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VCE Specialist Mathematics ½

Advanced Trigonometric Functions [3.4]

Homework

Admin Info & Homework Outline:

Student Name	
Questions You Need Help For	
Compulsory Questions	Pg 2- Pg 18
Supplementary Questions	Pg 19- Pg 34

Section A: Compulsory Questions

Sub-Section [3.4.1]: Trigonometric Identities and Solving Exact Values of Reciprocal Functions

Question 1

Evaluate the following:

a. $\text{cosec}\left(\frac{\pi}{4}\right)$

b. $\sec\left(\frac{\pi}{6}\right)$

c. $\cot\left(\frac{3\pi}{4}\right)$

Question 2

Evaluate the following:

a. $\text{cosec}\left(\frac{15\pi}{4}\right)$

b. $\sec\left(-\frac{7\pi}{6}\right)$

c. $\cot\left(\frac{7\pi}{3}\right)$

Question 3

a. If $\cos(x) = \frac{2}{3}$ and x is not in the first quadrant, find in simplest surd form, the value of:

$$\frac{\cos(x) - 2\cot(x)}{\tan(x) - 3\sin(x)}$$

b. Prove the trigonometric identity. Only use the Pythagorean identity.

$$(1 - \tan(x))^2 + (1 + \tan(x))^2 = 2\sec^2(x)$$

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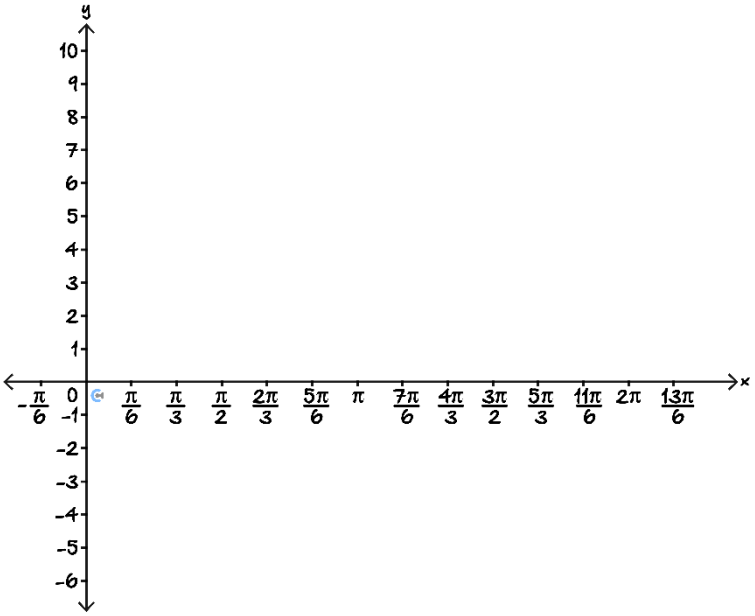
Sub-Section [3.4.2]: Graph Reciprocal Trigonometric Functions

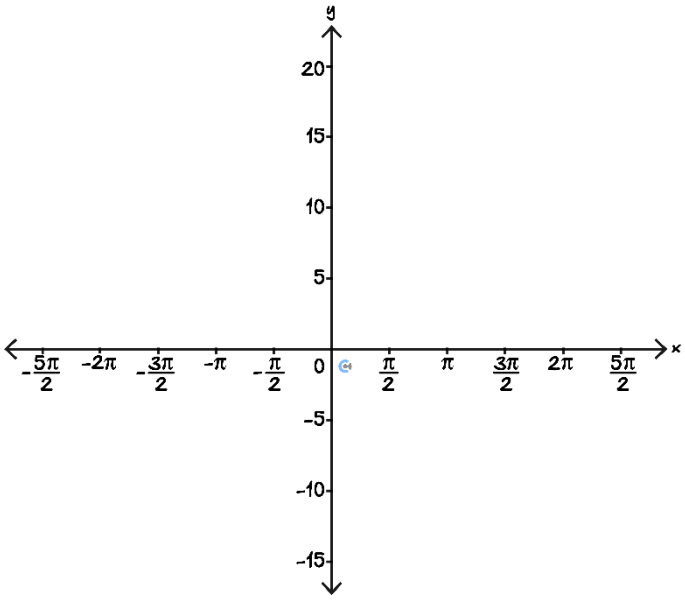
Question 4

Sketch the graphs of the following functions on the axes below. Label all axes intercepts, turning points and asymptotes.

a. $f(x) = \sec(2x) + 1$, for $x \in [0, 2\pi]$.



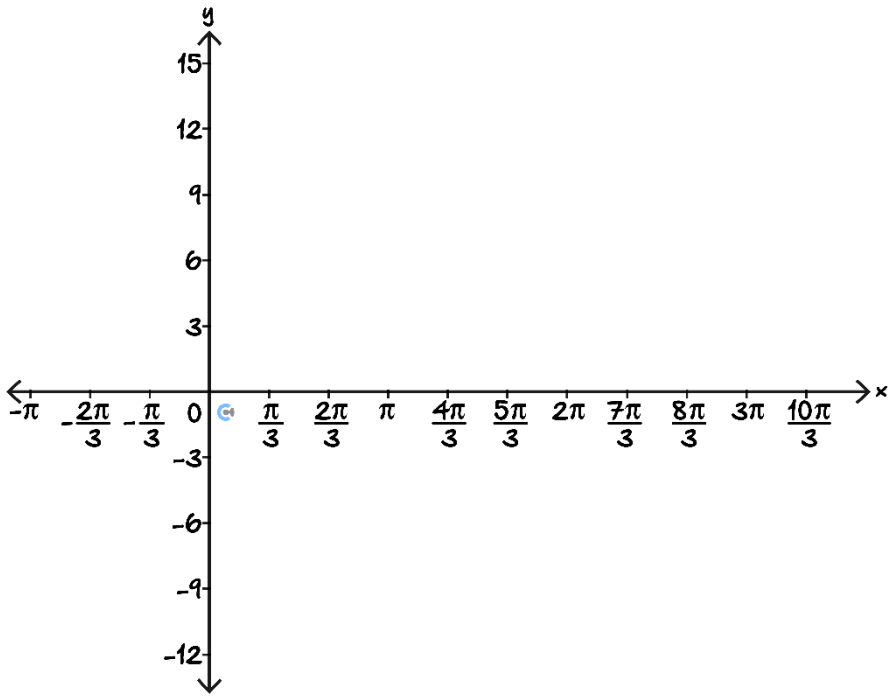
b. $f(x) = \text{cosec}\left(\frac{x}{2} + 2\right)$, for $x \in [-2\pi, 2\pi]$.



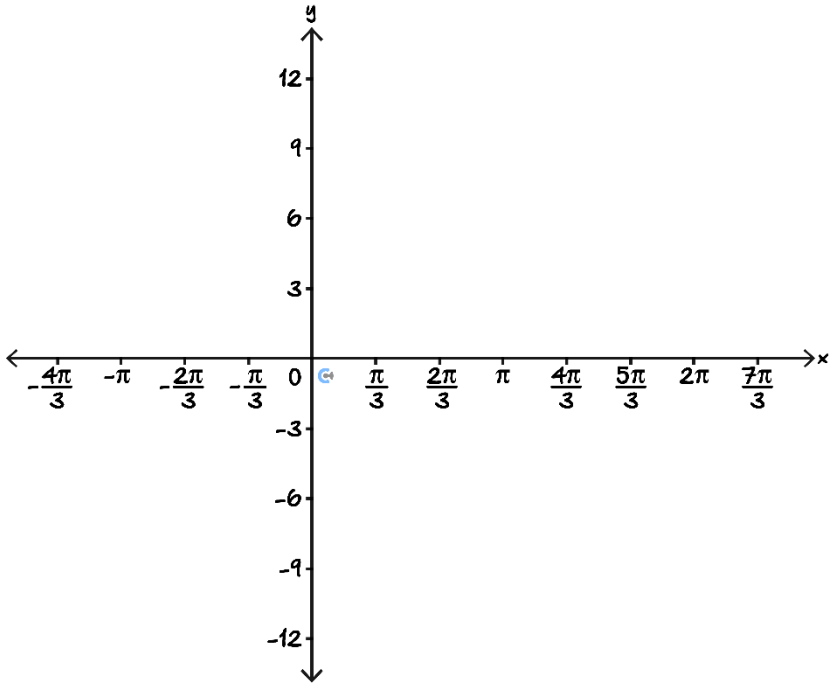
Question 5

Sketch the graphs of the following functions on the axes below. Label all axes intercepts, turning points and asymptotes.

a. $f(x) = 2\sec\left(x - \frac{\pi}{3}\right) - 1$, for $x \in [0, 3\pi]$.



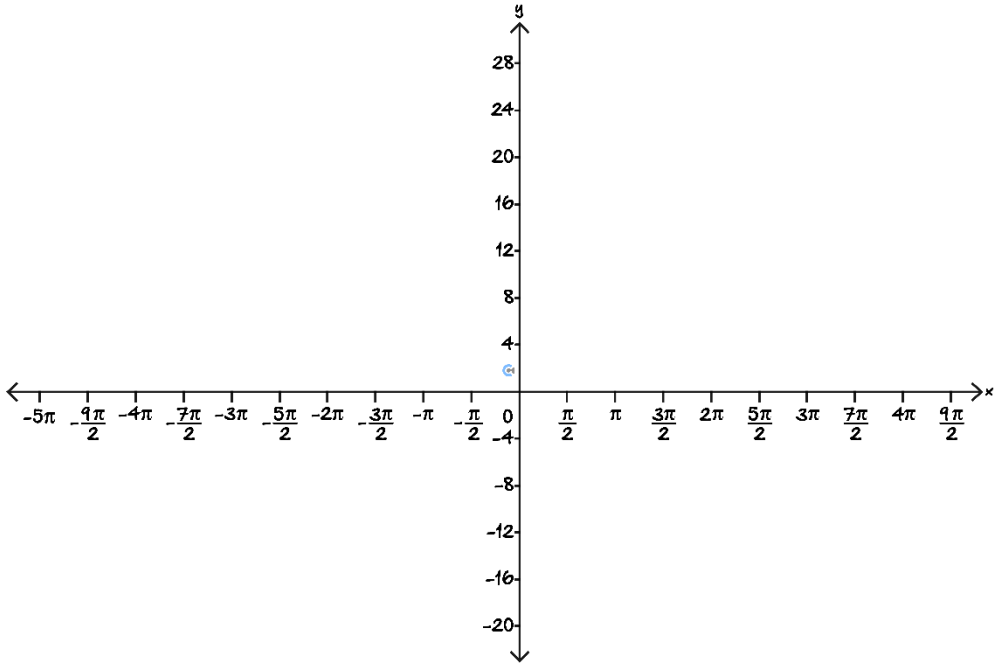
b. $f(x) = 2\cot\left(x - \frac{\pi}{3}\right) - 2$, for $x \in [-\pi, 2\pi]$.



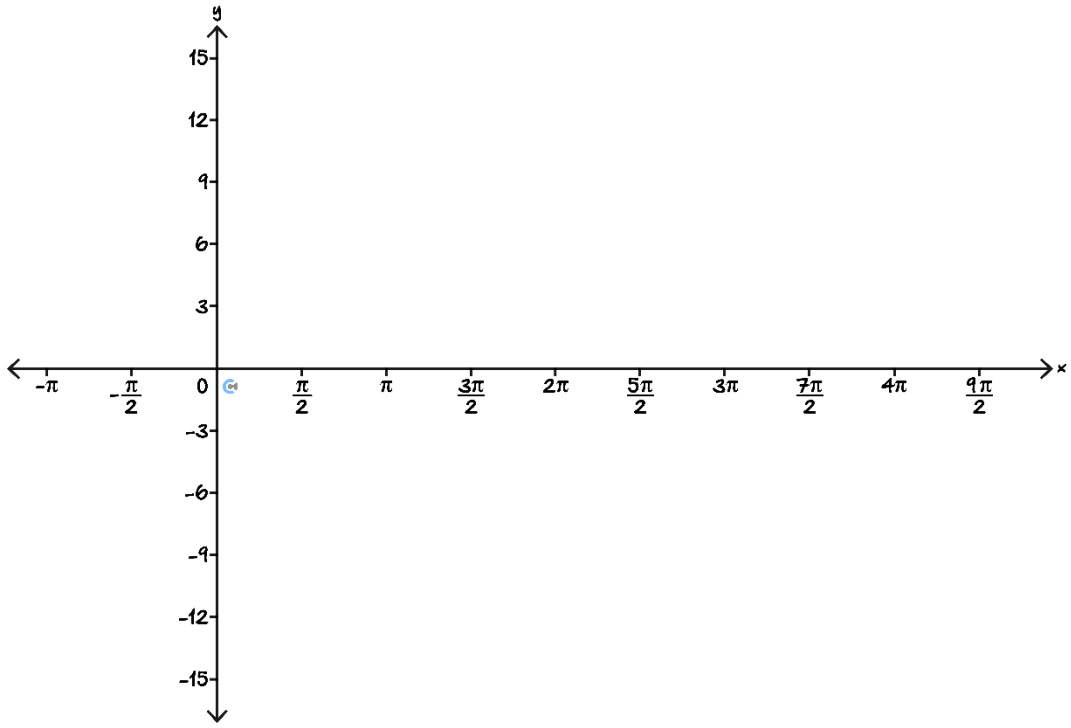
Question 6

Sketch the graphs of the following functions on the axes below. Label all axes intercepts, turning points and asymptotes.

a. $f(x) = 2\sec\left(\frac{x}{3} - \frac{\pi}{4}\right)$, for $x \in [-5\pi, 4\pi]$.



b. $f(x) = \cot\left(\frac{x}{2} - \frac{\pi}{4}\right) - \sqrt{3}$, for $x \in [0, 4\pi]$.



Sub-Section [3.4.3]: Apply Compound and Double Angle Formula to Solve Exact Values

Question 7

If $\sin(x) = \frac{4}{5}$ and $x \in \left[0, \frac{\pi}{2}\right]$, then find the value of $\cos(2x)$.

Question 8

Find the exact value of $\cos\left(\frac{7\pi}{12}\right)$.

Question 9

Find the exact value of $\sin\left(\frac{\pi}{8}\right)$.

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Sub-Section [3.4.4]: Find Domain, Range and Rule of the Inverse Trigonometric Function

Question 10

Suppose $f: \left[0, \frac{\pi}{2}\right] \to \mathbb{R}, f(x) = \cos(2x)$.

a. Find the domain and range of the inverse function, f^{-1}.

b. Hence, define f^{-1}.

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Question 11

Suppose $f: \left[0, \frac{\pi}{2}\right] \to \mathbb{R}, f(x) = 2\sin\left(2x - \frac{\pi}{6}\right)$.

a. Find the domain and range of the inverse function, f^{-1}.

b. Hence, define f^{-1}.

Question 12

Suppose $f: \left[\frac{3\pi}{4}, \pi\right] \to \mathbb{R}, f(x) = \tan\left(2x + \frac{\pi}{4}\right) + \sqrt{3}$.

a. Find the domain and range of the inverse function, f^{-1}.

b. Hence, define f^{-1}.

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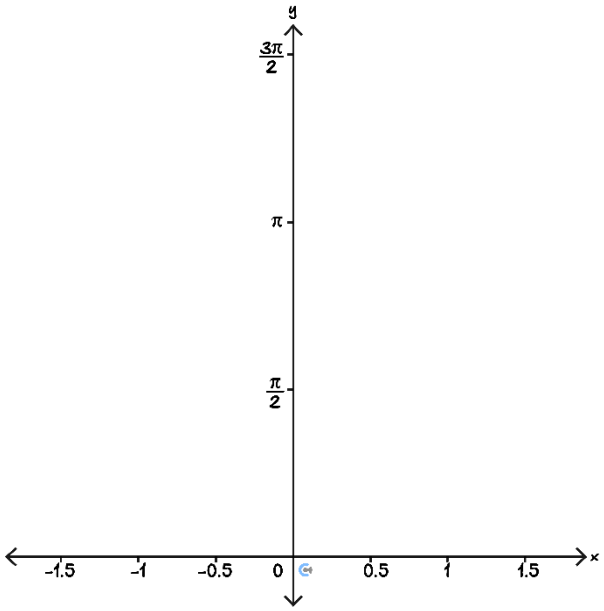
Sub-Section [3.4.5]: Graphing Inverse Trigonometric Functions

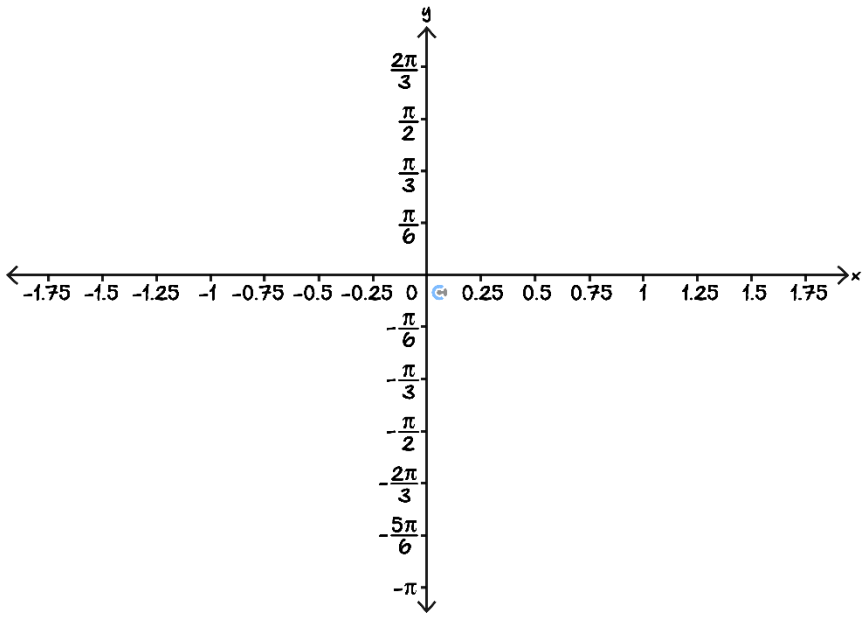
Question 13

Sketch the graphs of the following inverse trigonometric functions over their maximal domain on the axes below. Label all axes intercepts and endpoints with coordinates, and asymptotes with their equations.

a. $f(x) = \arccos(2x) + \frac{\pi}{2}$.



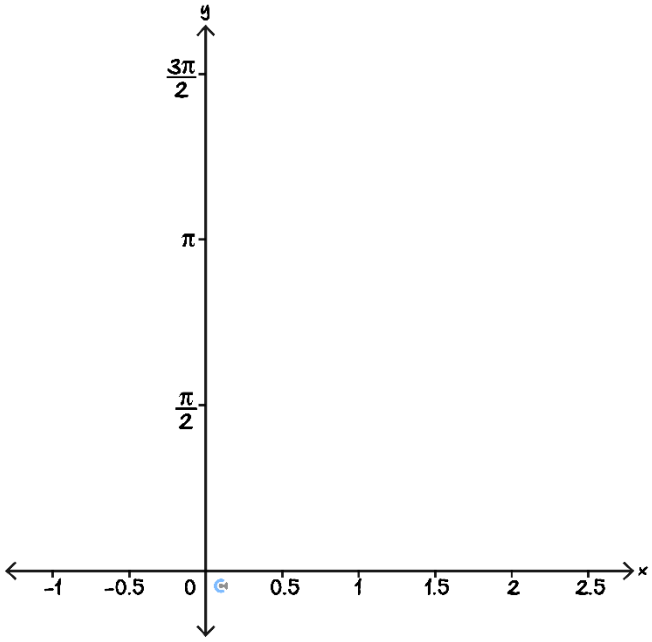
b. $f(x) = \arctan(2x) - \frac{\pi}{3}$.



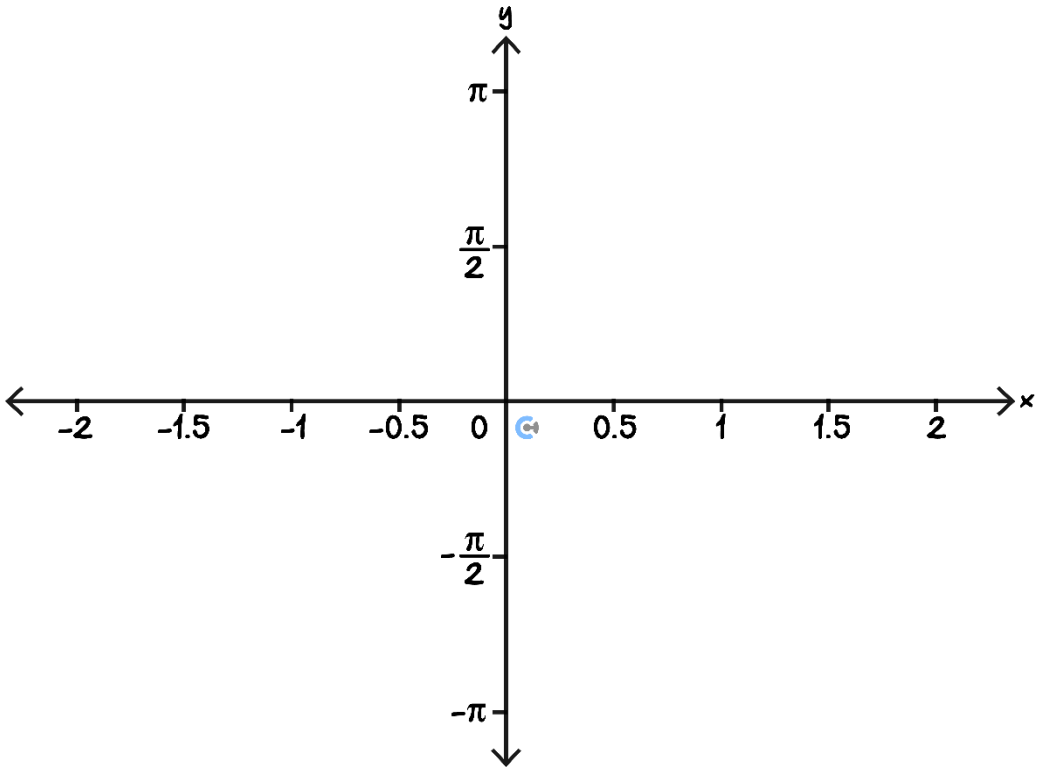
Question 14

Sketch the graphs of the following inverse trigonometric functions over their maximal domain on the axes below. Label all axes intercepts and endpoints with coordinates, and asymptotes with their equations.

a. $f(x) = -\arcsin(x-1) + \frac{\pi}{2}$.



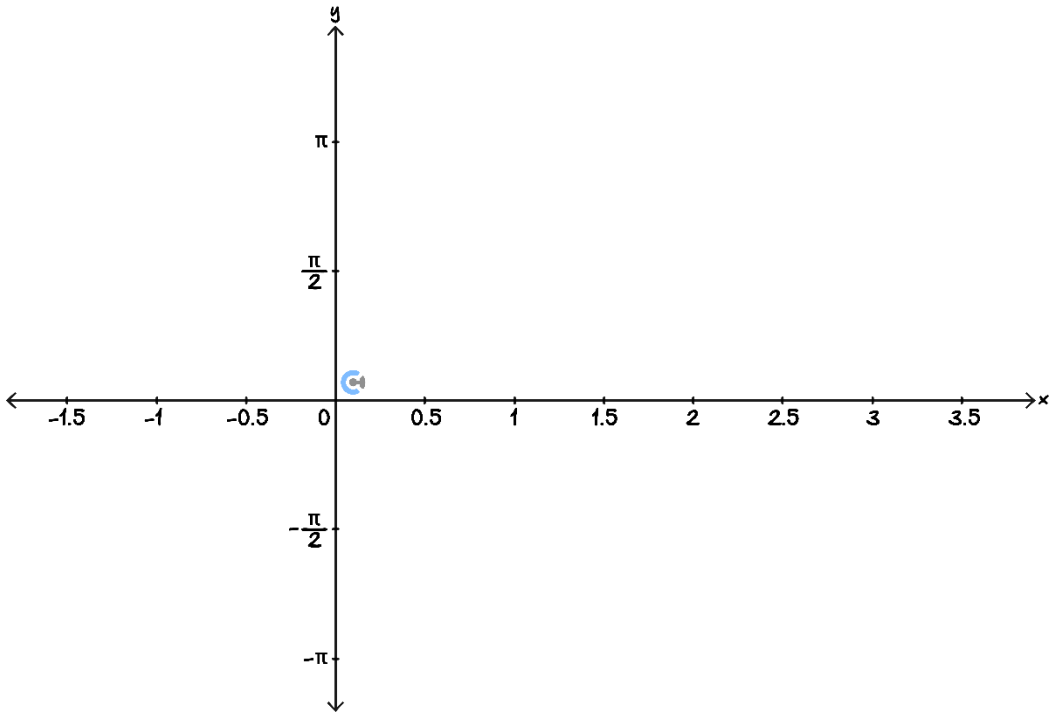
b. $f(x) = 2\arccos(2x+1) - \pi$.



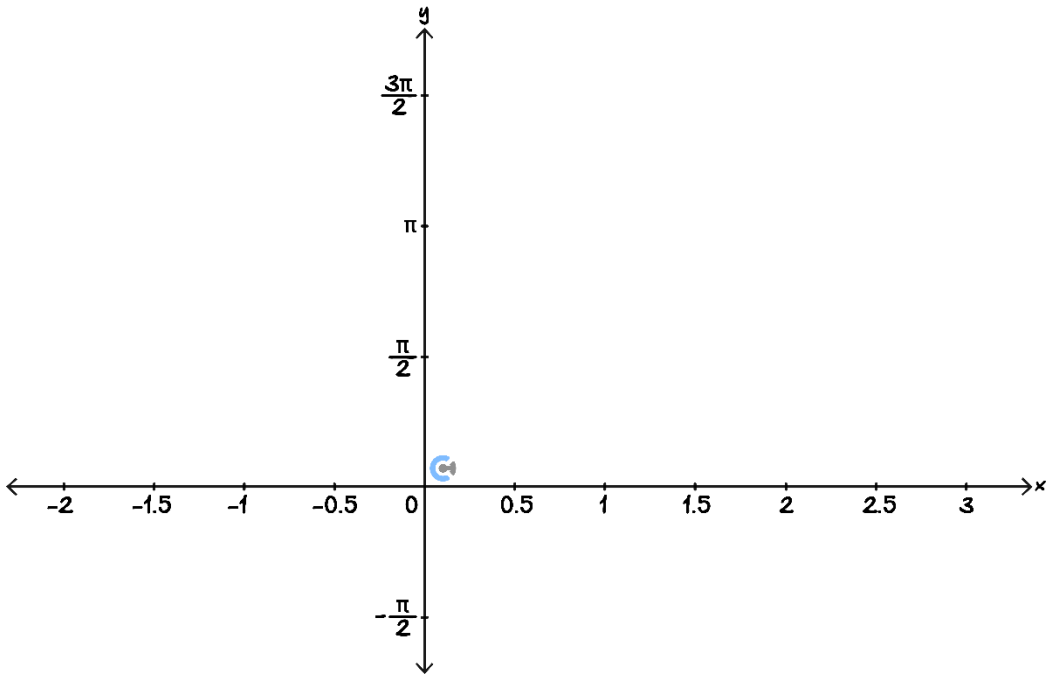
Question 15

Sketch the graphs of the following inverse trigonometric functions over their maximal domain on the axes below. Label all axes intercepts and endpoints with coordinates and asymptotes with their equations.

a. $f(x) = -\frac{1}{2}\arccos\left(\frac{x}{2} - \frac{1}{2}\right) + \frac{\pi}{4}$.



b. $f(x) = -2\arctan(3x - \sqrt{3}) + \frac{\pi}{2}$.



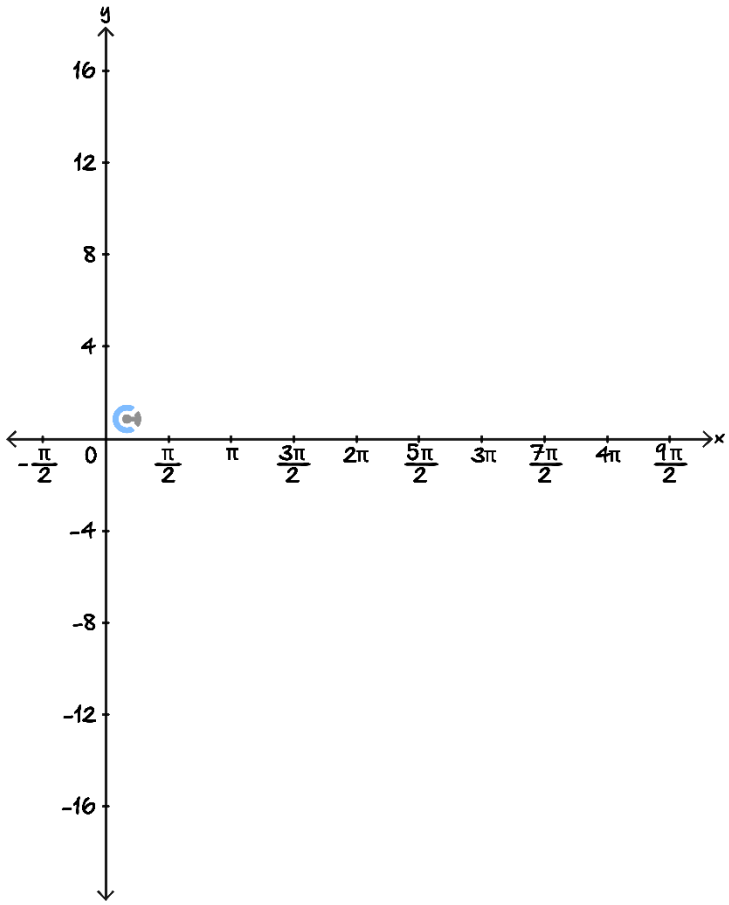
Sub-Section: Final Boss

Question 16

a. Use a double-angle formula to show that $\cos\left(\frac{3\pi}{8}\right) = \frac{\sqrt{2-\sqrt{2}}}{2}$.

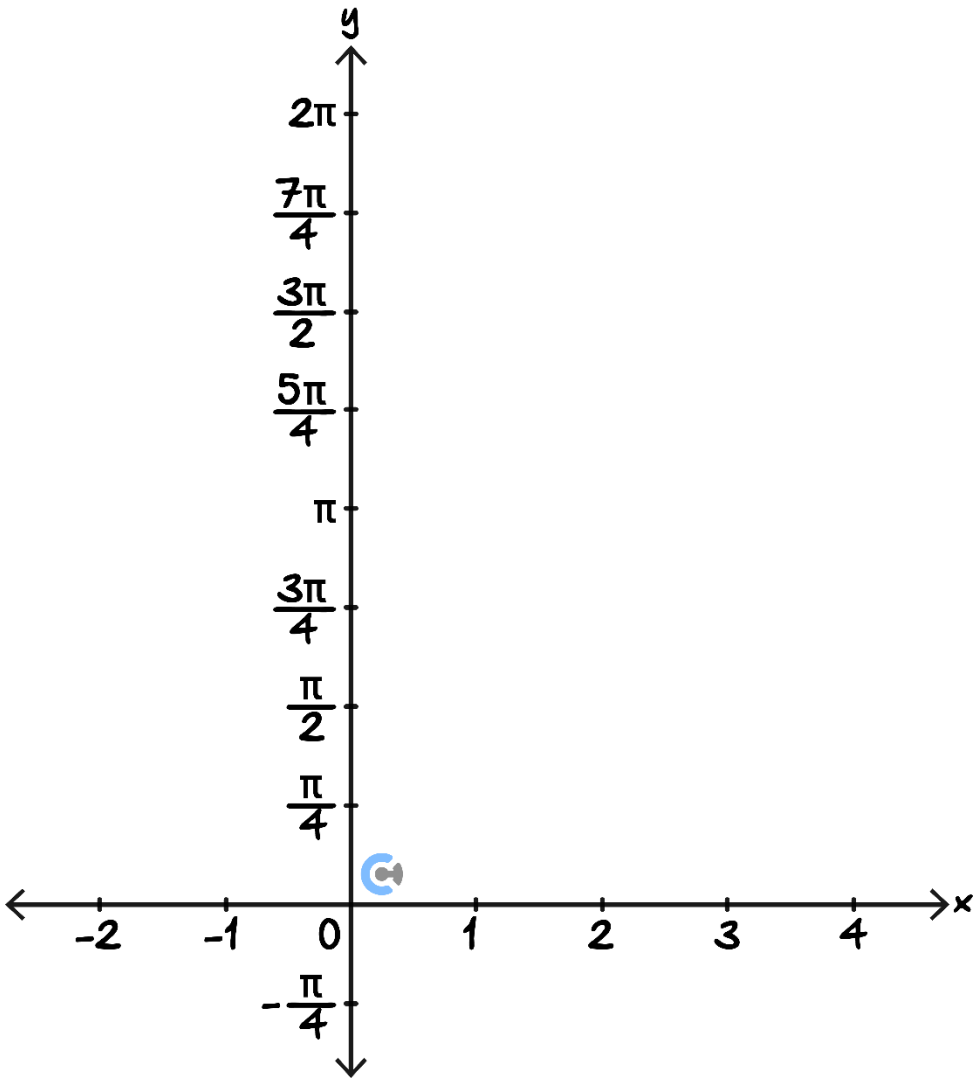
b. Hence, state the value of $\sec\left(\frac{3\pi}{8}\right)$.

c. Sketch the graph of $f(x) = 2\sec\left(\frac{x}{2} - \frac{3\pi}{8}\right)$, for $x \in \left[0, \frac{15\pi}{4}\right]$. Label all axes intercepts and turning points.



d. State the domain and range of the function $g(x) = 2\arccos(x - 2)$.

e. Sketch the graph of $y = 2\arccos(x - 2)$ on the axes below. Label all endpoints and points of inflection with coordinates.



f. Use the Pythagorean identity to evaluate $\sin\left(\arccos\left(\frac{1}{\sqrt{3}}\right)\right)$.

Section B: Supplementary Questions

Sub-Section [3.4.1]: Trigonometric Identities and Solving Exact Values of Reciprocal Functions

Question 17

Evaluate the following:

a. $\sec\left(\frac{\pi}{4}\right)$

b. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

c. $\tan^{-1}(1)$

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Question 18

Evaluate the following:

a. $\cot\left(\frac{11\pi}{6}\right)$

b. $\text{cosec}\left(\frac{7\pi}{3}\right)$

c. $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

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Question 19

Prove the identity $(\cot x + \operatorname{cosec} x)^2 = \frac{1+\cos x}{1-\cos x}$.

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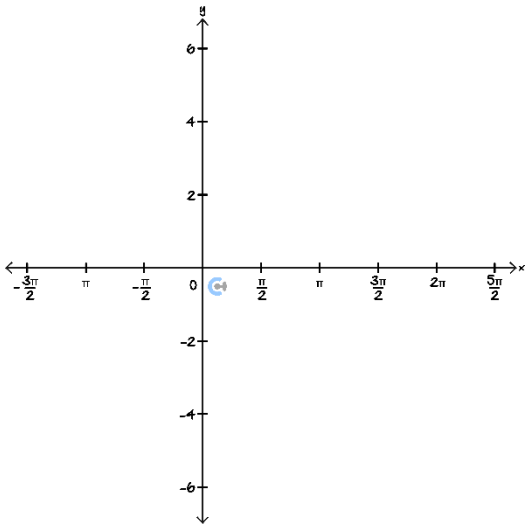
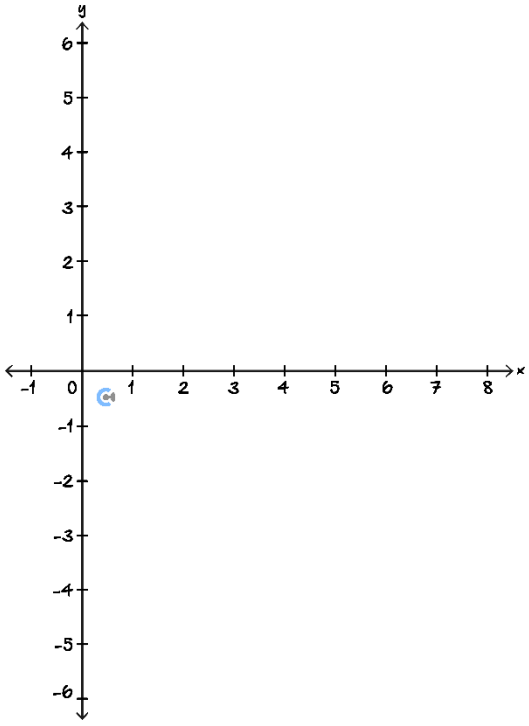
Sub-Section [3.4.2]: Graph Reciprocal Trigonometric Functions

Question 20

a. Sketch the graph of $y = 2\sec\left(x - \frac{\pi}{2}\right)$ for $-\pi < x < 2\pi$, labelling all stationary points, axes intercepts and asymptotes with their equations.

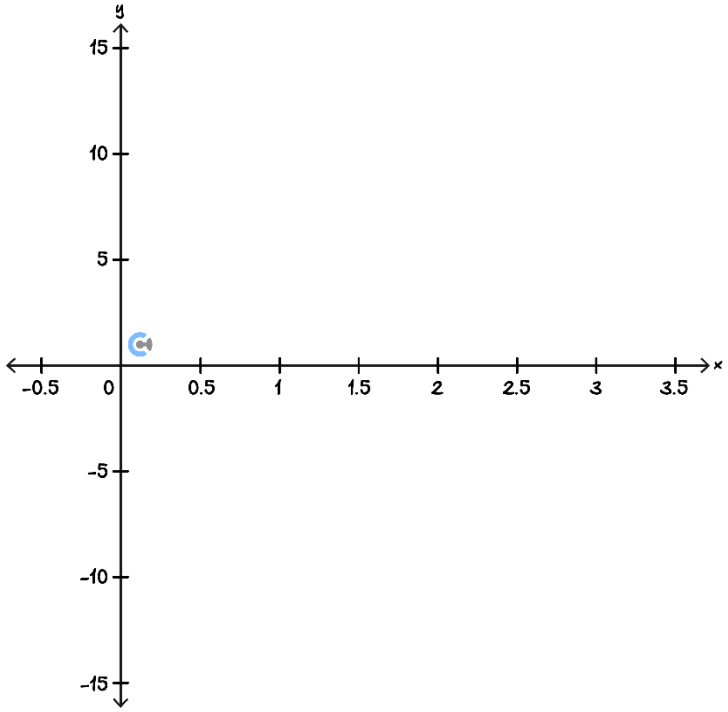


b. Sketch the graph of $\frac{\operatorname{cosec}(x)}{2} - \frac{1}{2}$ for $0 < x < 2\pi$, labelling all stationary points, axes intercepts and asymptotes with their equations.

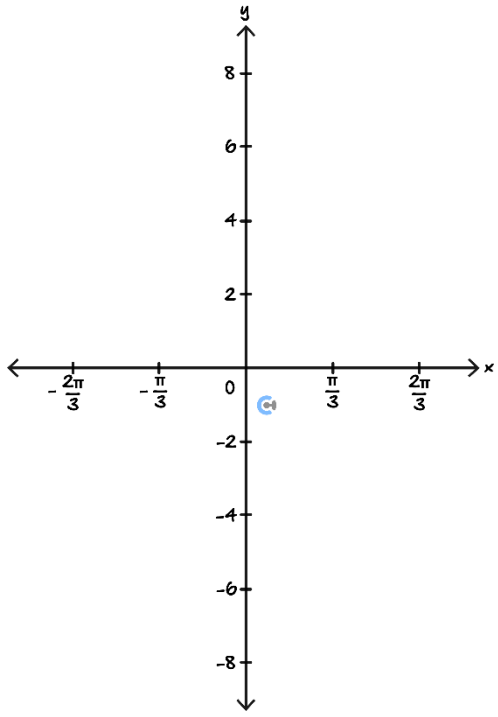


Question 21

a. Sketch the graph of $y = 4\text{cosec}\left(7\pi x - \frac{2\pi}{3}\right)$ for $-1 \leq x \leq 3$, labelling all stationary points, axes intercepts and asymptotes with their equations.

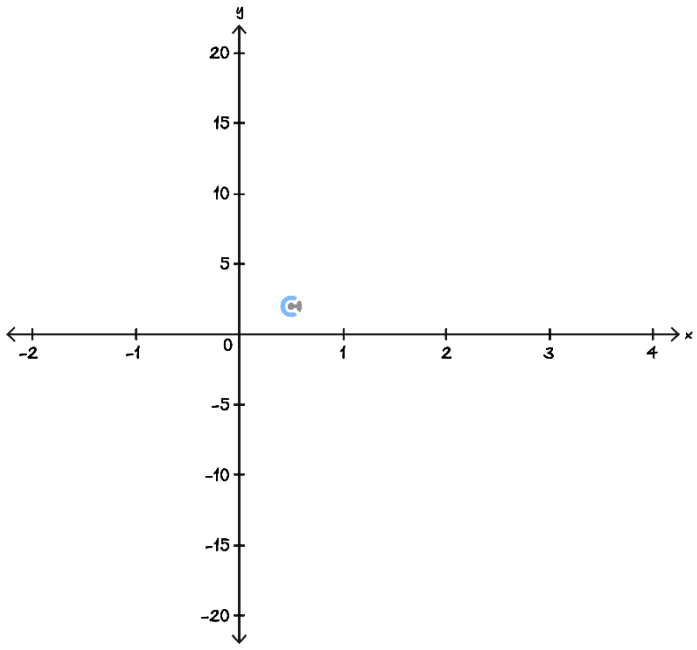


b. Sketch the graph of $y = -\cot(\pi - 3x)$ for $-\frac{2\pi}{3} < x < \frac{2\pi}{3}$, labelling all stationary points, axes intercepts and asymptotes with their equations.

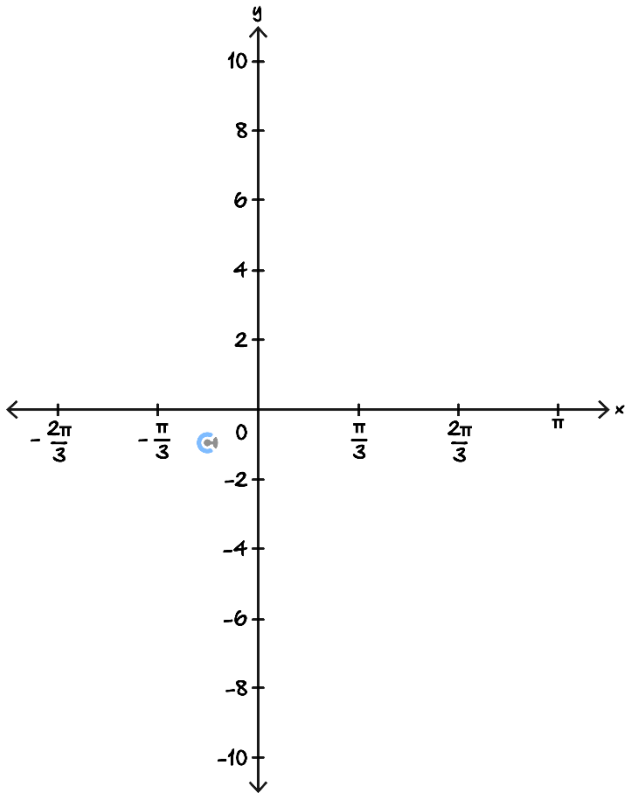


Question 22

a. Sketch the graph of $y = 1 - \sqrt{3}\cot\left(\pi x - \frac{\pi}{3}\right)$ for $-1 \leq x \leq 3$, labelling all stationary points, axes intercepts and asymptotes with their equations.



b. Sketch the graph of $y = \cot\left(2x - \frac{\pi}{4}\right) + \sqrt{3}$ for $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{4}$, labelling all stationary points, axes intercepts and asymptotes with their equations.



Sub-Section [3.4.3]: Apply Compound and Double Angle Formula to Solve Exact Values

Question 23

Use a compound angle formula to evaluate $\sin\left(\frac{5\pi}{12}\right)$.

Question 24

Use a double-angle formula to evaluate $\tan\left(-\frac{\pi}{8}\right)$.

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Question 25

Use a compound angle formula to evaluate $\cos\left(\frac{19\pi}{12}\right)$.

Question 26

Given that $\cos(x-y)=\frac{7}{25}$ and $\cot(x)\cot(y)=\frac{4}{3}$, find $\cos(x+y)$.

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Sub-Section [3.4.4]: Find Domain, Range and Rule of the Inverse Trigonometric Function

Question 27

Consider the function $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R: f(x) = \frac{\tan(x)}{3}$.

a. State the domain of $f^{-1}(x)$.

b. State the range of $f^{-1}(x)$.

c. Hence, or otherwise, find the rule of $f^{-1}(x)$.

Question 28

Consider the function $f: \left[-\frac{9\pi}{4}, \frac{3\pi}{4}\right] \to R: f(x) = 2\sin\left(\frac{x}{3} + \frac{\pi}{4}\right) - \sqrt{2}$.

a. State the domain of $f^{-1}(x)$.

b. State the range of $f^{-1}(x)$.

c. Hence, or otherwise, find the rule of $f^{-1}(x)$.

Question 29

Consider the function $f: \left[\frac{5\pi}{3}, \frac{8\pi}{3}\right] \to R: f(x) = \sqrt{5}\cos\left(x + \frac{\pi}{3}\right)$.

a. State the domain of $f^{-1}(x)$.

b. State the range of $f^{-1}(x)$.

c. Hence, or otherwise, find the rule of $f^{-1}(x)$.

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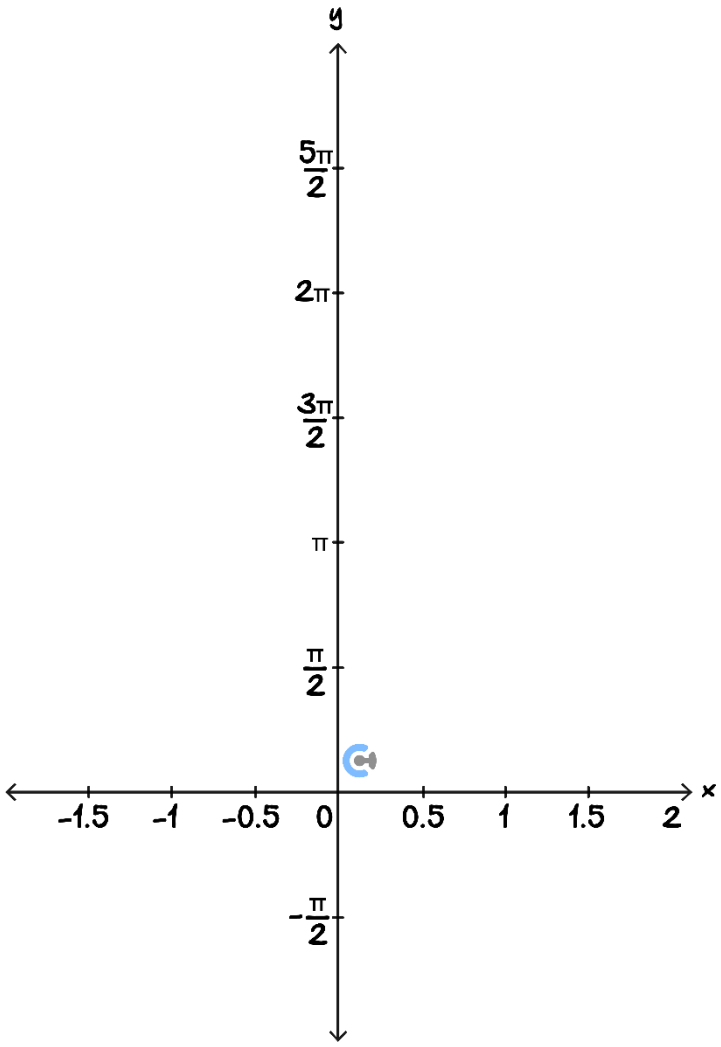
Sub-Section [3.4.5]: Graphing Inverse Trigonometric Functions

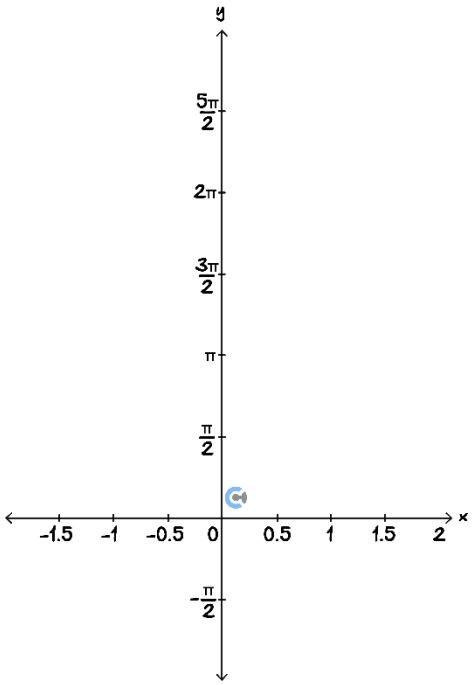
Question 30

a. Sketch the graph of $y = 2\sin^{-1}(x) + \pi$ on the axes below. Label all endpoints and axes intercepts.



b.

i. Sketch the graph of $y = 2\cos^{-1}(-x)$ below.



ii. What do you notice?

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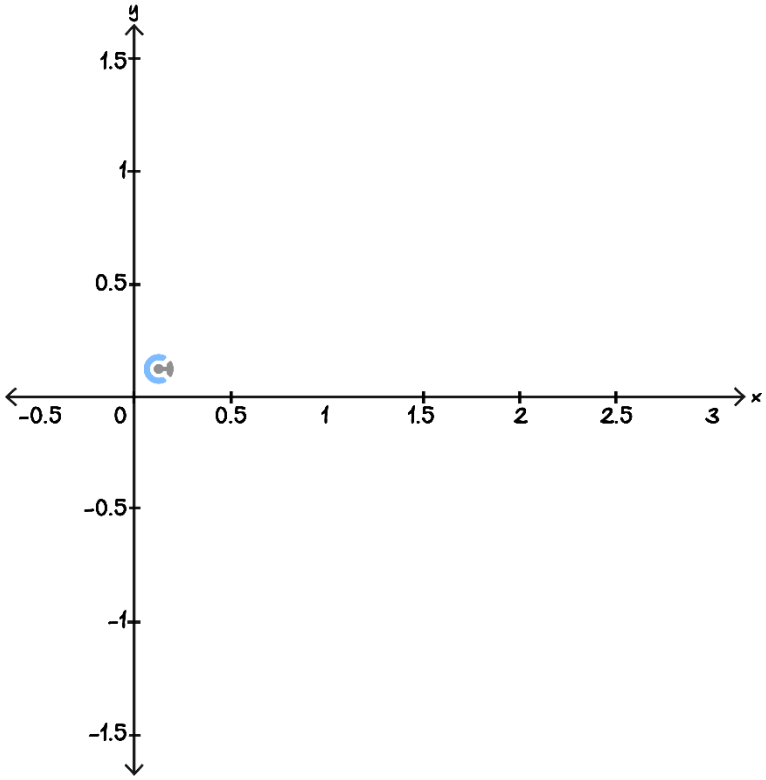
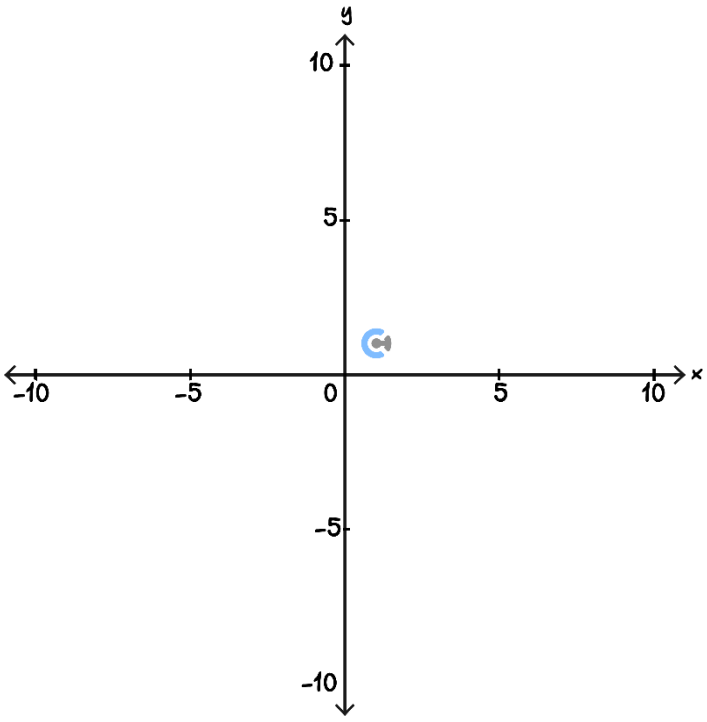
Question 31

a. Sketch the graph of $y = -\frac{2}{\pi}\cos^{-1}(4 - 2x) + 1$ on the axes below, labelling all endpoints and axes intercepts.

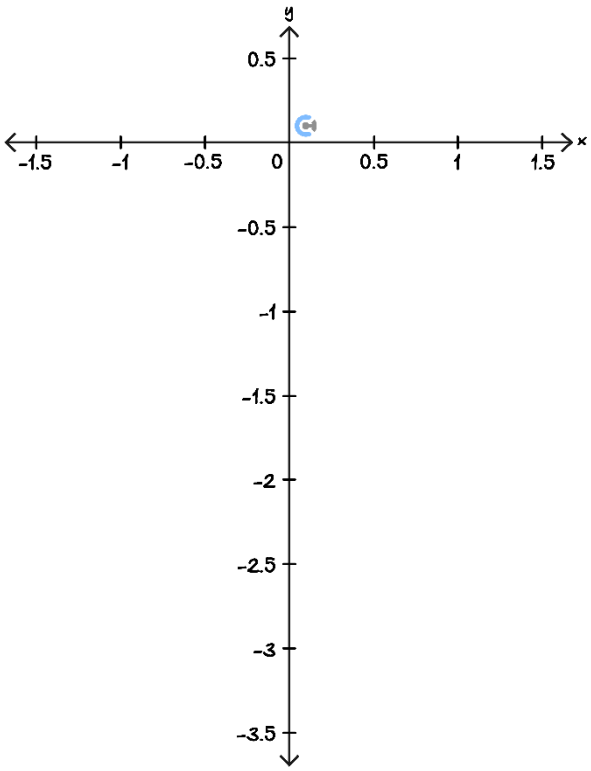


b. Sketch the graph of $y = -3\tan^{-1}(2x + 1)$ below, labelling all key points and asymptotes.

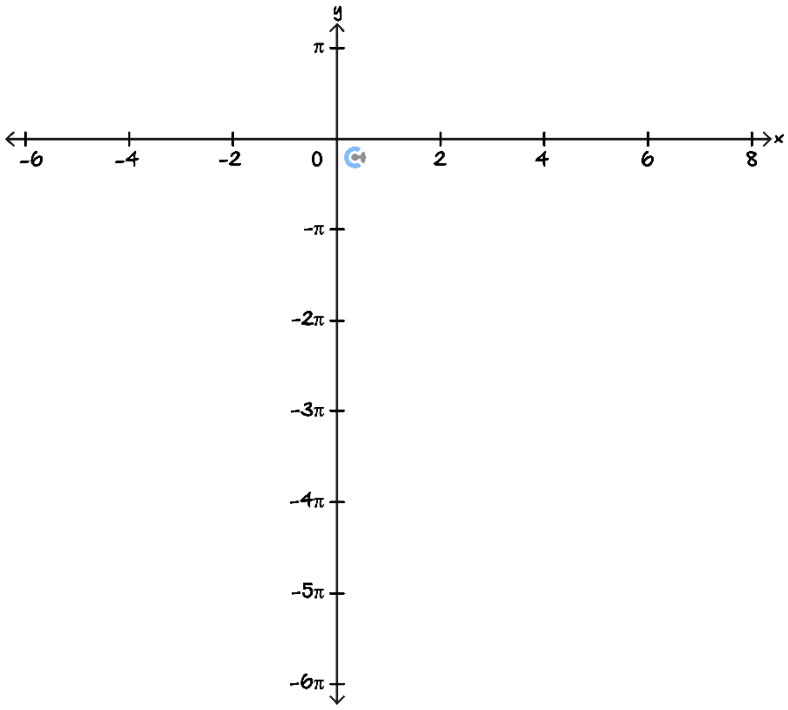


Question 32

a. Sketch the graph of $y = \sin^{-1}(2x) - \sqrt{3}$ on the axes below. Label all endpoints.



b. Sketch the graph of $y = \pi \tan^{-1}\left(\frac{x}{2} - 1\right) - \pi^2$ on the axes below. Label all axes intercepts and asymptotes with their equation.



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