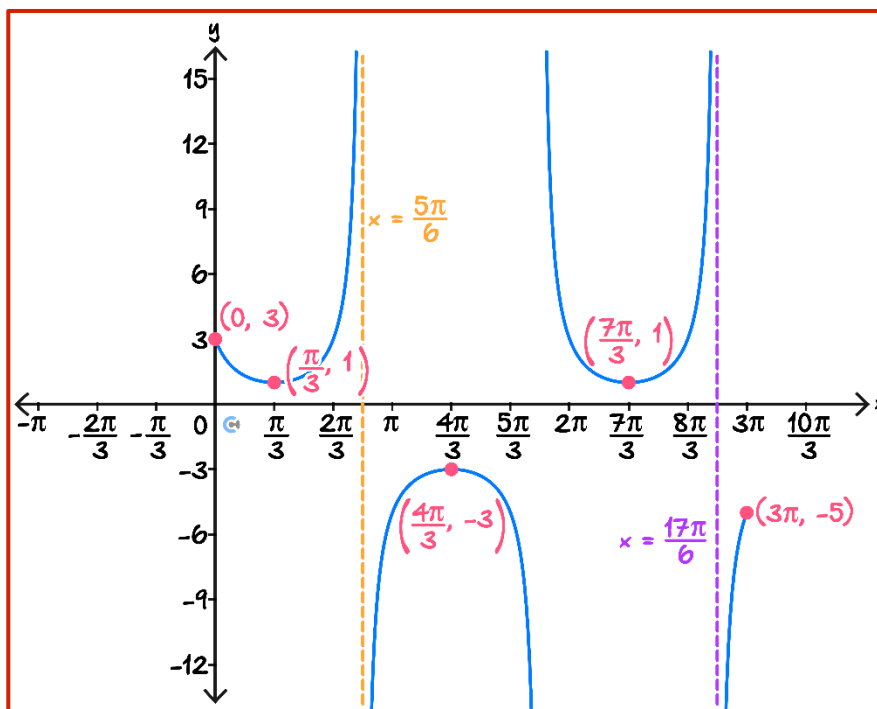




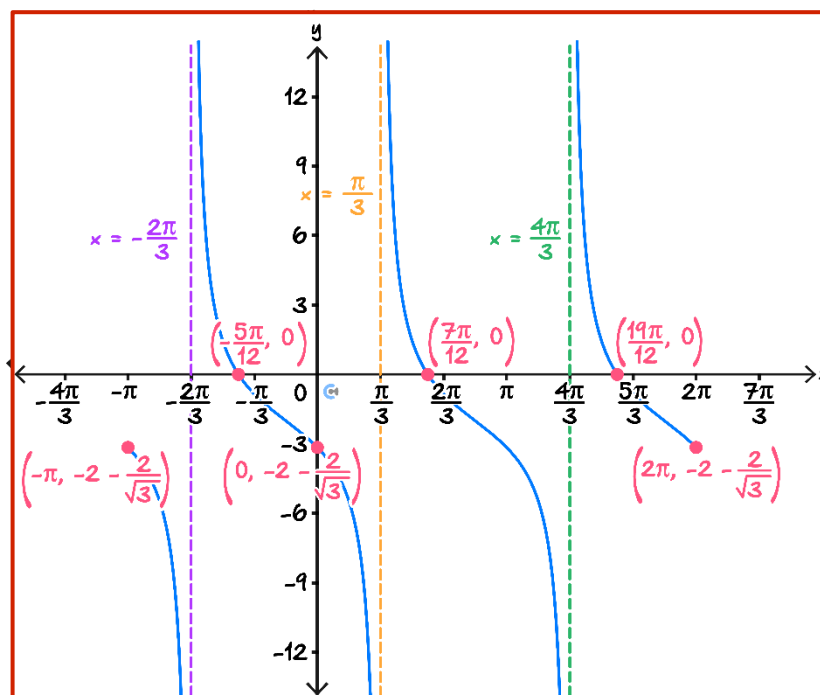
Question 5

Sketch the graphs of the following functions on the axes below. Label all axes intercepts, turning points and asymptotes.

a. $f(x) = 2 \sec\left(x - \frac{\pi}{3}\right) - 1$, for $x \in [0, 3\pi]$.



b. $f(x) = 2 \cot\left(x - \frac{\pi}{3}\right) - 2$, for $x \in [-\pi, 2\pi]$.

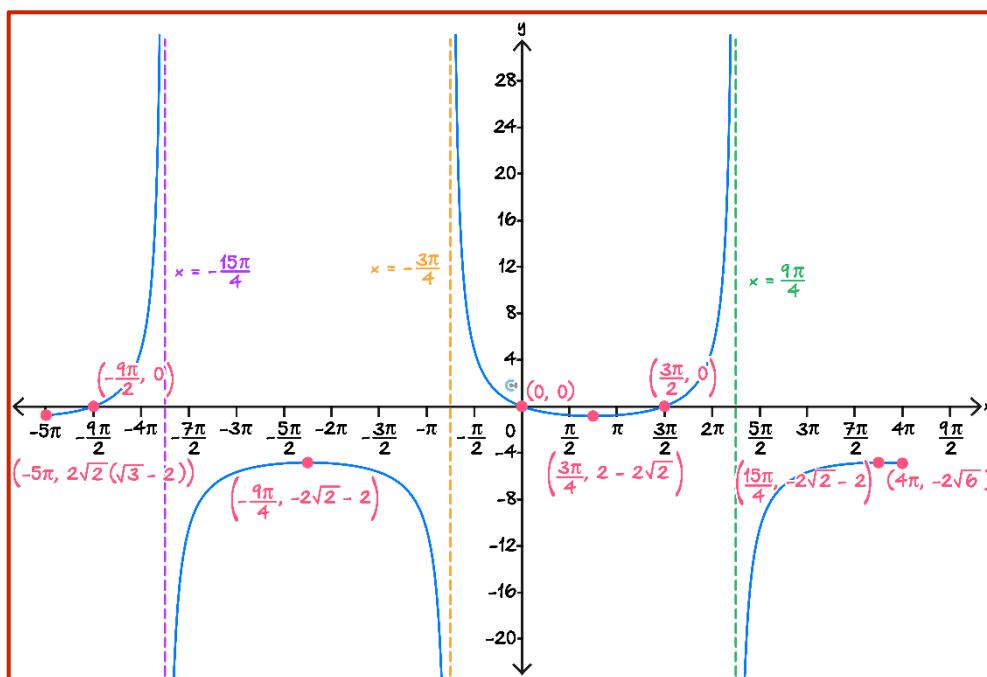




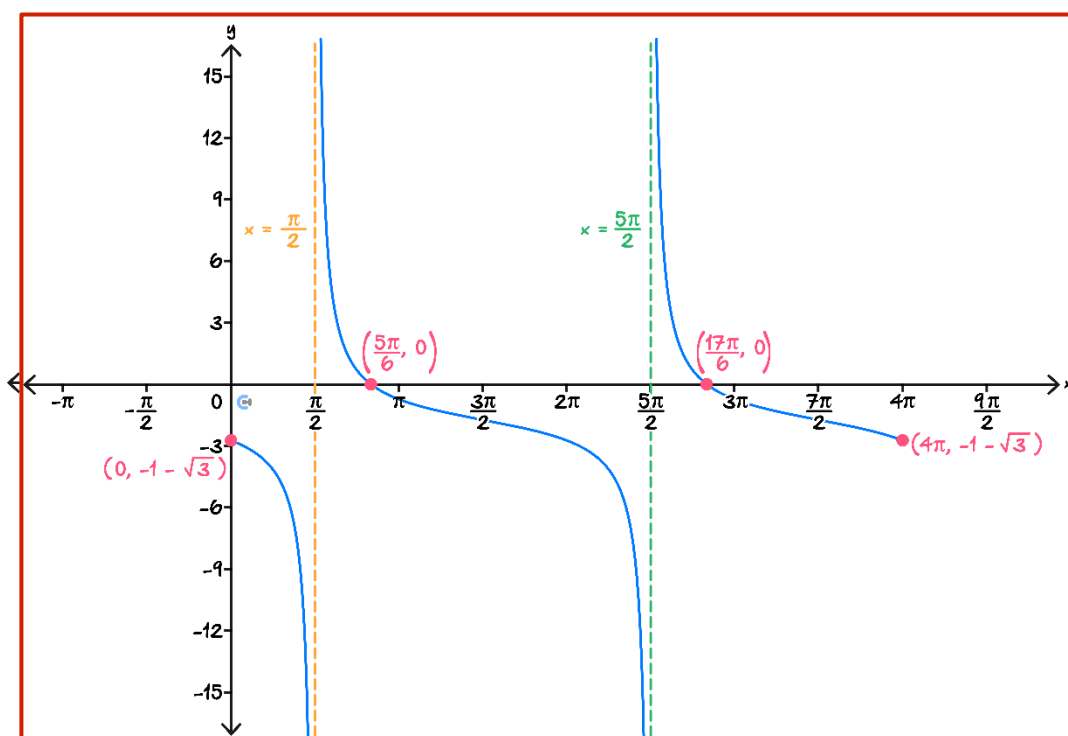
Question 6

Sketch the graphs of the following functions on the axes below. Label all axes intercepts, turning points and asymptotes.

a. $f(x) = 2 \sec\left(\frac{x}{3} - \frac{\pi}{4}\right)$, for $x \in [-5\pi, 4\pi]$.



b. $f(x) = \cot\left(\frac{x}{2} - \frac{\pi}{4}\right) - \sqrt{3}$, for $x \in [0, 4\pi]$.





Sub-Section [3.4.3]: Apply Compound and Double Angle Formula to Solve Exact Values

Question 7



If $\sin(x) = \frac{4}{5}$ and $x \in \left[0, \frac{\pi}{2}\right]$, then find the value of $\cos(2x)$.

$$\cos(2x) = 1 - 2\sin^2(x) = 1 - 2 \times \frac{16}{25} = -\frac{7}{25}$$

Question 8



Find the exact value of $\cos\left(\frac{7\pi}{12}\right)$.

$$\begin{aligned}\cos\left(\frac{7\pi}{12}\right) &= \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) \\ &= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) \\ &= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \\ &= \frac{1 - \sqrt{3}}{2\sqrt{2}}\end{aligned}$$



Question 9

Find the exact value of $\sin\left(\frac{\pi}{8}\right)$.

Let $a = \sin\left(\frac{\pi}{8}\right)$. Then

$$\cos\left(2 \times \frac{\pi}{8}\right) = 1 - 2a^2$$

$$2a^2 = 1 - \frac{\sqrt{2}}{2}$$

$$a^2 = \frac{2 - \sqrt{2}}{4}$$

$$a = \pm \frac{\sqrt{2 - \sqrt{2}}}{2}$$

But first quadrant, therefore $\sin\left(\frac{\pi}{8}\right) = \frac{\sqrt{2 - \sqrt{2}}}{2}$

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Sub-Section [3.4.4]: Find Domain, Range and Rule of the Inverse Trigonometric Function

Question 10



Suppose $f : \left[0, \frac{\pi}{2}\right] \rightarrow \mathbb{R}, f(x) = \cos(2x)$.

- a. Find the domain and range of the inverse function, f^{-1} .

$$\text{dom } f^{-1} = \text{ran } f = [-1, 1] \text{ and } \text{ran } f^{-1} = \left[0, \frac{\pi}{2}\right]$$

- b. Hence, define f^{-1} .

$$x = \cos(2y) \implies 2y = \arccos(x). \text{ Therefore}$$

$$f^{-1} : [-1, 1] \rightarrow \mathbb{R}, f^{-1}(x) = \frac{1}{2} \arccos(x)$$

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Question 11

Suppose $f : \left[0, \frac{\pi}{2}\right] \rightarrow \mathbb{R}, f(x) = 2 \sin\left(2x - \frac{\pi}{6}\right)$.

- a. Find the domain and range of the inverse function, f^{-1} .

$$\text{dom } f^{-1} = \text{ran } f = [-1, 2] \text{ and } \text{ran } f^{-1} = \left[0, \frac{\pi}{3}\right]$$

- b. Hence, define f^{-1} .

$$x = 2 \sin\left(2y - \frac{\pi}{6}\right) \implies 2y - \frac{\pi}{6} = \arcsin\left(\frac{x}{2}\right). \text{ Therefore}$$

$$f^{-1} : [-1, 2] \rightarrow \mathbb{R}, f^{-1}(x) = \frac{1}{2} \arcsin\left(\frac{x}{2}\right) + \frac{\pi}{12}$$

Question 12



Suppose $f : \left[\frac{3\pi}{4}, \pi\right] \rightarrow \mathbb{R}, f(x) = \tan\left(2x + \frac{\pi}{4}\right) + \sqrt{3}$.

- a. Find the domain and range of the inverse function, f^{-1} .

$$\text{dom } f^{-1} = \text{ran } f = [\sqrt{3} - 1, \sqrt{3} + 1] \text{ and } \text{ran } f^{-1} = \left[\frac{3\pi}{4}, \pi\right]$$

b. Hence, define f^{-1} .

$$x = \tan\left(2y + \frac{\pi}{4}\right) + \sqrt{3} \implies 2y + \frac{\pi}{4} = n\pi + \arctan(x - \sqrt{3}).$$

$$\text{Therefore } y = \frac{1}{2} \arctan(x - \sqrt{3}) + \frac{4n\pi}{8} - \frac{\pi}{8} \text{ for some } n \in \mathbb{Z}.$$

$$\text{Now must have } f^{-1}(\sqrt{3} - 1) = \frac{3\pi}{4}.$$

$$\frac{1}{2} \arctan(-1) - \frac{\pi}{8} + \frac{4n\pi}{8} = \frac{6\pi}{8}$$

$$-\frac{2\pi}{8} + \frac{4n\pi}{8} = \frac{6\pi}{8} \implies n = 2$$

Therefore,

$$f^{-1} : [\sqrt{3} - 1, \sqrt{3} + 1] \rightarrow \mathbb{R}, f^{-1}(x) = \frac{1}{2} \arctan(x - \sqrt{3}) + \frac{7\pi}{8}$$

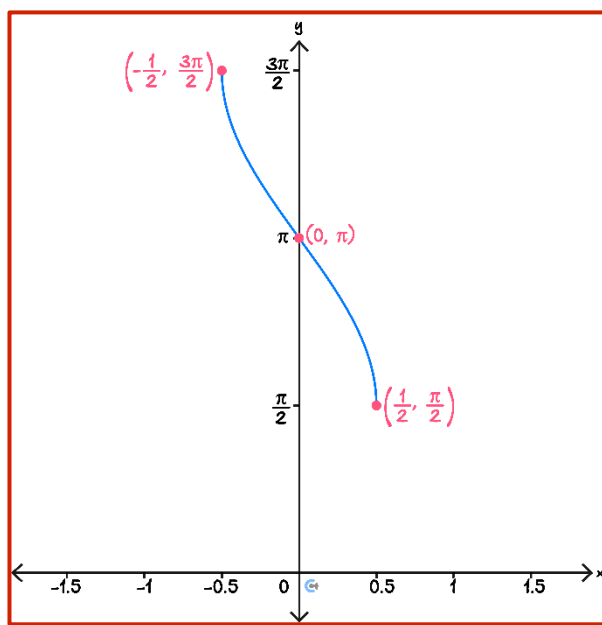
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Sub-Section [3.4.5]: Graphing Inverse Trigonometric Functions

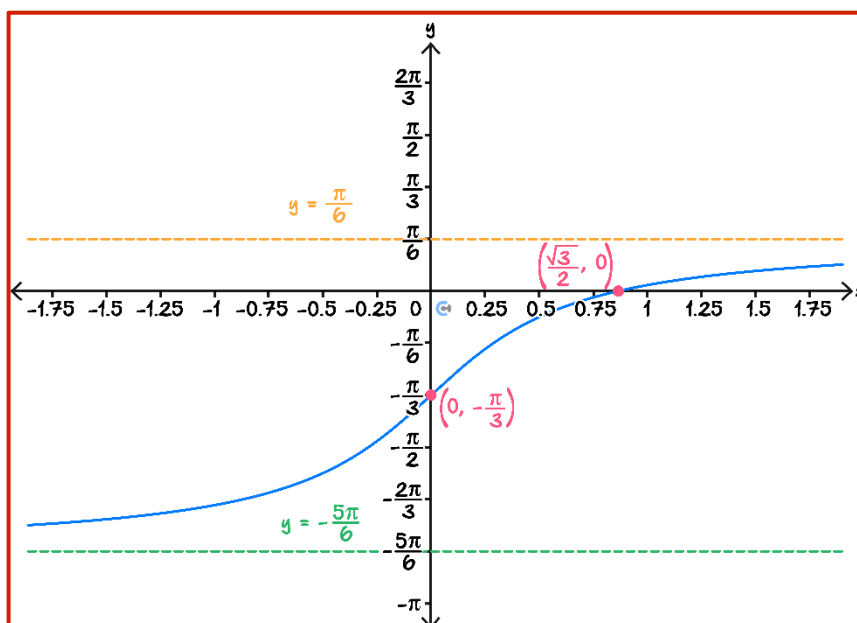
Question 13

Sketch the graphs of the following inverse trigonometric functions over their maximal domain on the axes below. Label all axes intercepts and endpoints with coordinates, and asymptotes with their equations.

a. $f(x) = \arccos(2x) + \frac{\pi}{2}$.



b. $f(x) = \arctan(2x) - \frac{\pi}{3}$.

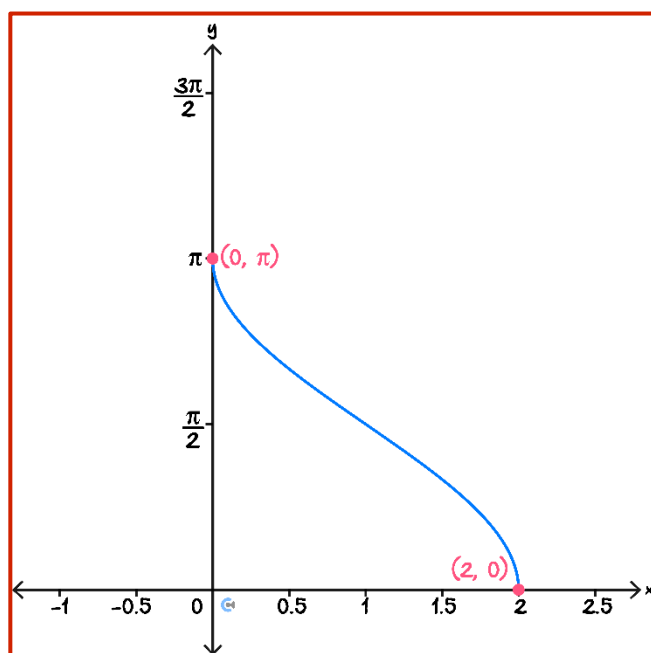




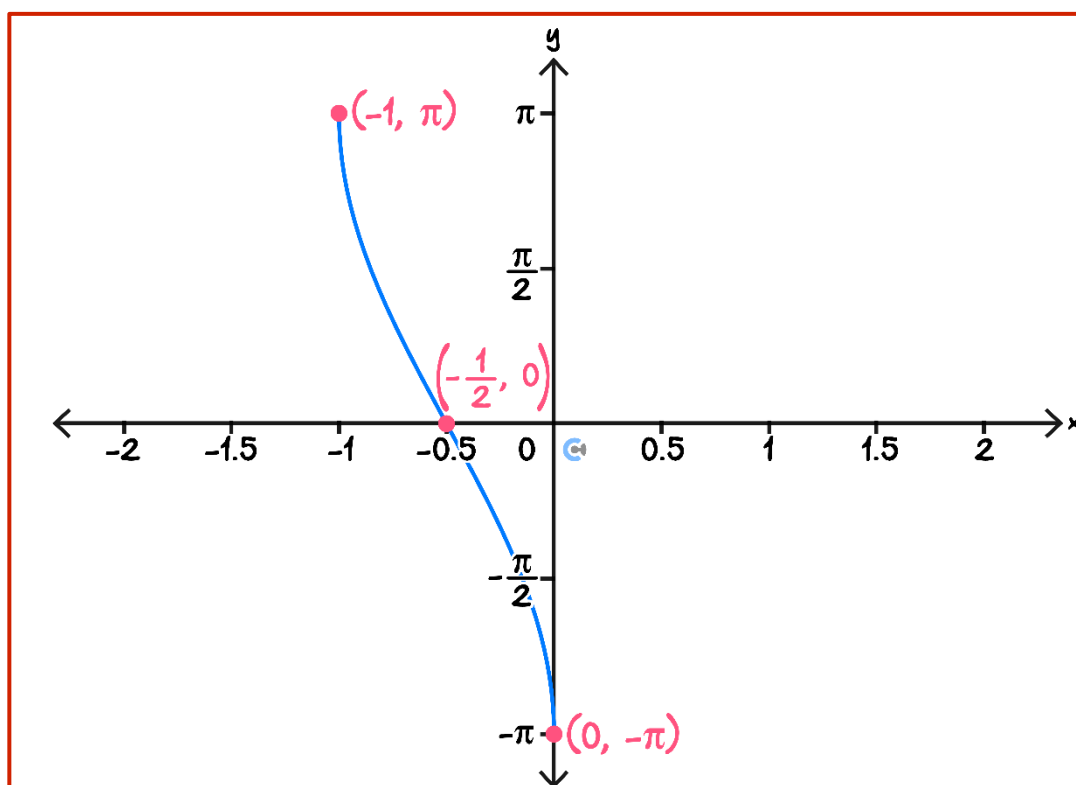
Question 14

Sketch the graphs of the following inverse trigonometric functions over their maximal domain on the axes below. Label all axes intercepts and endpoints with coordinates, and asymptotes with their equations.

a. $f(x) = -\arcsin(x - 1) + \frac{\pi}{2}$.



b. $f(x) = 2 \arccos(2x + 1) - \pi$.

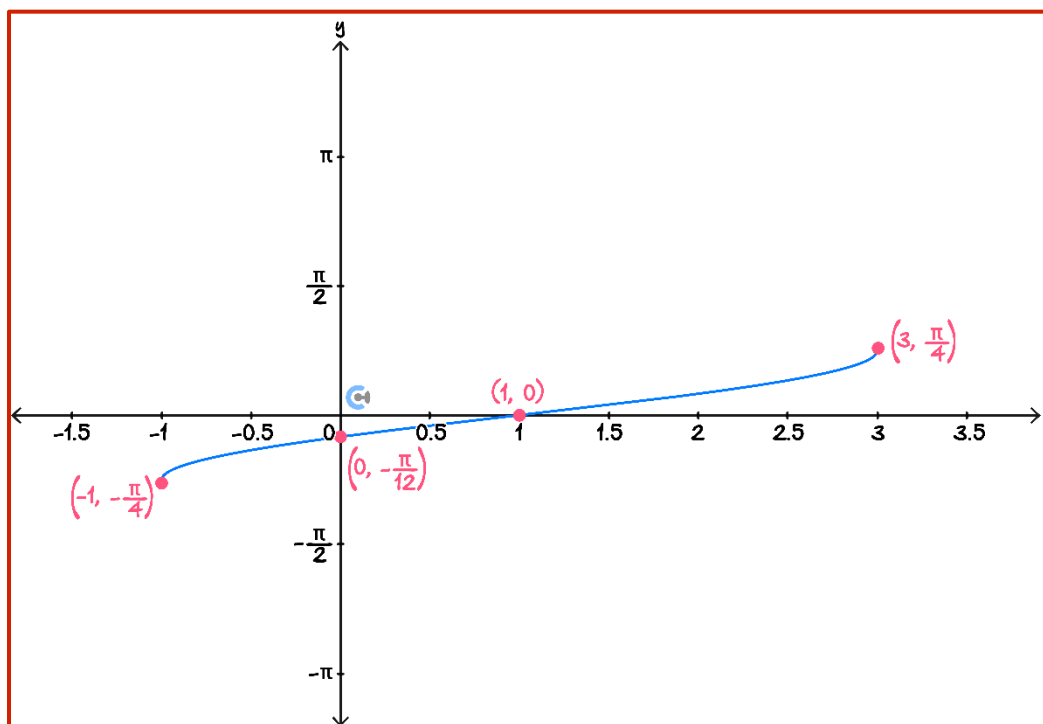




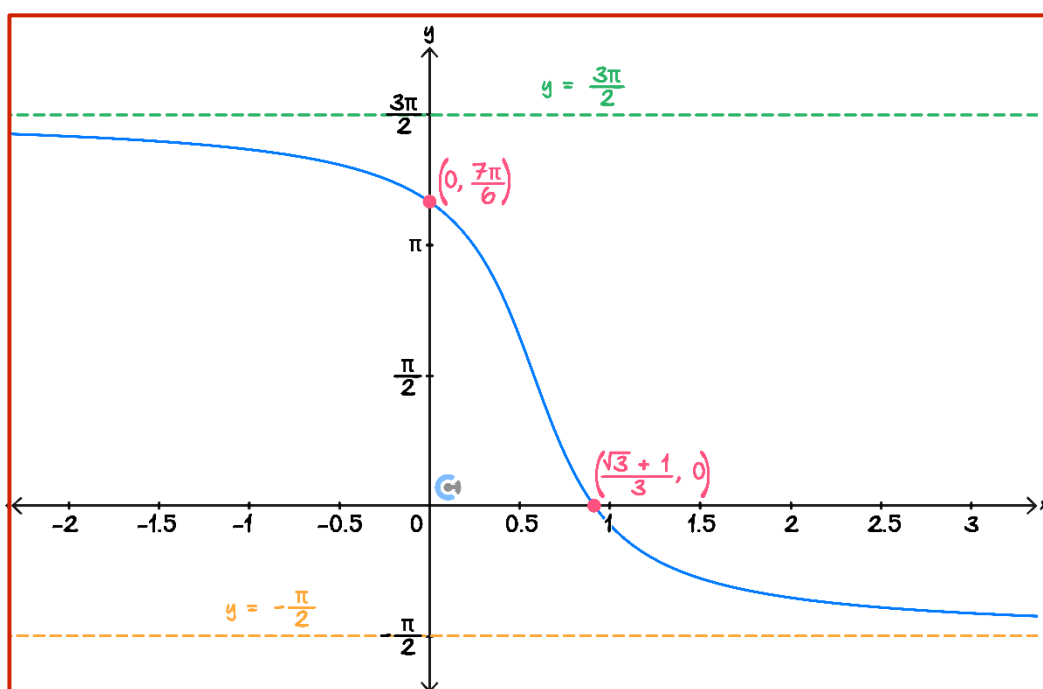
Question 15

Sketch the graphs of the following inverse trigonometric functions over their maximal domain on the axes below. Label all axes intercepts and endpoints with coordinates and asymptotes with their equations.

a. $f(x) = -\frac{1}{2}\arccos\left(\frac{x}{2} - \frac{1}{2}\right) + \frac{\pi}{4}$.



b. $f(x) = -2\arctan(3x - \sqrt{3}) + \frac{\pi}{2}$.





Sub-Section: Final Boss

Question 16

- a. Use a double-angle formula to show that $\cos\left(\frac{3\pi}{8}\right) = \frac{\sqrt{2-\sqrt{2}}}{2}$.

Let $a = \cos\left(\frac{3\pi}{8}\right)$, then we have

$$\cos\left(2 \times \frac{3\pi}{8}\right) = 2a^2 - 1$$

$$2a^2 = \frac{2}{2} - \frac{\sqrt{2}}{2}$$

$$a^2 = \frac{2 - \sqrt{2}}{4}$$

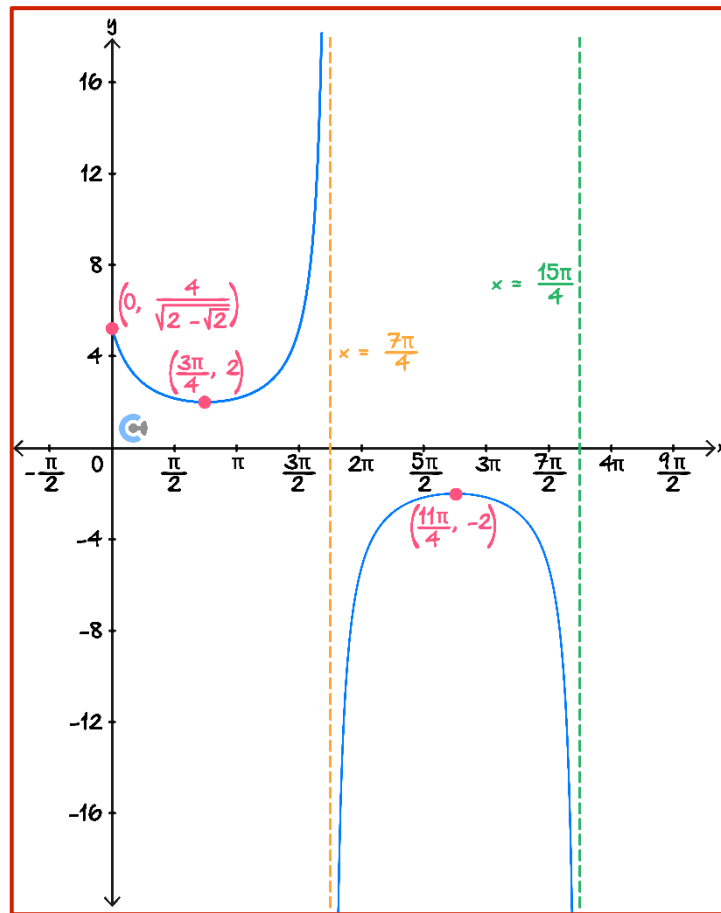
$$a = \pm \frac{\sqrt{2 - \sqrt{2}}}{2}$$

But a is in the first quadrant. Hence $\cos\left(\frac{3\pi}{8}\right) = \frac{\sqrt{2 - \sqrt{2}}}{2}$

- b. Hence, state the value of $\sec\left(\frac{3\pi}{8}\right)$.

$$\frac{4}{\sqrt{2 - \sqrt{2}}}$$

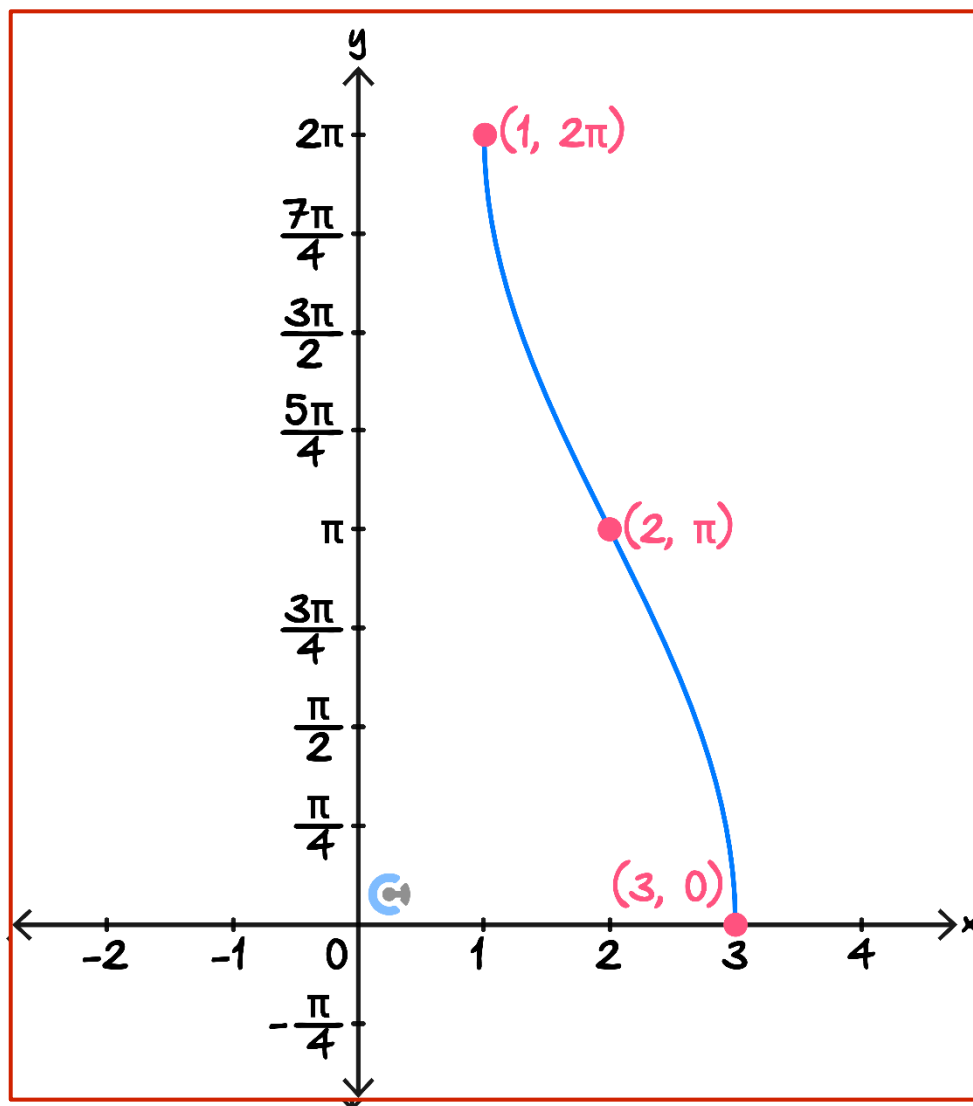
- c. Sketch the graph of $f(x) = 2 \sec\left(\frac{x}{2} - \frac{3\pi}{8}\right)$, for $x \in \left[0, \frac{15\pi}{4}\right]$. Label all axes intercepts and turning points.



- d. State the domain and range of the function $g(x) = 2\arccos(x - 2)$.

Domain = $[1, 3]$ and range = $[0, 2\pi]$

- e. Sketch the graph of $y = 2\arccos(x - 2)$ on the axes below. Label all endpoints and points of inflection with coordinates.



- f. Use the Pythagorean identity to evaluate $\sin\left(\arccos\left(\frac{1}{\sqrt{3}}\right)\right)$.

Let $a = \sin\left(\arccos\left(\frac{1}{\sqrt{3}}\right)\right)$. Then we have that

$$a^2 + \frac{1}{3} = 1$$

$$a^2 = \frac{2}{3}$$

$$a = \pm\sqrt{\frac{2}{3}}$$

But range of \arccos is $[0, \pi]$. So we have $\sin\left(\arccos\left(\frac{1}{\sqrt{3}}\right)\right) = \sqrt{\frac{2}{3}}$

Section B: Supplementary Questions

Sub-Section [3.4.1]: Trigonometric Identities and Solving Exact Values of Reciprocal Functions



Question 17



Evaluate the following:

a. $\sec\left(\frac{\pi}{4}\right)$

$$\frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$$

b. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$\frac{\pi}{6}$$

c. $\tan^{-1}(1)$

$$\frac{\pi}{4}$$

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Question 18

Evaluate the following:

a. $\cot\left(\frac{11\pi}{6}\right)$

$$\frac{1}{\tan\left(-\frac{\pi}{6}\right)} = -\sqrt{3}$$

b. $\operatorname{cosec}\left(\frac{7\pi}{3}\right)$

$$\frac{1}{\sin\left(\frac{\pi}{3}\right)} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

c. $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

$$-\frac{\pi}{6}$$

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Question 19

Prove the identity $(\cot x + \operatorname{cosec} x)^2 = \frac{1+\cos x}{1-\cos x}$.

$$\begin{aligned} (\cot x + \operatorname{cosec} x)^2 &= \left(\frac{\cos x + 1}{\sin x} \right)^2 \\ &= \frac{(\cos x + 1)^2}{\sin^2 x} \\ &= \frac{(\cos x + 1)^2}{1 - \cos^2 x} \\ &= \frac{(\cos x + 1)^2}{(1 + \cos x)(1 - \cos x)} \\ &= \frac{1 + \cos x}{1 - \cos x} \end{aligned}$$

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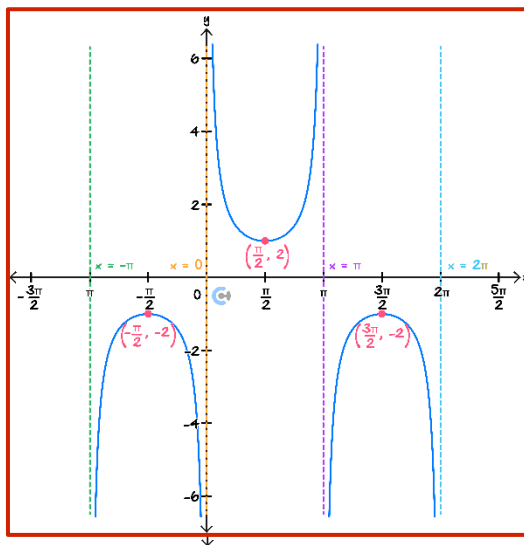
Sub-Section [3.4.2]: Graph Reciprocal Trigonometric Functions



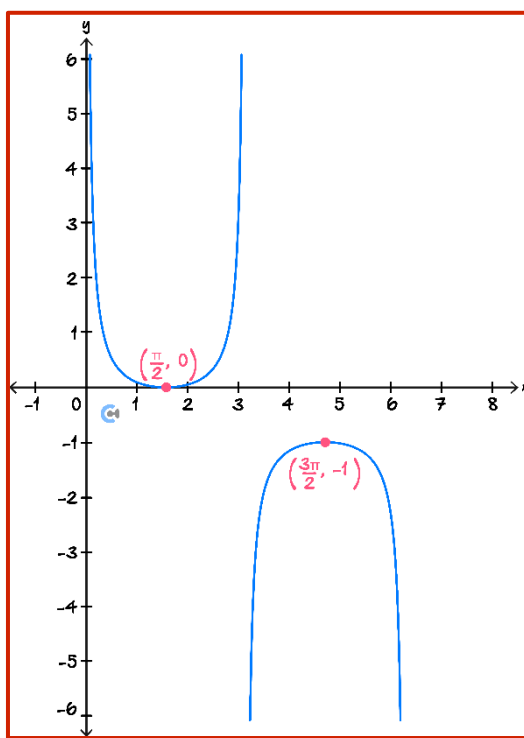
Question 20



- a. Sketch the graph of $y = 2\sec\left(x - \frac{\pi}{2}\right)$ for $-\pi < x < 2\pi$, labelling all stationary points, axes intercepts and asymptotes with their equations.



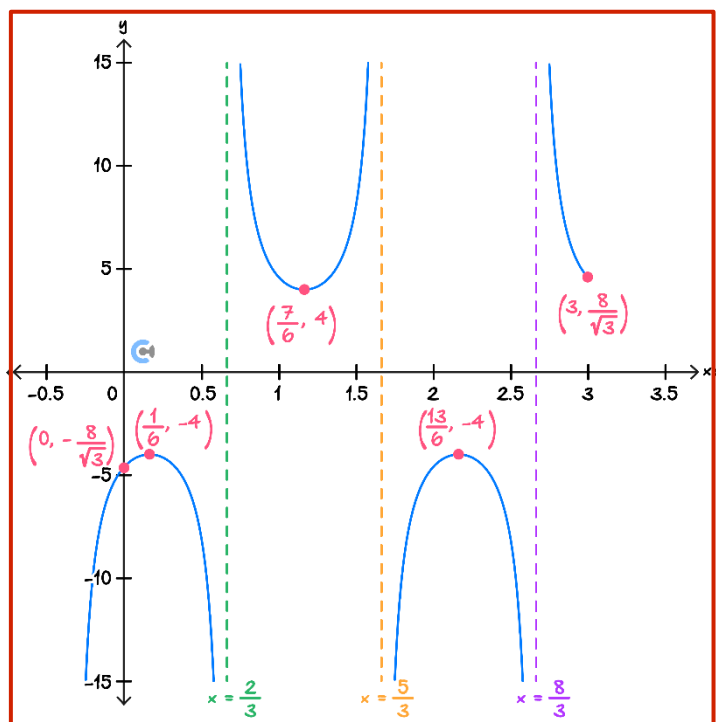
- b. Sketch the graph of $\frac{\operatorname{cosec}(x)}{2} - \frac{1}{2}$ for $0 < x < 2\pi$, labelling all stationary points, axes intercepts and asymptotes with their equations.



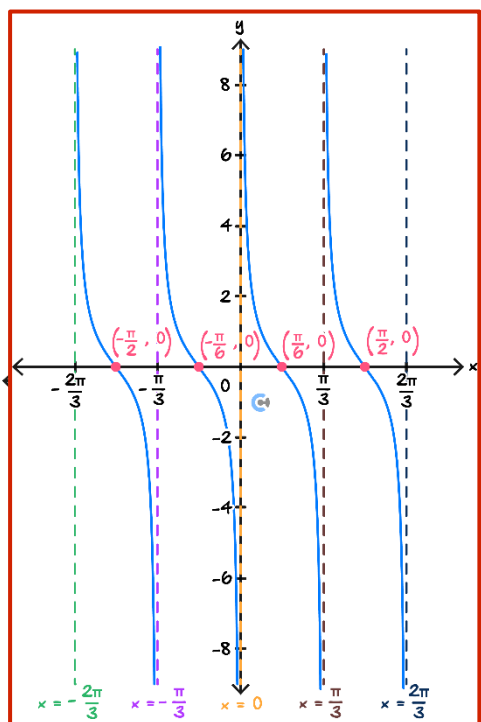


Question 21

- a. Sketch the graph of $y = 4\operatorname{cosec}\left(7\pi x - \frac{2\pi}{3}\right)$ for $-1 \leq x \leq 3$, labelling all stationary points, axes intercepts and asymptotes with their equations.



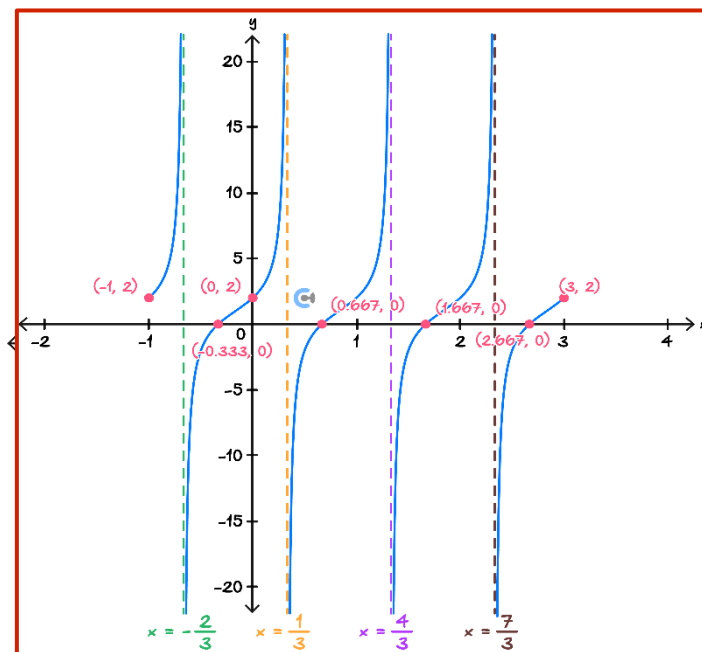
- b. Sketch the graph of $y = -\cot(\pi - 3x)$ for $-\frac{2\pi}{3} < x < \frac{2\pi}{3}$, labelling all stationary points, axes intercepts and asymptotes with their equations.



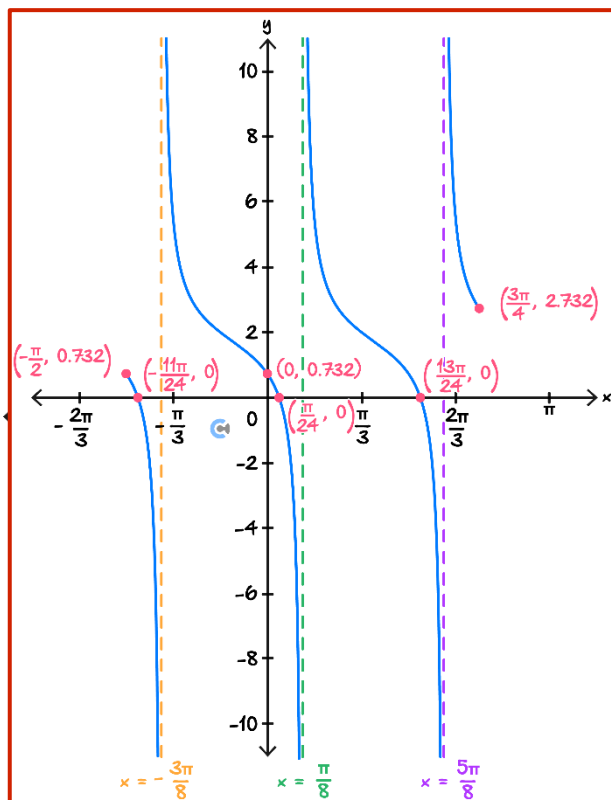


Question 22

- a. Sketch the graph of $y = 1 - \sqrt{3} \cot\left(\pi x - \frac{\pi}{3}\right)$ for $-1 \leq x \leq 3$, labelling all stationary points, axes intercepts and asymptotes with their equations.



- b. Sketch the graph of $y = \cot\left(2x - \frac{\pi}{4}\right) + \sqrt{3}$ for $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{4}$, labelling all stationary points, axes intercepts and asymptotes with their equations.



Sub-Section [3.4.3]: Apply Compound and Double Angle Formula to Solve Exact Values

Question 23

Use a compound angle formula to evaluate $\sin\left(\frac{5\pi}{12}\right)$.

$$\begin{aligned}\sin\left(\frac{5\pi}{12}\right) &= \sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right) \\ &= \sin\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right) \sin\left(\frac{\pi}{6}\right) \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{\sqrt{3} + 1}{2\sqrt{2}} \\ &= \frac{\sqrt{2} + \sqrt{6}}{4}\end{aligned}$$

Question 24

Use a double-angle formula to evaluate $\tan\left(-\frac{\pi}{8}\right)$.

Use the formula $\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$

$$\begin{aligned}\tan\left(-2 \times \frac{\pi}{8}\right) &= \frac{2 \tan\left(-\frac{\pi}{8}\right)}{1 - \tan^2\left(-\frac{\pi}{8}\right)} \\ -1 &= \frac{2 \tan\left(-\frac{\pi}{8}\right)}{1 - \tan^2\left(-\frac{\pi}{8}\right)}\end{aligned}$$

let $a = \tan\left(-\frac{\pi}{8}\right)$

$$\begin{aligned}-1 + a^2 &= 2a \\ a^2 - 2a &= 1 \\ (a - 1)^2 &= 2 \\ a - 1 &= \pm\sqrt{2} \\ a &= 1 - \sqrt{2}\end{aligned}$$

is the only solution since the solution must be < 0 because $-\frac{\pi}{8}$ is in the fourth quadrant
Therefore,

$$\tan\left(-\frac{\pi}{8}\right) = 1 - \sqrt{2}.$$

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Question 25



Use a compound angle formula to evaluate $\cos\left(\frac{19\pi}{12}\right)$.

$$\begin{aligned}\cos\left(\frac{19\pi}{12}\right) &= \cos\left(\frac{15\pi}{12}\right) + \cos\left(\frac{4\pi}{12}\right) \\ &= \cos\left(\frac{5\pi}{4}\right) \cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{5\pi}{4}\right) \sin\left(\frac{\pi}{3}\right) \\ &= -\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} \\ &= \frac{\sqrt{3} - 1}{2\sqrt{2}} \\ &= \frac{\sqrt{6} - \sqrt{2}}{4}\end{aligned}$$

Question 26



Given that $\cos(x - y) = \frac{7}{25}$ and $\cot(x)\cot(y) = \frac{4}{3}$, find $\cos(x + y)$.

From the information given we have

$$\begin{aligned}\cos(x - y) &= \cos(x) \cos(y) + \sin(x) \sin(y) = \frac{7}{25}, \\ \cos(x) \cos(y) &= \frac{4}{3} \sin(x) \sin(y).\end{aligned}$$

Therefore,

$$\begin{aligned}\frac{4}{3} \sin(x) \sin(y) + \sin(x) \sin(y) &= \frac{7}{25}, \\ \implies \sin(x) \sin(y) &= \frac{3}{25}, \\ \implies \cos(x) \cos(y) &= \frac{4}{3} \cdot \frac{3}{25} = \frac{4}{25},\end{aligned}$$

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Now we have

$$\cos(x + y) = \cos(x) \cos(y) - \sin(x) \sin(y) = \frac{4}{25} - \frac{3}{25} = \frac{1}{25}$$



Sub-Section [3.4.4]: Find Domain, Range and Rule of the Inverse Trigonometric Function

Question 27



Consider the function $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R} : f(x) = \frac{\tan(x)}{3}$.

a. State the domain of $f^{-1}(x)$.

$$\text{dom } f^{-1} = \text{ran } f = \mathbb{R}$$

b. State the range of $f^{-1}(x)$.

$$\text{ran } f^{-1} = \text{dom } f = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

c. Hence, or otherwise, find the rule of $f^{-1}(x)$.

Swap x and y .

$$x = \frac{1}{3} \tan(y) \implies y = \tan^{-1}(3x). \text{ Therefore,}$$

$$f^{-1}(x) = \tan^{-1}(3x)$$

Question 28



Consider the function $f : \left[-\frac{9\pi}{4}, \frac{3\pi}{4}\right] \rightarrow \mathbb{R} : f(x) = 2 \sin\left(\frac{x}{3} + \frac{\pi}{4}\right) - \sqrt{2}$.

a. State the domain of $f^{-1}(x)$.

$$f\left(-\frac{9\pi}{4}\right) = -2 - \sqrt{2} \text{ and } f\left(\frac{3\pi}{4}\right) = 2 - \sqrt{2}.$$

$$\text{Therefore, } \text{dom } f^{-1} = \text{ran } f = [-2 - \sqrt{2}, 2 - \sqrt{2}]$$

b. State the range of $f^{-1}(x)$.

$$\text{ran } f^{-1} = \text{dom } f = \left[-\frac{9\pi}{4}, \frac{3\pi}{4}\right]$$

- c. Hence, or otherwise, find the rule of $f^{-1}(x)$.

Swap x and y .

$$x = 2 \sin\left(\frac{y}{3} + \frac{\pi}{4}\right) - \sqrt{2} \implies \frac{y}{3} + \frac{\pi}{4} = \sin^{-1}\left(\frac{x + \sqrt{2}}{2}\right). \text{ Therefore,}$$

$$f^{-1}(x) = 3 \sin^{-1}\left(\frac{x + \sqrt{2}}{2}\right) - \frac{3\pi}{4}$$

Question 29



Consider the function $f : \left[\frac{5\pi}{3}, \frac{8\pi}{3}\right] \rightarrow \mathbb{R} : f(x) = \sqrt{5} \cos\left(x + \frac{\pi}{3}\right)$.

- a. State the domain of $f^{-1}(x)$.

$$f\left(\frac{5\pi}{3}\right) = \sqrt{5} \text{ and } f\left(\frac{8\pi}{3}\right) = -\sqrt{5}$$

$$\text{dom } f^{-1} = \text{ran } f = [-\sqrt{5}, \sqrt{5}]$$

- b. State the range of $f^{-1}(x)$.

$$\text{ran } f^{-1} = \text{dom } f = \left[\frac{5\pi}{3}, \frac{8\pi}{3}\right]$$

- c. Hence, or otherwise, find the rule of $f^{-1}(x)$.

Swap x and y .

$$x = \sqrt{5} \cos\left(y + \frac{\pi}{3}\right)$$

$$y + \frac{\pi}{3} = 2\pi + \cos^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

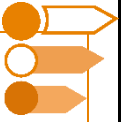
$$y = \frac{5\pi}{3} + \cos^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

where we added the period 2π so that $y \in \text{ran } f^{-1} = \left[\frac{5\pi}{3}, \frac{8\pi}{3}\right]$ Therefore,

$$f^{-1}(x) = \frac{5\pi}{3} + \cos^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Space for Personal Notes

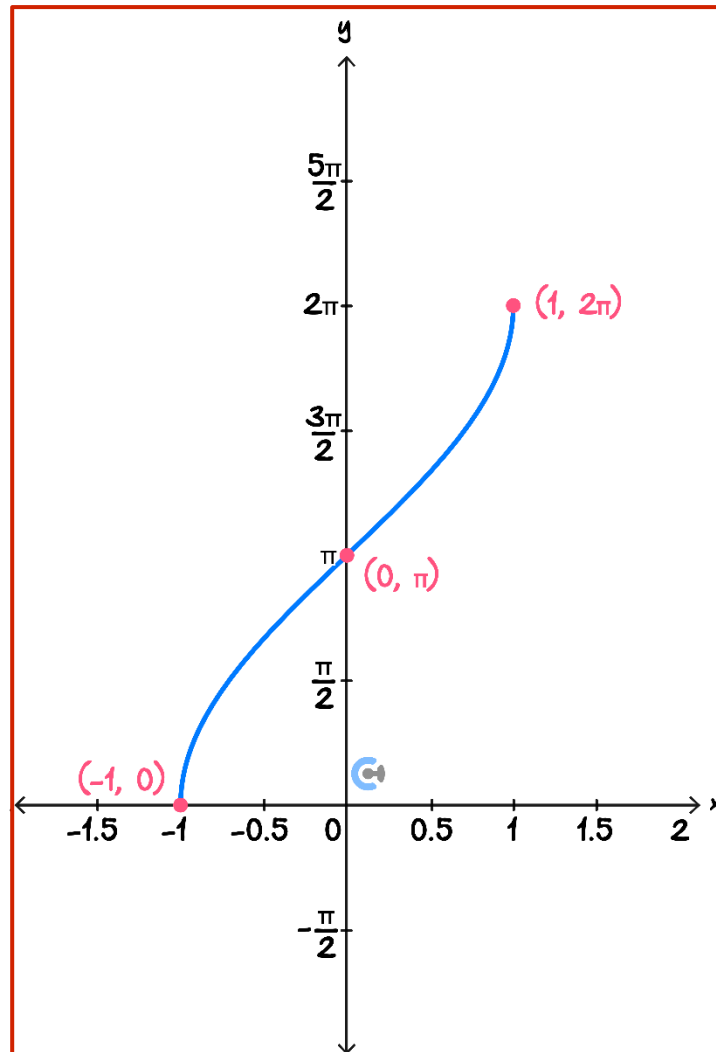
Sub-Section [3.4.5]: Graphing Inverse Trigonometric Functions



Question 30

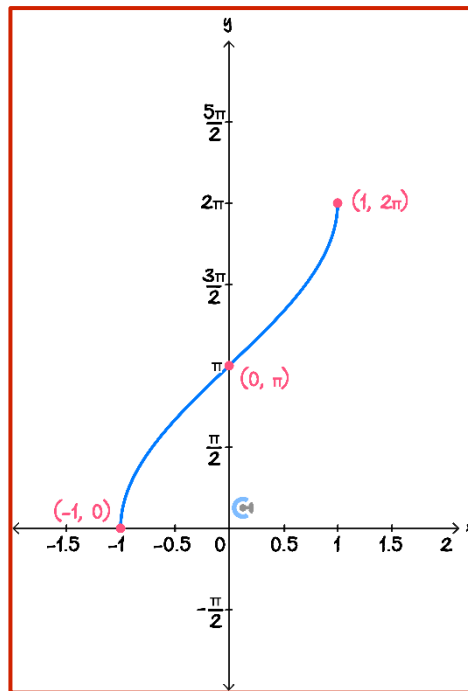


- a. Sketch the graph of $y = 2 \sin^{-1}(x) + \pi$ on the axes below. Label all endpoints and axes intercepts.



b.

i. Sketch the graph of $y = 2 \cos^{-1}(-x)$ below.



ii. What do you notice?

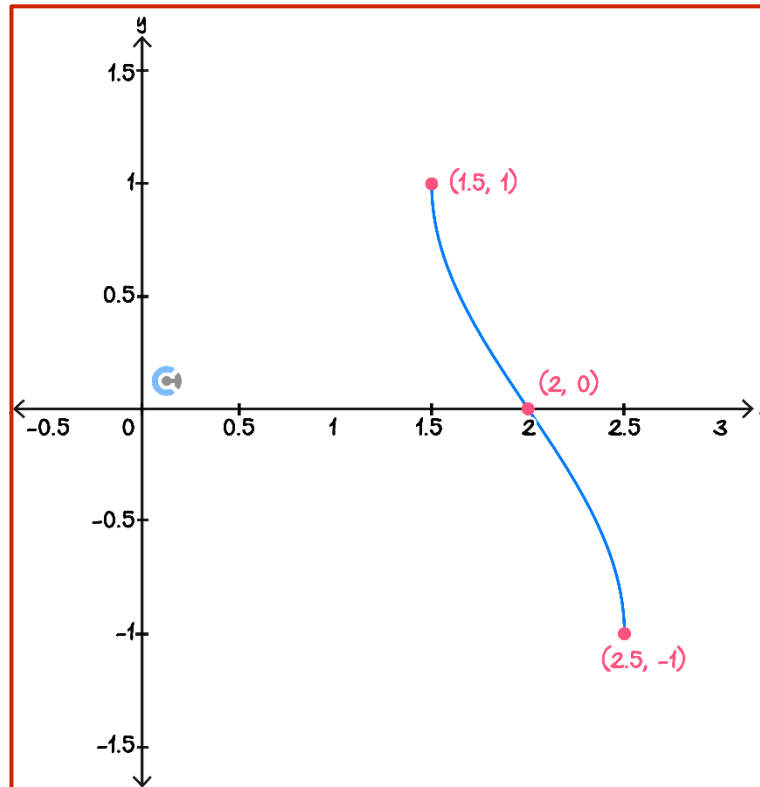
Same graph as **part a.**

Space for Personal Notes

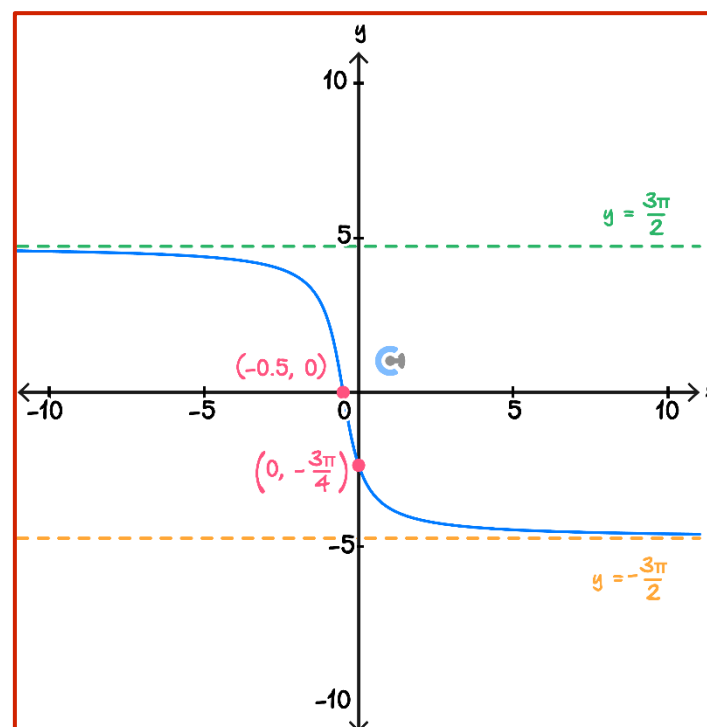


Question 31

- a. Sketch the graph of $y = -\frac{2}{\pi} \cos^{-1}(4 - 2x) + 1$ on the axes below, labelling all endpoints and axes intercepts.



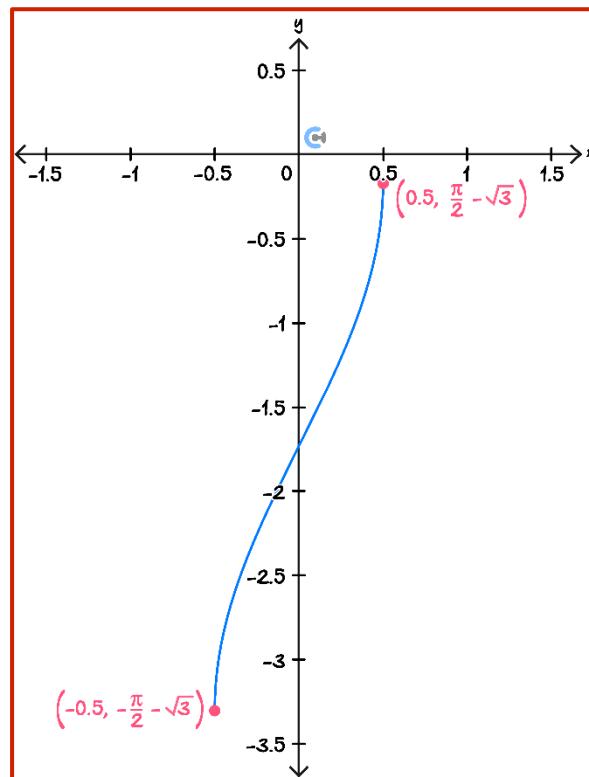
- b. Sketch the graph of $y = -3 \tan^{-1}(2x + 1)$ below, labelling all key points and asymptotes.



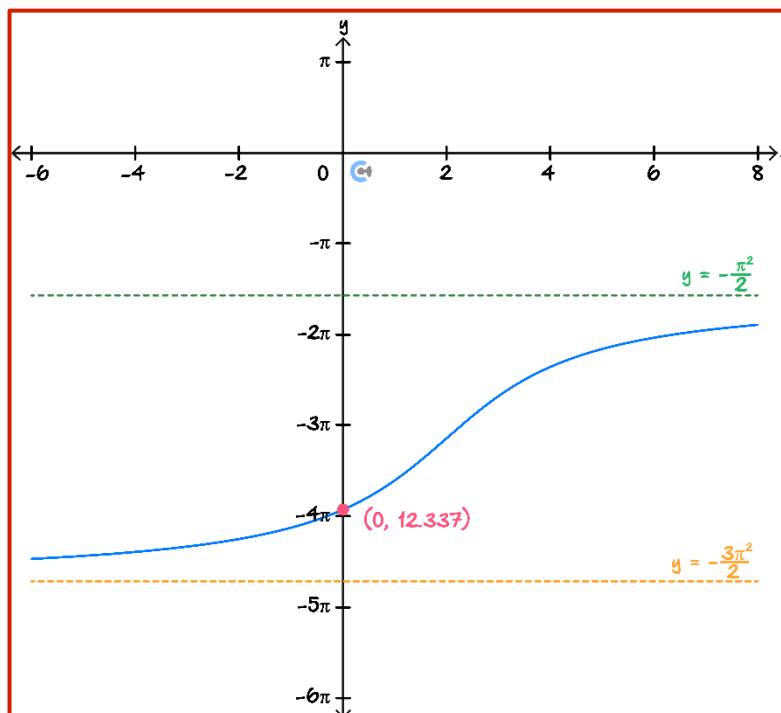


Question 32

- a. Sketch the graph of $y = \sin^{-1}(2x) - \sqrt{3}$ on the axes below. Label all endpoints.



- b. Sketch the graph of $y = \pi \tan^{-1}\left(\frac{x}{2} - 1\right) - \pi^2$ on the axes below. Label all axes intercepts and asymptotes with their equation.





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